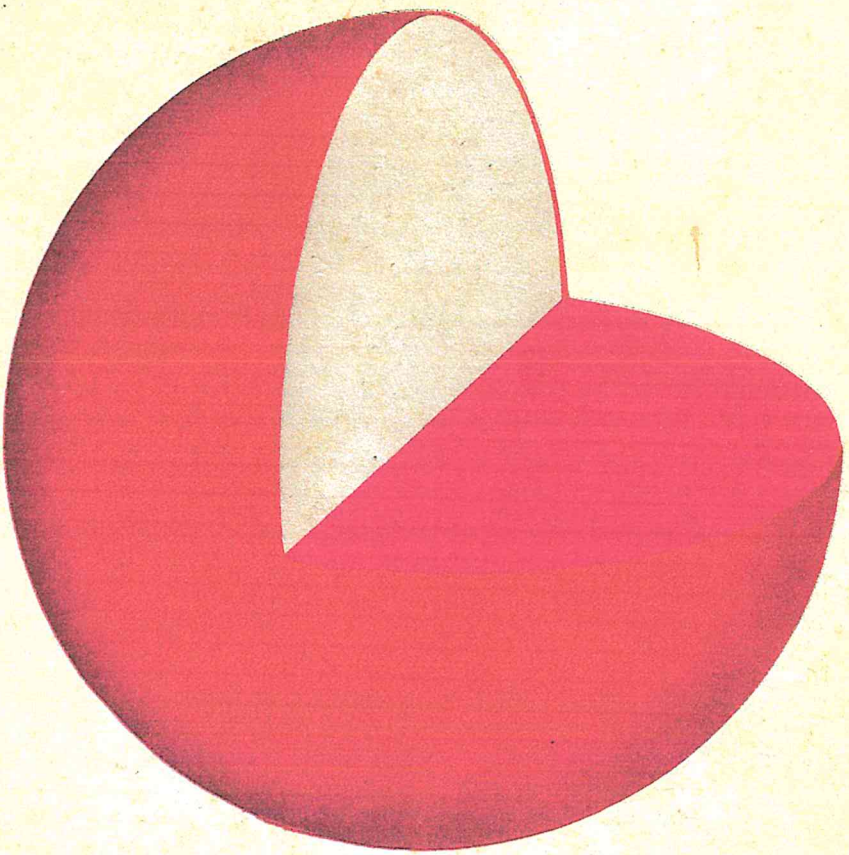
# Modern School Mathematics

## Structure and Use



CALIFORNIA STATE SERIES

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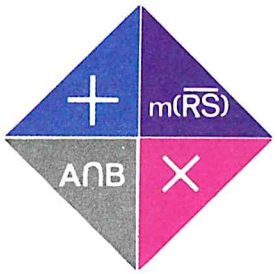


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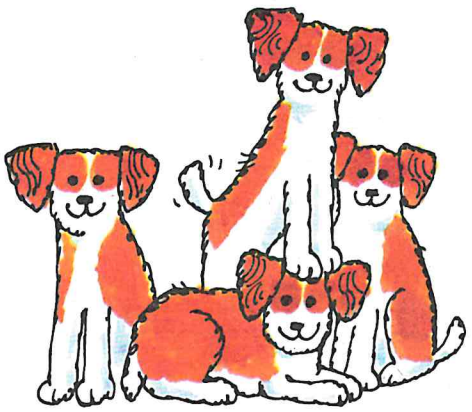
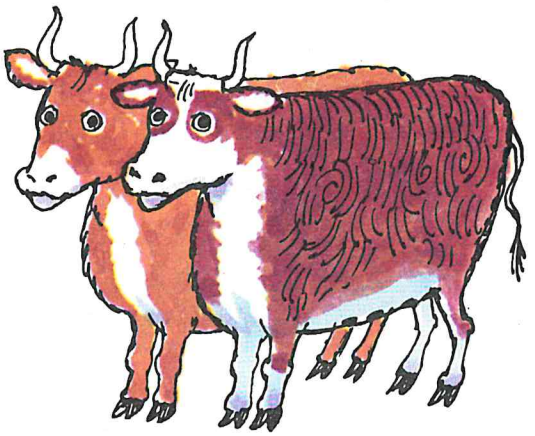
# CHAPTER 1 • *Sets, Numbers, Numerals*

## Sets



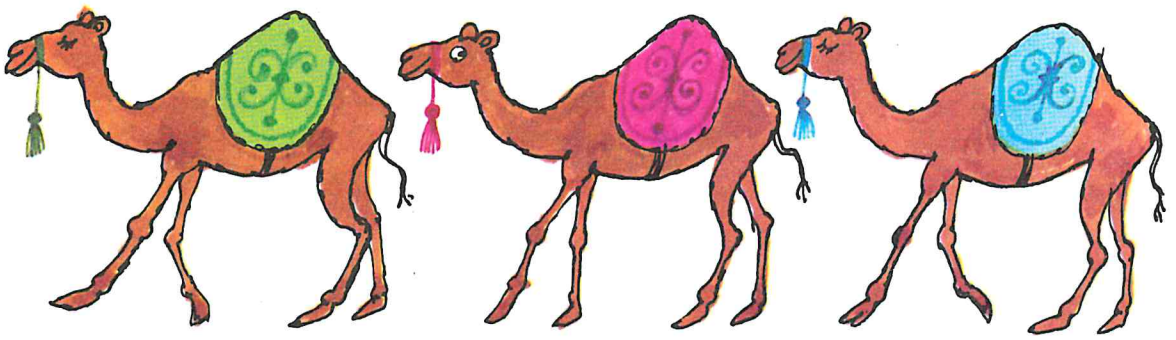
A **litter** of puppies



A **yoke** of oxen

A **flight** of geese

A **drove** of sheep



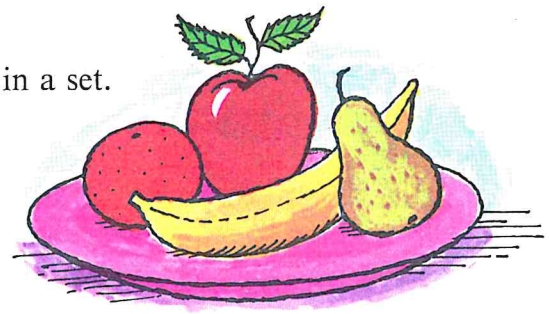
A **train** of camels

Each of these is a **set.** Can you think of other sets?

## Sets

1. We may **list** the names of the objects in a set.
   $A$ = {banana, orange, apple, pear}

2. We may **describe** the objects in a set.
   $A$ = {pieces of fruit on the plate}



The objects in a set are called its **members.**
We may use **braces,** { }, to show a set.

### EXERCISES

List the members of each set.

1. {the students in your row}
2. {your teacher}
3. {the subjects you will study in fourth grade}
4. {the last three months of the year}
5. {the capital of the U.S.A.}
6. {three main meals in a day}
7. {the first eight letters of the alphabet}
8. {the counting numbers between 10 and 20}

Describe the members of each set.

9. {penny, nickel, dime, quarter, half-dollar}
10. {v, w, x, y, z}
11. {summer, winter, spring, fall}
12. {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

*13. {March, May}

*14. {0, 2, 4, 6, 8}

Answer the questions.

15. Is 4 a member of {even numbers less than 10}?
16. Is $R$ a member of {letters in your first name}?
17. If $M$ = {0, 2, 4, 6, 8, 10} and $N$ = {0, 3, 6, 9, 12},
    **a.** is 6 a member of $M$?
    **b.** is 6 a member of $N$?

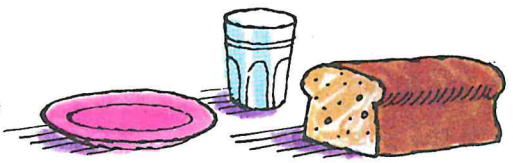
Two sets are **equal** if they have the same members.

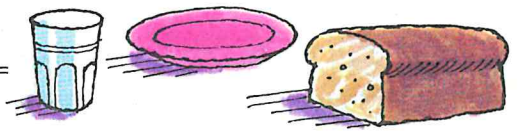
$A$ = {pencil, eraser, book}
$B$ = {book, pencil, eraser}
$A = B$

A set with no members is called the **empty set,** written { }.

{giraffes that fly} = { } since there are no giraffes that fly.

## EXERCISES

$S$ = 

$T$ = 

**1.** Is it true that $S = T$?

Write a set equal to

**2.** {Tom, Dick, Harry}
**3.** {horse, cow, goat}
**4.** {1, 2, 3, 4}
**5.** {210, 410, 510}

Pair the equal sets. For example, $A = E$.

**6.** $A$ = {dog, cat, car, bicycle}
**7.** $B$ = {a, b, c}
**8.** $C$ = {the whole numbers between 0 and 6}
$D$ = {5, 4, 3, 2, 1}
$E$ = {cat, car, dog, bicycle}
$F$ = {the first three letters in the alphabet}

Which of these sets are empty?

**9.** {men who have traveled to the moon}
**10.** {girls in your class who can jump 67 feet}
**11.** {men who are 50 feet tall}
**12.** {the whole numbers between 5 and 6}
**13.** {even numbers which are also odd numbers}
***14.** {0}

## Matching sets

The cardinal number of a set tells **how many** members are in the set.

The cardinal number of each of these sets is 3. $A$ and $B$ can be matched one-to-one.

$$A = \{a, b, c\} \quad n(A) = 3$$
$$\updownarrow \quad \updownarrow \quad \updownarrow$$
$$B = \{1, 2, 3\} \quad n(B) = 3$$

$A$ and $B$ are **equivalent sets.**

$$n(A) = n(B)$$

## EXERCISES

Name the cardinal number of each set. For example, $n(A) = 6$.

1. $A = \{\text{Tom, Dick, Harry, Kate, Jean, Martha}\}$
2. $B = \{a, e, i, o, u\}$
3. $C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
4. $D = \{\text{the fingers of one hand}\}$
5. $E = \{\text{the first six months of the year}\}$
6. $F = \{\text{the days of the week}\}$
7. $G = \{\text{the even numbers from 2 to 20}\}$
8. $H = \{\text{the seven continents}\}$

**9–12.** Name the pairs of equivalent sets in exercises 1 to 8. For example, write $n(A) = n(E)$.

Name the cardinal number of each set.

13. $W = \{b, o, y, a, n, d, g, i, r, l\}$
14. $X = \{0, 1, 2, a, b, c, 3, 4, 5\}$
15. $Y = \{\text{the states in the U.S.A.}\}$
16. $Z = \{\text{the highest mountain in the world}\}$

Answer the questions.

17. What is the cardinal number of the empty set?
18. If set $P$ = set $Q$,
    **a.** are $P$ and $Q$ equivalent sets?
    **b.** is it true that $n(P) = n(Q)$?

*19. If set $R$ is equivalent to set $S$,
    **a.** is it true that $n(R) = n(S)$?
    **b.** is it true that $R = S$?

$A = \{0, 2, 4, 6, 8\}$
$A = \{\text{the even numbers less than } 10\}$
We **can** list all the members of set $A$.

If all the members of a set can be listed and counted, the set is **finite.**

$B = \{0, 2, 4, 6, 8, 10, \ldots\}$
$B = \{\text{the even numbers}\}$

We **cannot** list all the members of set $B$. The three dots show that the list of members goes on and on without stopping.
If all the members of a set cannot be listed and counted, the set is **infinite.**

The numbers we use to count with are called the **counting numbers.** The first counting number is 1.
The set of counting numbers is $\{1, 2, 3, 4, 5, 6, \ldots\}$. This is an infinite set.

## EXERCISES

List the members of each set.

1. $A = \{\text{the days of the week}\}$
2. $B = \{\text{the first six months of the year}\}$
3. $C = \{\text{the members of your family}\}$
4. $D = \{\text{counting numbers between 17 and 32}\}$
5. $E = \{\text{the odd numbers from 1 to 11}\}$
6. $F = \{\text{the even numbers from 0 to 10}\}$
7. Which of the sets $A$ to $F$ are finite and which are infinite?
8. Name the cardinal numbers of the sets $A$ to $F$.

For each set, write *F* or *I* to show whether it is *finite* or *infinite*.

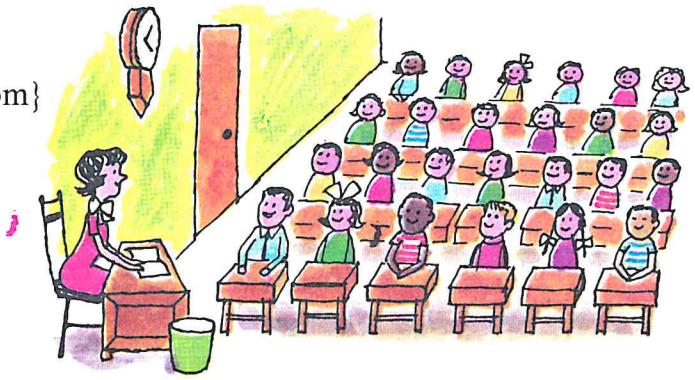
9. $G = \{300 \text{ horses and a dog}\}$
10. $H = \{\text{all the odd numbers}\}$
11. $I = \{\text{all the people alive in the world}\}$
12. $J = \{\text{the animals in the nearest zoo}\}$
13. $K = \{\text{the counting numbers}\}$
14. $L = \{\text{all books ever written}\}$
15. $M = \{\text{all geometric shapes that can be drawn}\}$
16. $N = \{76 \text{ trombones and the rest of the band}\}$

## Part of a set

$L$ = {the children in the classroom}
$M$ = {the girls in the classroom}

Each member of set $M$ is also a member of set $L$.

We say that $M$ is a **subset** of $L$.



## EXERCISES

Pair each set with its subset. For example, $J$ is a subset of $A$.

1. $A$ = {Hatty, Sue, Carol}
2. $B$ = {dogs, cats, horses}
3. $C$ = {winter sports}
4. $D$ = {boys in your classroom}
5. $E$ = {◇, ©, □, ○, ☆}

$F$ = {animals with four legs}
$G$ = {children in your classroom}
$H$ = {hockey, football, basketball}
$I$ = {©, □, ☆}
$J$ = {Carol, Sue}

Answer the questions.

6. Is {1, 2} a subset of {1, 2, 3, 4, 5}?
7. If $M$ = {the counting numbers},
   a. is {2, 4, 6} a subset of $M$?
   b. is {100} a subset of $M$?

If $N$ = {the odd numbers less than 10}, which of the following sets are subsets of $N$?

8. $P$ = {1, 3, 5, 7}
9. $Q$ = {5, 7, 3, 6, 1}
10. $R$ = {3}
11. $S$ = {1, 3, 5, 7, 9}
12. $T$ = {1, 5, 8, 9}
13. $U$ = {11}

John brings his lunch to school. In his lunch box he has a sandwich, an apple, and a cookie. He can make several choices of what he will eat.

14. Can John eat everything in his lunch box?
15. Can he eat nothing from his lunch box?
16. What other choices does John have from his lunch box?

{Joe, Bob} is a subset of many sets.

{children in the classroom}

{children in the world}

Each of these sets is a **universal set** for {Joe, Bob}. We name the universal set $U$.

Joe and Bob are tall if we think of {children in the classroom} as $U$.
Joe and Bob may not be tall if we think of {children in the world} as $U$.

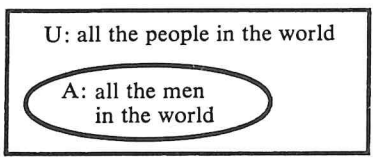
## EXERCISES

Name a universal set for

1. {cat, giraffe, cow}
2. {football shoes}
3. {Jim, Joan}
4. {4}
5. {1, 3, 5, 7}
6. {2, 4, 6, 8, 10, . . .}

If $U$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, list the members in

7. {the odd numbers}
8. {numbers less than 4}
9. {the even numbers}
10. {numbers greater than 20}

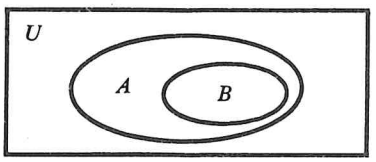



$U$ = {all people in the world}
$A$ = {all men in the world}
This diagram shows that $A$ is a subset of $U$.

Draw diagrams like the one above to show

11. $U$ = {all airplanes} and $A$ = {all jet airplanes}
12. $U$ = {all even numbers} and $B$ = {2, 4, 6, 8}
13. $U$ = {all letters in the alphabet} and $C$ = {a, b, c}

Study the diagram.




14. Is $B$ a subset of $A$?
15. Is $B$ a subset of $U$?
16. Is $A$ a subset of $U$?

## Joining sets

These children are making a bulletin board for the hall.
$A = \{\text{Tom, Bill, Sally, Betty}\}$

These children are making a bulletin board for the classroom.
$B = \{\text{Bill, Joan, Harvey, Sally}\}$

These children are making bulletin boards.
$C = \{\text{Tom, Bill, Sally, Betty, Joan, Harvey}\}$

> Set $C$ was formed of members of $A$ or $B$ or both. $C$ is called the **union** of $A$ and $B$. The **union of $A$ and $B$** is written $A \cup B$.
> $C = A \cup B$

### EXERCISES

Dan, Helen, Bob, and Louise belong to the Jay Hawks Club.
Tony, Nancy, Rita, and Bob belong to the Beavers Club.

1. List the members of the Jay Hawks Club as a set.
2. List the members of the Beavers Club as a set.
3. These clubs joined to form a new club. List the members of the new club as a set.
4. How many members are in the new set?
5. The new club is the ___?___ of the first two sets.

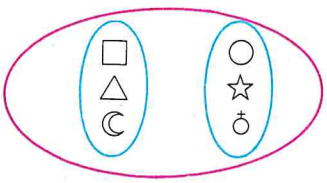
For each pair of sets, list the members of the union.

6. $J = \{\text{Jane, Jerry, Jack}\}$ and $K = \{\text{Karl, Ken, Kathy}\}$
7. $L = \{\text{car, bus, bicycle}\}$ and $M = \{\text{boat, car, train, bus, scooter}\}$
8. $T = \{\text{shoes, socks, shirt}\}$ and $V = \{\text{tie, shirt, coat, hat, scarf, sweater}\}$
9. $C = \{1, 2, 3, 4, 5\}$ and $D = \{2, 3, 4, 5, 6, 7\}$
10. $E = \{0, 5, 10, 15, 20, 25\}$ and $F = \{0, 10, 20, 30, 40, 50, 60\}$
11. $R = \{\text{the first four even numbers}\}$ and $S = \{\text{the first five odd numbers}\}$

Study the sets. List the members of each union.

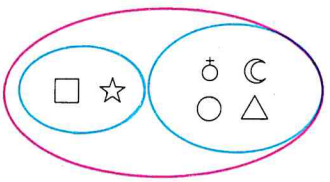
$A = \{0, 1, 2, 3, 4\}$ $C = \{\text{s, t, u}\}$ $E = \{\text{b, u, n, s}\}$
$B = \{\text{r, u, s, t}\}$ $D = \{1, 3, 5, 7, 9\}$ $F = \{\text{a, b, c, d, e}\}$

12. $A \cup D$ 13. $B \cup E$ 14. $C \cup F$ 15. $A \cup F$ 16. $B \cup C$ 17. $C \cup E$



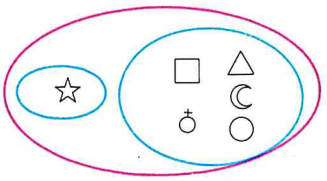
$n(A) = 3 \quad n(B) = 3$

$n(A \cup B) = 6$



$n(C) = 2 \quad n(D) = 4$

$n(C \cup D) = 6$



$n(E) = 1 \quad n(F) = 5$

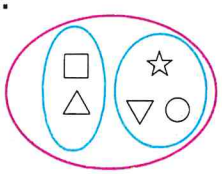
$n(E \cup F) = 6$

Names for a number are called **numerals.** The sets show that $3 + 3$, $2 + 4$, $1 + 5$, and 6 all name the same number.

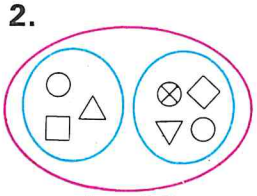
## EXERCISES

Write two names for the number shown in each set.

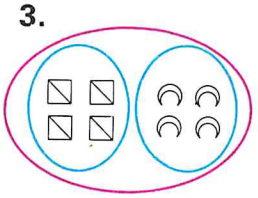
**1.**



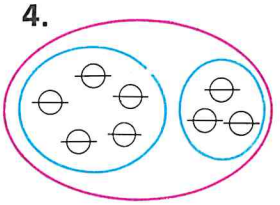
**2.**



**3.**



**4.**



Which are other names for 4?

**5.** $1 + 1 + 1 + 1$

**6.** 1111

**7.** 40

**8.** $25 - 21$

**9.** $2005 - 2001$

**10.** $4 + 4 - 4$

**11.** $2 + 3 - 1$

**12.** $(2 \times 3) - 2$

Write the simplest numeral for

**13.** $16 - 11$

**14.** $8001 - 8000$

**15.** $31 - 24$

**16.** $1 + 1 + 2 + 1 + 1 + 2 + 1 + 1 + 2 + 1 + 1 + 2$

**17.** $63 \div 7$

**18.** $3 \times 20$

Match the two names for the same number. For example, write (**19, a**)

**19.** $3\overline{)12}$

**20.** 64

**21.** $4 + 3$

**22.** $2 \times 9$

**a.** $2 + 2$

**b.** $12 + 6$

**c.** $8 \times 8$

**d.** $63 \div 9$

Complete the names for the cardinal number of the empty set.

**23.** $a$

**24.** $2 - b$

**25.** $3 \times c$

**26.** $d \div 4$

## Number sentences

$$6 + 1 = 7$$

This is an **equation.**

$6 + 1 = 7$ means,
"6 + 1 and 7 name the same number."

$$3 < 4$$
$$6 + 2 > 4 + 2$$

These are **inequalities.**

$3 < 4$ means, "3 *is less than* 4."
$6 + 2 > 4 + 2$ means,
"6 + 2 *is greater than* 4 + 2."

### EXERCISES

Name a number to make each sentence true.

**1.** $7 + 2 = a$
**2.** $4 + 3 > b$
**3.** $8 \div 4 = c$
**4.** $8 + 5 = 5 + d$
**5.** $8 + e < 13$
**6.** $9 + f = 15$
**7.** $3 \times 4 < 10 + g$
**8.** $9 > 5 + h$
**9.** $i + 3 < 7$

Complete each sentence. Write $<$, $>$, or $=$ to replace the ◯.

**10.** $7 \bigcirc 9$
**11.** $3 \times 2 \bigcirc 2 \times 3$
**12.** $12 - 3 \bigcirc 6 + 2$
**13.** $6 + 2 \bigcirc 3 + 2$
**14.** $4 + 4 + 4 \bigcirc 3 \times 4$
**15.** $10 \div 2 \bigcirc 2 \times 2$
**16.** $9 - 4 \bigcirc 8 - 3$
**17.** $(3 + 4) + 5 \bigcirc 3 + (4 + 5)$

$4 + 1 \neq 6$ means "4 + 1 and 6 do **not** name the same number,"
or "4 + 1 **is not equal to** 6."
If $4 + 1 \neq 6$, then either $4 + 1 < 6$ or $4 + 1 > 6$.
We know that $4 + 1 < 6$.

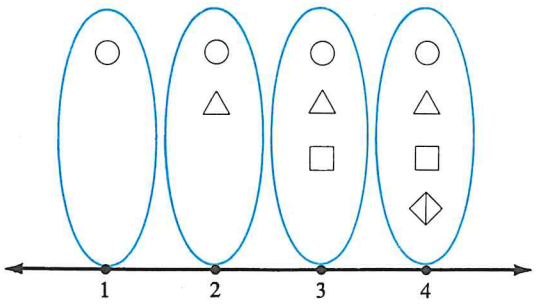
Name the matching pairs of sentences. For example, write (18, **b**).

**18.** $4 + 3 \neq 6$
**19.** $5 + 2 \neq 8$
**20.** $6 + 4 \neq 9$
**21.** $8 + 2 > 7$
**22.** $4 + 5 \neq 8$
**23.** $3 + 2 > 4$

**a.** $9 < 6 + 4$
**b.** $4 + 3 > 6$
**c.** $10 \neq 7$
**d.** $5 + 2 < 8$
**e.** $3 + 2 \neq 4$
**f.** $4 + 5 > 8$

## Order and the number line

The number line shows the **order of the counting numbers.**
We begin at 1 and count to the right by ones. The arrows show that the line and the counting numbers go on and on.
3 is to the right of 2 since 3 is *greater than* 2.




### EXERCISES

Complete each sentence. Write $<$ or $>$ to replace the ◯.

**1.** 2 is to the left of 3 on the number line since $2 \bigcirc 3$.

**2.** 5 is to the right of 4 on the number line since $5 \bigcirc 4$.

**3.** 1 is to the left of 2 on the number line since $1 \bigcirc 2$.

**4.** 8 is to the right of 6 on the number line since $8 \bigcirc 6$.

**5.** 15 is to the right of 7 on the number line since $15 \bigcirc 7$.

Complete the sentences.

**6.** If $x$ is to the left of $y$ on the number line, then $x$ is __?__ than $y$.

**7.** If $x$ is to the right of $y$ on the number line, then $x$ is __?__ than $y$.

**8.** If $x$ is less than $y$, then $x$ is to the __?__ of $y$ on the number line.

**9.** If $x$ is greater than $y$, then $x$ is to the __?__ of $y$ on the number line.

$W = \{0, 1, 2, 3, 4, 5, \ldots\}$ is the set of **whole numbers.**

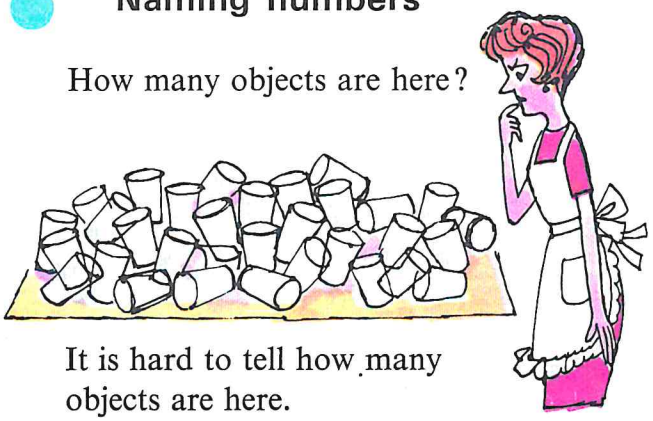
**10.** Is $W$ an infinite set?

**11.** How is $W$ different from the set of counting numbers?

**12.** Is the set of counting numbers a subset of the set of whole numbers?

## Naming numbers

How many objects are here?



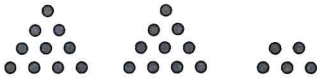
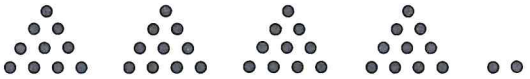
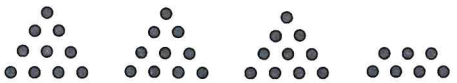
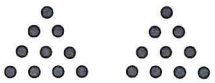
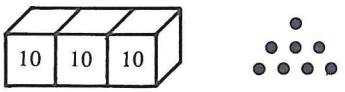
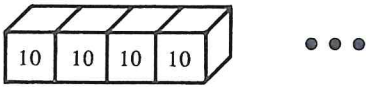
It is hard to tell how many objects are here.

It is easier to count these objects. Why?

We can see there are three sets of ten and four more in the second picture. *Three tens and four more* may be shown by the numeral 30 + 4, or 34. 34 means *three tens four*, or *thirty-four*.

### EXERCISES

Copy and complete the pattern.

| | | | | |
|---|---|---|---|---|
| 1. |  | t \| o<br>2 \| 5 | 20 + 5 | 25 |
| 2. |  | t \| o<br>*a* \| *b* | *c* | *d* |
| 3. |  | t \| o<br>*e* \| *f* | *g* | *h* |
| 4. |  | t \| o<br>*i* \| *j* | *k* | *l* |
| 5. |   | t \| o<br>*m* \| *n* | *z* | *p* |
| 6. |   | t \| o<br>*q* \| *r* | *s* | *t* |

There are ten **digits:** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. We use the digits to name numbers.
Every digit has three values:

1. Face Value — The *face value* of the middle digit in 465 is *6*.
2. Place Value — The *place value* of the 6 in 465 is *tens*.
3. Total Value — The *total value* of the 6 in 465 is *6 tens*, or *60*.

The **compact numeral** for this number is 465. The **expanded numeral** is 400 + 60 + 5. The expanded numeral names the number as the sum of the total values of its digits.

## EXERCISES

Name the total value of the 2 in

**1.** 62 **2.** 29 **3.** 209 **4.** 824

Write an expanded numeral for

**5.** 43 **7.** 18 **9.** 72 **11.** 99
**6.** 403 **8.** 180 **10.** 1072 **12.** 999

Write a two or three digit compact numeral for

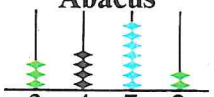
**13.** 30 + 6 **15.** 70 + 1 **17.** 700 + 10 + 1
**14.** 200 + 8 **16.** 700 + 10 **18.** 700 + 1
**19.** the greatest number named by a two digit numeral
**20.** the greatest number named by a three digit numeral
**21.** the least number named by a three digit numeral

Complete each sentence. Write $<$, $>$, or $=$ to replace the ◯.

**22.** 35 ◯ 53 **24.** 123 ◯ 213 **26.** 400 + 30 + 9 ◯ 439
**23.** 87 ◯ 78 **25.** 707 ◯ 770 **27.** 600 + 90 + 8 ◯ 600 + 80 + 9

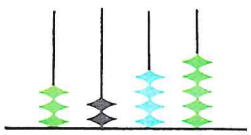
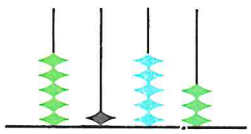
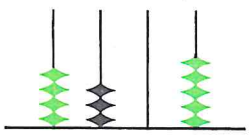
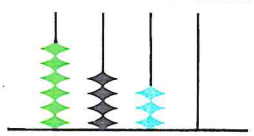
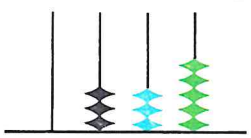
## Sets of tens, hundreds, and thousands

Here are four ways to show a number.

| Abacus | Columns | Expanded numeral | Compact numeral |
|---|---|---|---|
|  3 4 7 2 | th \| h \| t \| o<br>3 \| 4 \| 7 \| 2 | $3000 + 400 + 70 + 2$ | 3472 |

### EXERCISES

Copy and complete the pattern.

| | Abacus | Columns | Expanded numeral | Compact numeral |
|---|---|---|---|---|
| 1. |  | th \| h \| t \| o<br>3 \| 2 \| 4 \| 5 | $3000 + 200 + 40 + 5$ | 3245 |
| 2. |  | th \| h \| t \| o<br>*a* \| *b* \| *c* \| *d* | *e* | *f* |
| 3. |  | th \| h \| t \| o<br>*g* \| *h* \| *i* \| *j* | *k* | *l* |
| 4. |  | th \| h \| t \| o<br>*m* \| *n* \| *z* \| *p* | *q* | *r* |
| 5. |  | th \| h \| t \| o<br>*s* \| *t* \| *u* \| *v* | *w* | *x* |

Write a compact four digit numeral for

6. the greatest number named using the digits 3, 7, 2, and 6 one time each
7. the least number named using the digits 3, 7, 2, and 6 one time each
8. the greatest number named by four digits
9. the least number named by four digits

## Order on number lines

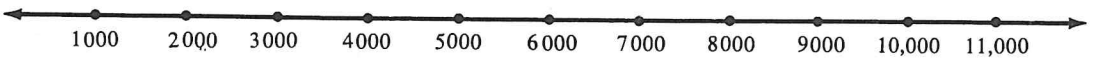
These number lines show the order of ones, of tens, of hundreds, and of thousands.

*A*

*B*

*C*

*D*




### EXERCISES

Use number line *A* to name the number

**1.** 2 greater than 4
**2.** 3 greater than 6
**3.** 4 less than 11
**4.** 2 less than 8

Use number line *B* to name the ten

**5.** 30 greater than 50
**6.** 50 greater than 60
**7.** 20 less than 110
**8.** 50 less than 60

Use number line *C* to name the hundred

**9.** 400 greater than 100
**10.** 700 greater than 200
**11.** 300 less than 1000
**12.** 200 less than 1100
**13.** between 100 and 300
**14.** between 300 and 500
**15.** between 700 and 900
**16.** between 900 and 1100

Use number line *D* to name the thousand

**17.** 1000 greater than 7000
**18.** 2000 greater than 3000
**19.** 2000 less than 3000
**20.** 7000 less than 11,000
**21.** between 2000 and 4000
**22.** between 5000 and 7000
**23.** between 6000 and 8000
**24.** between 9000 and 11,000

## Thinking of numbers in different ways

We may think of the number 4365 in many ways. These equations show some of the ways.

$$\begin{aligned} 4365 &= 4365 \text{ ones} \\ &= 436 \text{ tens} + 5 \text{ ones} \\ &= 43 \text{ hundreds} + 6 \text{ tens} + 5 \text{ ones} \\ &= 4 \text{ thousands} + 365 \text{ ones} \end{aligned}$$

### EXERCISES

Solve the equations.

1. $482 = 400 + a$ ones
2. $297 = c$ ones
3. $3486 = 348$ tens $+ e$ ones
4. $497 = 400 + 90 + g$
5. $340 = 300 + k$
6. $482 = 4$ hundreds $+ b$ ones
7. $4786 = d$ hundreds $+ 86$
8. $3486 = f$ thousands $+ 486$
9. $385 = h + 80 + 5$
10. $1427 = 1$ thousand $+ l$ ones

Write a compact numeral for

11. $400 + 85$
12. $3000 + 785$
13. $36 + 200$
14. $6000 + 90 + 7$
15. $4000 + 60 + 80 + 7$
16. $(45 \times 10) + 7$
17. $(87 + 10) + 4$
18. $400 + (8 \times 10)$
19. $(53 \times 100) + 46$

Copy and complete the tables.

20.

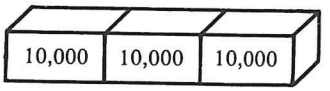
| th | h | t | o | compact numeral |
|---|---|---|---|---|
| 4 | 3 | 5 | 7 | *a* |
| 2 | 0 | 7 | 5 | *b* |
| 0 | 26 | 3 | 8 | *c* |
| 0 | 43 | 0 | 2 | *d* |
| 0 | 31 | 0 | 12 | *e* |

21.

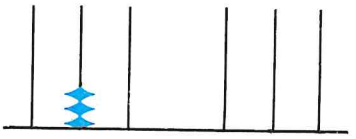
| th | h | t | o | compact numeral |
|---|---|---|---|---|
| *f* | *g* | *h* | *i* | 7211 |
| 0 | 0 | 276 | 3 | *j* |
| 6 | 0 | 0 | 13 | *k* |
| 6 | 0 | 15 | 7 | *l* |
| 29 | 3 | 9 | 7 | *m* |

22. Which names the greatest number?
   a. 6 thousands + 437
   b. 637 tens + 9 ones
   c. 62 hundreds + 8 tens + 7 ones
   d. 6327

## Tens of thousands



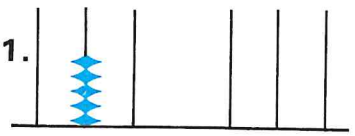

Each box contains *ten thousand* marbles. There are $3 \times 10{,}000 = 30{,}000$ marbles in all.
Remember to use a comma to mark off the thousands.
*Thirty thousand* may be shown on the abacus, using the fifth rod.



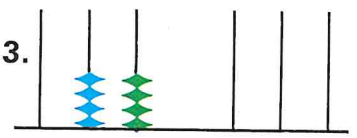
## EXERCISES

Name the number shown on each abacus.

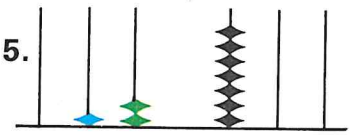
**1.**



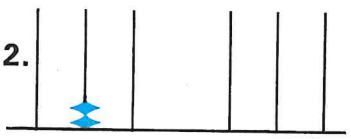
**3.**



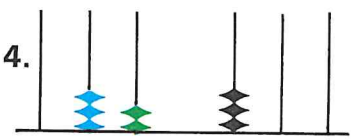
**5.**



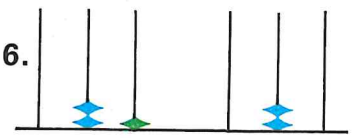
**2.**



**4.**



**6.**



Write a compact numeral for

**7.** seventy thousand
**8.** thirty-seven thousand
**9.** seventeen thousand
**10.** seventy-seven thousand

Write a compact numeral for

**11.** forty-three thousand, six hundred eighty-two
**12.** twenty-seven thousand, nine hundred fifty
**13.** eighty thousand, four hundred six
**14.** ninety-one thousand, twenty-eight

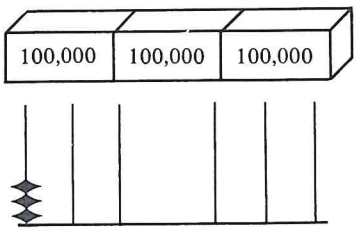
Write a compact numeral for

**15.** forty thousand + 683
**16.** 34 thousand + 861
**17.** fourteen thousand + 762
**18.** 67 thousand + 280

Write a compact numeral for

**19.** 30,000 + 4000 + 500 + 70 + 2
**20.** 60,000 + 8000 + 700 + 50 + 4
**21.** 80,000 + 9000 + 500 + 40
**22.** 30,000 + 8000 + 400 + 9

## Hundreds of thousands

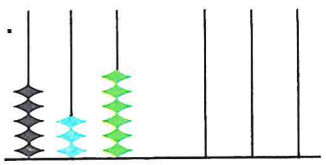



Each box contains *one hundred thousand* marbles. There are $3 \times 100{,}000 = 300{,}000$ marbles in all. *Three hundred thousand* may be shown on an abacus, using the sixth rod.

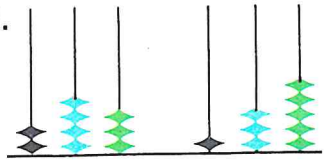
### EXERCISES

Name the number shown on each abacus.

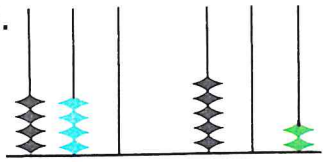
**1.**



**2.**



**3.**



Write a compact numeral for

**4.** six hundred thousand

**5.** nine hundred thousand

**6.** eight hundred thousand

**7.** four hundred twenty-three thousand

**8.** two hundred forty-three thousand, six hundred eighty-two

**9.** eight hundred twenty-seven thousand, nine hundred five

**10.** three hundred eighteen thousand, three hundred forty

**11.** six hundred seven thousand, twenty-one

**12.** two hundred twenty thousand + 541

**13.** three hundred fifty-one thousand + 698

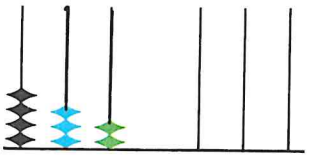
**14.** four hundred twenty-eight thousand + 574

**15.** five hundred seventy-six thousand + 821

*Just For Fun*

The system of numerals that we use is the Hindu-Arabic system. It was invented more than 1000 years ago, about the time of Harun al-Rashid. Do you know who he was?
Do you know the story of Aladdin and his lamp, Sinbad the Sailor, or Ali Baba and the Forty Thieves? Find out about them in *The Thousand and One Arabian Nights*.

## Hundreds of thousands



The numeral *432 thousand* may be written with digits as 432,000.
Write each of these numerals, using only digits.

1. 576 thousand
2. 42 thousand
3. seven hundred eighty-four thousand
4. nine hundred thousand
5. eighty-four thousand
6. eight thousand

Name the sums.

**7.**
3
40
200
8000
+ 30000

**8.**
6
50
200
9000
+ 30000

**9.**
8
70
400
9000
+ 50000

**10.**
9
80
200
5000
+ 60000

**11.**
6
70
4000
+ 70000

Complete the equations.

12. $13{,}000 + 400 + 80 + 6 = a$
13. $20{,}000 + 3000 + 476 = b$
14. $29{,}000 + 700 + 90 + 6 = c$
15. $40{,}000 + 8000 + 200 = d$
16. $7 + 20 + 500 + 13{,}000 = e$
17. $8 + 50 + 400 + 26{,}000 = f$

Name the sums.

**18.**
368
+ 475000

**20.**
316
+ 274000

**22.**
300000
+ 300

**24.**
400000
+ 8

**19.**
692
+ 356000

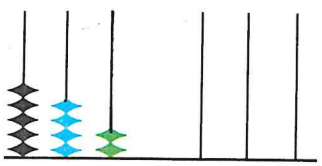
**21.**
756
+ 841000

**23.**
400000
+ 800

**25.**
61000
+ 610

Name the numbers shown here.

**26.**



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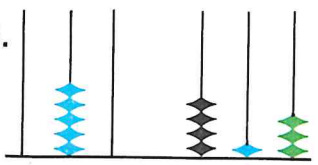

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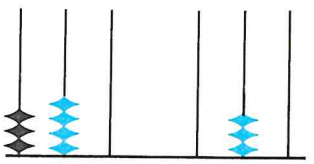

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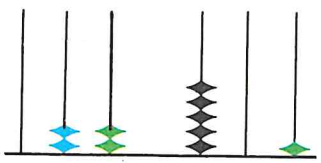

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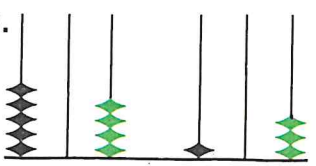
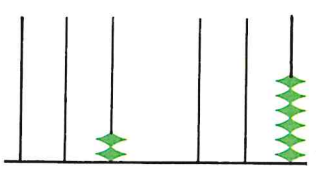


**31.**



## Millions

| Ones | Tens | Hundreds | Thousands | Ten Thousands | Hundred Thousands |
|---|---|---|---|---|---|
| 1 | 10 | 100 | 1000 | 10,000 | 100,000 |
| 2 | 20 | 200 | 2000 | 20,000 | 200,000 |
| 3 | 30 | 300 | 3000 | 30,000 | 300,000 |
| 4 | 40 | 400 | 4000 | 40,000 | 400,000 |
| 5 | 50 | 500 | 5000 | 50,000 | 500,000 |
| 6 | 60 | 600 | 6000 | 60,000 | 600,000 |
| 7 | 70 | 700 | 7000 | 70,000 | 700,000 |
| 8 | 80 | 800 | 8000 | 80,000 | 800,000 |
| 9 | 90 | 900 | 9000 | 90,000 | 900,000 |
| 10 | 100 | 1000 | 10,000 | 100,000 | 1,000,000 |

Another name for *ten hundred thousand* is **one million.** We write 1,000,000 for *one million*, 2,000,000 for *two million*, and so on.

### EXERCISES

Write a compact numeral for

**1.** 700,000 + 50,000 + 6000 + 800 + 20 + 3

**2.** 1,000,000 + 853,000 + 600 + 25

**3.** 90,000 + 800 + 60 + 4

**4.** 1,500,000 + 86,000 + 329

Write an expanded numeral for

**5.** 43,582

**6.** 673,219

**7.** 1,584,702

**8.** 3,408,650

**9.** 2,084,391

**10.** 5,005,005

**11.** 4,444,444

**12.** 9,900,900

**13–20.** Write numerals using only words for exercises 5 to 12.

Write a compact numeral for

**21.** three hundred forty-nine thousand, two hundred fifty-four

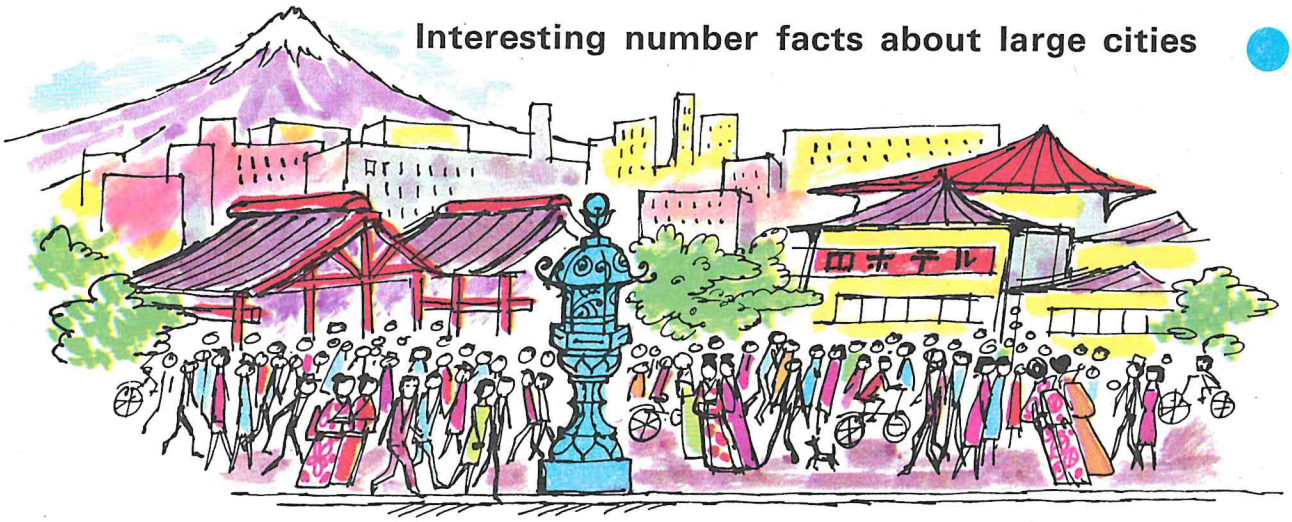
**22.** three million, five hundred twenty thousand, sixty-two

**23.** nine million, twenty-one thousand, seven hundred four

**24.** eight million, six hundred twenty-five thousand, 682

**25.** ten million, two hundred eight thousand, 702

## Interesting number facts about large cities




In 1962 the population of Tokyo, the capital of Japan, was more than 10 million people. It was the first city in history to reach the 10,000,000 mark and is the most populated city in the world.

### EXERCISES

1. In 1962 the population of New York was more than 8,000,000 people. By 1970 the population of New York is expected to be more than 10,000,000 people. How much larger will the population be in 1970?

2. In 1962 the population of London was more than 8 million people. Did New York and London have about the same population in 1962?

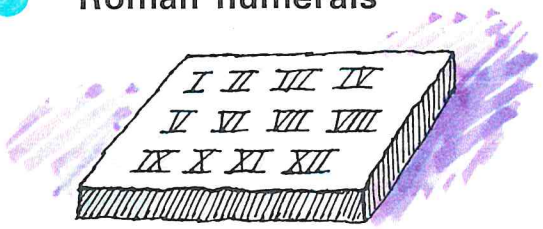
3. In 1962 the population of Peking was about 6,000,000 people. About how many more people were there in New York than in Peking in 1962?

4. In 1962 the population of Moscow was over 5,000,000. About how many people were there all together in the cities of London and Moscow in 1962?

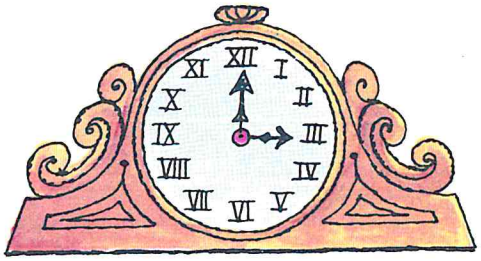
5. In 1962 the population of Paris was about 2 million, 8 hundred thousand. Use digits to name this population.

6. In 1962 the population of Melbourne, Australia was about one million, nine hundred fifty thousand. Use digits to name this population.

## Roman numerals



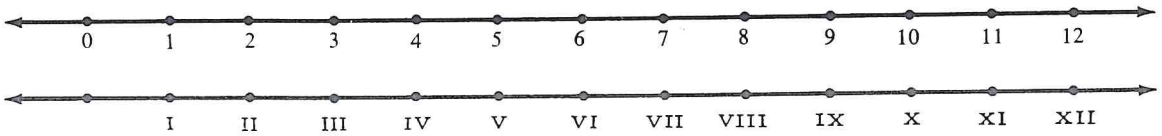

These numerals are not like the ones we use. They are the ones the early Romans used.




We still see **Roman numerals** on some clocks.

### EXERCISES

Compare the two number lines.




Rename these as Roman numerals.

**1.** 2 **2.** 4 **3.** 6 **4.** 7 **5.** 10 **6.** 11

**7.** Did the early Romans have a numeral for 0?

| | |
|---|---|
| I = 1 | V = 5 X = 10 |
| IV = 5 − 1 = 4 | The lesser number is on the *left*, so 1 is *subtracted* from 5. |
| VI = 5 + 1 = 6 | The lesser number is on the *right*, so 1 is *added* to 5. |
| XX = 10 + 10 = 20 | Equal numbers are added. |
| XXIV = 20 + 4 = 24 | |

Rename these as Hindu-Arabic numerals (the kind we use).

**8.** IX **9.** XI **10.** XIV **11.** XVI **12.** XXI **13.** XXVI

Rename these as Roman numerals.

**14.** a. 1 b. 11 c. 21

**15.** a. 9 b. 19 c. 29

**16.** a. 3 b. 13 c. 33

**17.** a. 8 b. 18 c. 38

**18.** a. 14 b. 24 c. 34

**19.** a. 16 b. 26 c. 36

## Roman numerals for greater numbers

| | |
|---|---|
| **L = 50** | **C = 100** |
| **D = 500** | **M = 1000** |

For greater numbers, the Romans used other symbols.

The same rules are used as for naming the lesser numbers.

LXXV = 50 + 20 + 5 = 75
CCCLXIX = 300 + 50 + 10 + (10 − 1) = 369
MCCCLXIX = 1000 + 300 + 50 + 10 + (10 − 1) = 1369
CDLX = (500 − 100) + 50 + 10 = 460
DCIX = 500 + 100 + (10 − 1) = 609
CDLXIX = (500 − 100) + 50 + 10 + (10 − 1) = 469

### EXERCISES

Solve the equations.

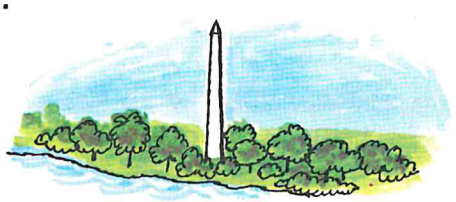
**1.** XXVII = 20 + 5 + 2 = *a*
**2.** LXXI = 50 + 20 + 1 = *b*
**3.** LIX = 50 + (10 − 1) = *c*
**4.** MCC = 1000 + 200 = *d*
**5.** CCXXVI = 200 + 20 + 5 + 1 = *e*
**6.** CCLXVI = 200 + 50 + 10 + 5 + 1 = *f*
**7.** XLVIII = (50 − 10) + 5 + 3 = *g*
**8.** MCM = 1000 + (1000 − 100) = *h*
**9.** MCMLX = 1000 + (1000 − 100) + 50 + 10 = *i*
**10.** MMCDXLIV = 2000 + (500 − 100) + (50 − 10) + (5 − 1) = *j*

Rename these as Roman numerals.

**11.** 45 **12.** 87 **13.** 97 **14.** 134 **15.** 278 **16.** 1278

Rename the dates as Hindu-Arabic numerals.

**17.**



**Washington Monument**
MDCCCLXXXV

**18.**

**Lincoln Memorial**
MCMXXII

## Number patterns

Complete the number patterns.

**1.**
$9 + 1 = a$
$99 + 1 = b$
$999 + 1 = c$
$9999 + 1 = d$
$99{,}999 + 1 = e$
$999{,}999 + 1 = f$

**2.**
$8 + 2 = g$
$98 + 2 = h$
$998 + 2 = i$
$9998 + 2 = j$
$99{,}998 + 2 = k$
$999{,}998 + 2 = l$

**3.**
$5 + 5 = m$
$95 + 5 = n$
$995 + 5 = z$
$9995 + 5 = p$
$99{,}995 + 5 = q$
$999{,}995 + 5 = r$

**4.**
$1{,}000{,}001 = a + 1$
$100{,}001 = b + 1$
$10{,}001 = c + 1$
$1001 = d + 1$
$101 = e + 1$
$11 = f + 1$

**5.**
$999{,}999 = g - 1$
$99{,}999 = h - 1$
$9999 = i - 1$
$999 = j - 1$
$99 = k - 1$
$9 = l - 1$

***6.**
$1{,}000{,}000 = 2 \times m$
$100{,}000 = 2 \times n$
$10{,}000 = 2 \times z$
$1000 = 2 \times p$
$100 = 2 \times q$
$10 = 2 \times r$

Name the sums and complete the number pattern.

**7.**
1
10
100
1000
10000
100000
1000000

**8.** $a = 1$

**9.** $b = 10 + 1$

**10.** $c = 100 + 10 + 1$

**11.** $d = 1000 + 100 + 10 + 1$

**12.** $e = 10{,}000 + 1000 + 100 + 10 + 1$

**13.** $f = 100{,}000 + 10{,}000 + 1000 + 100 + 10 + 1$

**14.** $g = 1{,}000{,}000 + 100{,}000 + 10{,}000 + 1000 + 100 + 10 + 1$

*Just For Fun*

Name the numbers.

**15.** This number has as many thousands as there are days in the week. In the hundreds' place is the solution to the equation $9 - 5 = n$. The number has a pair of tens and no ones.

**16.** This number is equal to fifteen thousands plus fifteen hundreds plus fifteen tens plus fifteen ones.

## CHECKING UP

For each set, write *F* or *I* to show whether it is *finite* or *infinite*.

1. $A$ = {all the counting numbers}
2. $B$ = {all the words in this book}
3. $C$ = {four hundred forty-four elephants}
4. $D$ = {all the counting numbers less than 4}
5. $E$ = {the words on this page}
6. $F$ = {a hop, a skip, a jump}
7. $G$ = {all the children in South America}
8. $H$ = {all the children in the world}

Name the set above that is

9. a subset of $A$
10. a subset of $B$
11. a subset of $H$

Name a counting number to complete each sentence.

12. $7 + a = 11$
13. $7 + b < 11$
14. $6 + 9 = c$
15. $6 + 9 < d$
16. $17 - 8 = e$
17. $17 - 8 > f$

Write an expanded numeral for

18. 34
19. 149
20. 2366
21. 9870
22. 40,872
23. 408,720
24. 467,045
25. 3,798,561

Write a compact numeral for

26. $30 + 7$
27. $60 + 6$
28. $400 + 7$
29. $810 + 8$
30. $500 + 76$
31. $2400 + 5$
32. $4000 + 500 + 60 + 6$
33. $30{,}000 + 800 + 30$

Write each of these, using only digits.

34. forty-three thousand, seven hundred twenty-six
35. three hundred fifty-four thousand, two hundred nine
36. four million, nine hundred seventy-three thousand, two hundred eighty

Rename these as Hindu-Arabic numerals.

37. XXIII
38. LXI
39. LXXVIII
40. CXIX
41. CCLI
42. CCLXXIV
43. MCCXI
44. MDCXX

Rename these as Roman numerals.

45. 12
46. 44
47. 57
48. 75
49. 333
50. 560
51. 748
52. 1000

## LOOKING BACK

> $A \cup B$, the **union** of sets $A$ and $B$, is the set that contains all members of $A$ or $B$, or both.

List the members of the union of each pair of sets.

1. $E = \{s, t, u, v, w\}$ and $F = \{v, w, x, y, z\}$
2. $G = \{4, 9, 8, 2, 7\}$ and $H = \{5, 8, 2, 9, 6, 3\}$

> Set $A$ is a **subset** of set $B$ if all members of $A$ are members of $B$.

For each set below, tell whether or not it is a subset of the set {a, b, c, d, e}.

3. $N = \{b, c\}$
4. $O = \{e, f, g, h\}$
5. $P = \{a\}$
6. $Q = \{b, d, e\}$
7. $R = \{a, b, c, d, e\}$
8. $S = \{\ \}$

> The **cardinal number** of a set tells how many objects are in the set. The cardinal number of the empty set is zero.

Name the cardinal number of each set.

9. {the letters in the alphabet}
10. {the months in a year}
11. {the odd numbers between 11 and 13}

> A **numeral** is a name for a number. We may use words, digits and place value, or expanded numerals to name numbers.

12. Which of these are other names for 5?
    **a.** five **b.** $5 + 0$ **c.** 50 **d.** $5 \times 0$ **e.** $4 + 1$ **f.** $20 \div 4$

Write a compact numeral for

13. $3000 + 700 + 60 + 8$
14. $50{,}000 + 800 + 90 + 7$
15. $700{,}000 + 50{,}000 + 2000 + 20$
16. $100{,}000 + 40{,}000 + 467$

**The Romans used two basic sets for their number system, sets of 5 and sets of 2.**

$5 = V$ $\quad$ $2 \times V = X$

$5 \times X = L$ $\quad$ $2 \times L = C$

$5 \times C = D$ $\quad$ $2 \times D = M$

We can count, using our numerals and the system of sets of 5 and 2. We count: 1, 2, 3, 4, 5, $5 + 1$, $5 + 2$, $5 + 3$, $5 + 4$, $2 \times 5$ (for 10), $10 + 1$, $10 + 2$, and so on.

Using the Roman system of sets of 5 and 2, write the numerals for

**1.** 7 **2.** 13 **3.** 18 **4.** 21 **5.** 28

The Romans conquered Britain about the year 50 B.C. Do you know what the letters B.C. stand for?
Can you tell what is wrong with this story: I have a real Roman coin that is dated *70 B.C.*

**The early Egyptians used a different system of numerals.**

1 = |

10 = ∩ (to stand for a cattle collar)

100 = 𓍢 (to stand for a coil of string)

1000 = 𓆼 (to stand for a plant)

They repeated each symbol as often as needed to show the numbers. They counted from one to nine like this:

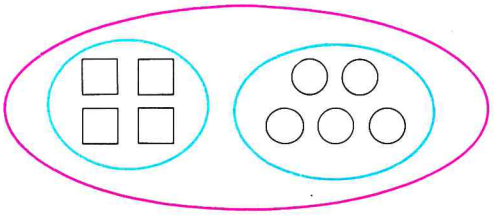
| | || | ||| | |||| | ||||| | |||| | |||| | |||| | |||| |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

They showed 23 as ∩∩ ||| and 456 as 𓍢𓍢 ∩∩ |||
𓍢𓍢 ∩∩∩ |||

Rename these as Egyptian numerals.

**6.** 7 **7.** 31 **8.** 46 **9.** 123 **10.** 1230 **11.** 93 **12.** 245 **13.** 1254 **14.** 2175 **15.** 9999

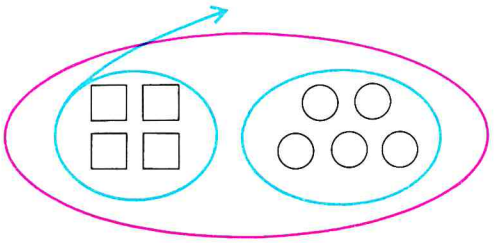
## Joining and separating sets



A set of 4 joined with a set of 5 forms a set of 9.

$$4 + 5 = 9$$

**Addition** is the renaming of two addends as the sum.



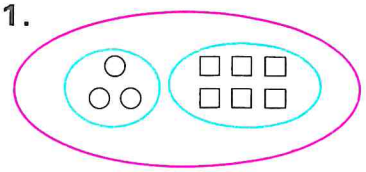
A set of 4 separated from a set of 9 leaves a set of 5.

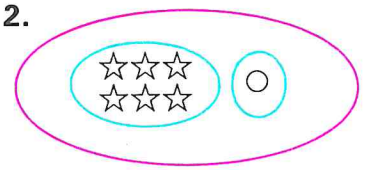
$$9 - 4 = 5$$

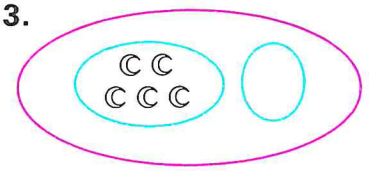
**Subtraction** is the renaming of a sum and one addend as the other addend.

### EXERCISES

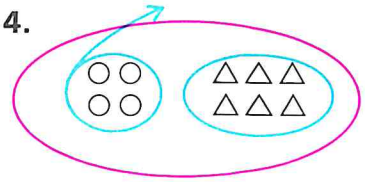
The subsets are being joined. Write an equation for the addition of the cardinal numbers.

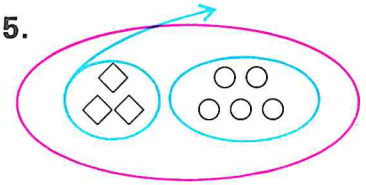
1. 

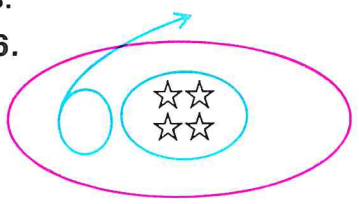
2. 

3. 

A subset is being separated from each set. Write an equation for the subtraction of the cardinal numbers.

4. 

5. 

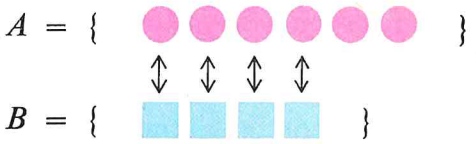
6. 

Name the missing sums and addends.

7.

| addend | 3 | 2 | 4 | 3 | 2 | 7 | *g* | *h* | *i* |
|---|---|---|---|---|---|---|---|---|---|
| addend | 6 | 8 | 5 | *d* | *e* | *f* | 3 | 6 | 8 |
| sum | *a* | *b* | *c* | 7 | 8 | 8 | 4 | 10 | 10 |

Set $B$ matches one-to-one with a subset of set $A$. In $A$ there are 2 unmatched elements.




The **difference** between n($A$) and n($B$) is named by subtraction.

$$\underset{\text{sum}}{n(A)} - \underset{\text{addends}}{n(B)} = \underset{\text{difference}}{2}$$

$$6 - 4 = 2$$

## EXERCISES

If you match the sets in each of the following pairs, which set contains unmatched members? Which set has more members?

**1.** $R = \{\bigcirc, \square, \triangle, \star\}$
$S = \{\triangle, \diamond\}$

**2.** $P = \{a, b, c, d, e, f, g, h\}$
$Q = \{X, Y, Z\}$

**3.** $F = \{\diamond, \bigcirc, \square, \star, \mathbb{C}, \bigcirc, \diamond\}$
$L = \{\triangle, \triangle, \triangle, \triangle\}$

**4.** $V = \{\star, \star, \star, \star, \star\}$
$G = \{\mathbb{C}, \mathbb{C}, \mathbb{C}, \mathbb{C}, \mathbb{C}, \mathbb{C}, \mathbb{C}\}$

**5–8.** Write an equation to name the difference between the cardinal numbers of the sets for each pair in exercises 1 to 4.

Name the differences.

**9.** $6 - 2 = a$ **11.** $7 - 3 = c$ **13.** $12 - 3 = e$ **15.** $14 - 9 = g$

**10.** $17 - 8 = b$ **12.** $13 - 8 = d$ **14.** $14 - 7 = f$ **16.** $12 - 4 = h$

Name the differences.

**17.**

| **sum** | 9 | 5 | 12 | 14 | 16 | 15 | 7 | 14 | 15 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|
| **addend** | 6 | 1 | 6 | 8 | 9 | 9 | 7 | 9 | 8 | 7 |
| **difference** | *a* | *b* | *c* | *d* | *e* | *f* | *g* | *h* | *i* | *j* |

## Basic addition-subtraction facts

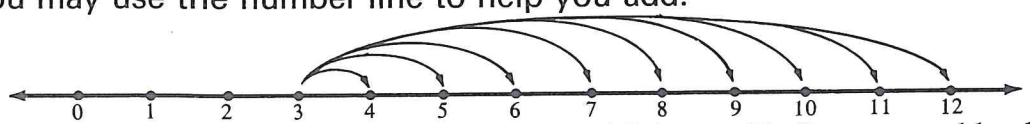
Additions and subtractions with two addends less than ten are **basic facts.** We call them basic because we use them in all additions and subtractions. There are 45 basic facts which you must know.

### EXERCISES

Write equations to name these sums.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | $1 + 1$ | **10.** | $2 + 2$ | **19.** | $3 + 4$ | **28.** | $4 + 7$ | **37.** | $6 + 7$ |
| **2.** | $1 + 2$ | **11.** | $2 + 3$ | **20.** | $3 + 5$ | **29.** | $4 + 8$ | **38.** | $6 + 8$ |
| **3.** | $1 + 3$ | **12.** | $2 + 4$ | **21.** | $3 + 6$ | **30.** | $4 + 9$ | **39.** | $6 + 9$ |
| **4.** | $1 + 4$ | **13.** | $2 + 5$ | **22.** | $3 + 7$ | **31.** | $5 + 5$ | **40.** | $7 + 7$ |
| **5.** | $1 + 5$ | **14.** | $2 + 6$ | **23.** | $3 + 8$ | **32.** | $5 + 6$ | **41.** | $7 + 8$ |
| **6.** | $1 + 6$ | **15.** | $2 + 7$ | **24.** | $3 + 9$ | **33.** | $5 + 7$ | **42.** | $7 + 9$ |
| **7.** | $1 + 7$ | **16.** | $2 + 8$ | **25.** | $4 + 4$ | **34.** | $5 + 8$ | **43.** | $8 + 8$ |
| **8.** | $1 + 8$ | **17.** | $2 + 9$ | **26.** | $4 + 5$ | **35.** | $5 + 9$ | **44.** | $8 + 9$ |
| **9.** | $1 + 9$ | **18.** | $3 + 3$ | **27.** | $4 + 6$ | **36.** | $6 + 6$ | **45.** | $9 + 9$ |

You may use the number line to help you add.




On our number line we have shown basic additions with 3 as one addend. You may arrange your sums to form a number pattern like this:

| + | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

Complete the number patterns.

**46.**

| + | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 7 | 8 | 9 | *a* | *b* | *c* | *d* | *e* | *f* |

**47.**

| + | 1 | 2 | 6 | 4 | 3 | 8 | 9 | 7 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 6 | 7 | 11 | *g* | *h* | *i* | *j* | *k* | *l* |

$A = \{0, 1, 2, 3, 4\}$
$B = \{2, 3, 4, 5, 6\}$
$C = \{(0, 2), (1, 3), (2, 4), (3, 5), (4, 6)\}$

Each member of $A$ is 2 less than a member of $B$. In $C$ we formed number pairs where the first number in each pair, from $A$, is 2 less than the second number, from $B$.

## EXERCISES

Use the rule to form a set of number pairs from each pair of sets.

**1.** The second number is 3 greater than the first.
$A = \{0, 1, 2, 3, 4, 5\}$
$B = \{3, 4, 5, 6, 7, 8\}$

**2.** The second number is 5 greater than the first.
$C = \{4, 5, 6, 7, 8, 9\}$
$D = \{9, 10, 11, 12, 13, 14\}$

**3.** The second number is 5 less than the first.
$E = \{5, 6, 7, 8, 9, 10\}$
$F = \{0, 1, 2, 3, 4, 5\}$

**4.** The second number is 4 greater than the first.
$G = \{0, 1, 2, 3, 4, 5\}$
$H = \{4, 5, 6, 7, 8, 9\}$

**5.** The second number is 3 less than the first.
$I = \{4, 5, 6, 7, 8, 9\}$
$J = \{1, 2, 3, 4, 5, 6\}$

**6.** The second number is 2 less than the first.
$K = \{8, 9, 10, 11, 12, 13\}$
$L = \{9, 8, 11, 6, 10, 7\}$

Use the rule to complete each set of number pairs.

**7.** The second number is 6 greater than the first.
$\{(0, a), (1, b), (2, c), (3, d), (4, e), (5, f)\}$

**8.** The second number is 8 greater than the first.
$\{(0, g), (1, h), (2, i), (3, j), (4, k), (5, l)\}$

**9.** The second number is 7 less than the first.
$\{(7, m), (8, n), (9, z), (10, p), (11, q), (12, r)\}$

**10.** The second number is 9 less than the first.
$\{(10, s), (11, t), (12, u), (13, v), (14, w), (15, x)\}$

**11.** The second number is 4 greater than the first.
$\{(a, 6), (b, 7), (c, 8), (d, 9), (e, 10), (f, 11)\}$

**12.** The second number is 8 less than the first.
$\{(g, 9), (h, 8), (i, 7), (j, 6), (k, 5), (l, 4)\}$

## Number sentences

Bob bought 4 apples and Milly bought 5. Together they bought 9 apples.

$4 + 5 = 9$

Milly ate 3 apples and Bob ate 2. Milly ate more apples than Bob.

$3 > 2$

Word sentences use **words** like *Bob*, *apples*, *bought*, and *ate*. Words like *bought* and *ate* are **verbs.**

Number sentences use **symbols** like 4, +, =, and >. The symbols =, <, and > are **verbs.**

### EXERCISES

Use =, >, or < and the other symbols to write these as number sentences.

1. The two addends 4 and 3 may be renamed as the sum 7.
2. The sum of 19 and 11 is 30.
3. 14 + 13 and 27 are two names for the same number.
4. 3 less than 14 is 11.
5. 9 is one greater than 8.
6. The difference between 8 and 6 is 2.
7. 4 + 3 is greater than 6.
8. 346 is less than 350.
9. The sum of 3 and 5 is less than the sum of 12 and 5.
10. 1000 + 30 + 2 and 1032 name the same number.
11. 14 is to the right of 12 on the number line.
12. The sum of 9 and 2 is greater than 10.
13. The difference between 9 and 2 is less than 10.

*14. There are more objects in set $A$ than in set $B$.

*15. The difference between $a$ and $b$ is $r$. $(a > b)$

It is possible to have true and false number sentences. For each number sentence, write *T* or *F* to show whether it is *true* or *false*.

16. $14 + 3 = 8 + 8 + 1$
17. $16 > 10 + 6$
18. $3 - 3 = 43 - 43$
19. $13 = 4 + 6 + 6$
20. $(4 + 5) + 2 = 4 + (5 + 2)$
21. $3 + 4 + 5 < 5 + 4 + 3$

| | | |
|---|---|---|
| Open Sentences | He was a king.<br>He was more than six feet tall. | $n = 14 + 3$<br>$n > 7$ |
| Closed Sentences<br>true | Henry the Eighth was a king.<br>Abraham Lincoln was more than six feet tall. | $17 = 14 + 3$<br>$8 > 7$ |
| false | Elizabeth was a king.<br>Karen's cat was more than six feet tall. | $18 = 14 + 3$<br>$6 > 7$ |

We cannot decide whether an open sentence is true or false.

When we replace *he* by a name, we can decide if the sentence is true or false.

To make the open equation $14 + 3 = n$ true, we replace the $n$ by 17. The number that makes the open equation true is called the **solution** of the equation. So, 17 is the solution of the equation $14 + 3 = n$.

## EXERCISES

Study these sentences. Replace each *he*, *she*, or *it* with a name to make the sentence true.

1. *He* was a reindeer with a red nose.
2. *It* is filled with ink.
3. *She* wore glass slippers.
4. *He* is a cartoon character who flies and is not hurt by bullets.
5. *She* was a friend of the Seven Dwarfs.
6. *He* grew a beanstalk and killed a giant.
7. *He* is a good football player.
8. *She* likes to go to the movies.
9. *She* visited the home of three bears.
10. *He* fell off a wall and all the King's men couldn't put him together again.

## True and false sentences

John has brown hair.

This sentence is about John. We can tell if it is true or false by looking at John.

$4 + 1 = 7$

This sentence is about numbers. We can tell it is **false** because $4 + 1$ and 7 do not name the same number.

$5 > 3$

This sentence is about numbers. We can tell it is **true** because 5 is to the right of 3 on the number line.

### EXERCISES

For each sentence, write *T* or *F* to show whether it is *true* or *false*.

1. George Washington was the first President of the United States of America.
2. Elephants live in trees.
3. The expanded numeral for 458 is $400 + 80 + 5$.
4. $2 + 9 = 11$
5. $6 + 8 > 7 + 8$
6. $16 - 9 > 2 + 5$
7. $2 + 4 + 6 < 1 + 3 + 5$
8. $(3 + 4) + 5 = 3 + (4 + 5)$
9. $10 - 3 < 17 - 8$

The sentence $4 + 3 = 8$ is false. We can make this a true sentence by changing it to $4 + 3 \neq 8$, $4 + 4 = 8$, or $4 + 3 < 8$.

**10–14.** Change the five false sentences in exercises 1 to 9 to make them true.

Solve the equations.

15. $u + 3 = 11$
16. $3 + v + 4 = 9$
17. $9 + w = 18$
18. $8 + x = 8$
19. $7 + 8 = y$
20. $6 + 6 = z$

Name a whole number to make each sentence true.

21. $a < 3$
22. $b - 4 < 13$
23. $c < 12 - 5$
24. $d + 3 < 8$
25. $e > 4 + 7$
26. $16 < f + 3$

**27–30.** For exercises 21 to 24, name all the whole numbers that make each sentence true.

To solve the equation $4 + 9 = n$, we replace $n$ by 13 because $4 + 9$ is equal to 13. There is only one number that makes the equation a true statement. The solution is 13.

Sentences like $n < 4$ and $n > 9$ are called **inequalities**. Often, more than one number will make an inequality true.
The whole numbers 0, 1, 2, and 3 all make the sentence $n < 4$ true. We may show this by writing the solutions as a set: $\{0, 1, 2, 3\}$. The **set of solutions** for the inequality $n > 9$ may be written $\{10, 11, 12, \ldots\}$ if our universal set is the set of whole numbers.

## EXERCISES

Write the set of solutions for each inequality. The universal set for each inequality is the set of whole numbers, $\{0, 1, 2, 3, 4, \ldots\}$

**1.** $n < 3$
**2.** $n < 9$
**3.** $n > 6$
**4.** $n > 3$
**5.** $n > 9$
**6.** $n > 100$
**7.** $n > 4$
**8.** $n < 10$
**9.** $n > 16$
**10.** $n < 23$
**11.** $n > 4$ and $n < 10$
**12.** $n > 16$ and $n < 23$
**13.** $n > 12$ and $n < 14$
**14.** $n > 38$ and $n < 48$
***15.** $n > 16$ and $n < 12$

Complete each number sentence. Write $<$ or $>$ to replace the ◯.

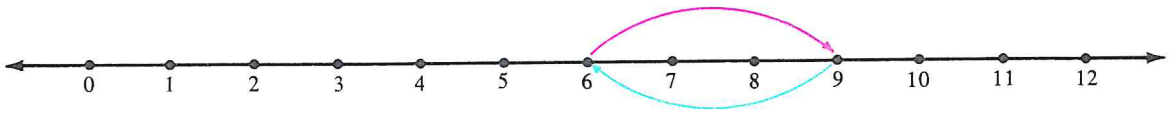
**16.** $4 \bigcirc 5$
**17.** $7 \bigcirc 17$
**18.** $71 \bigcirc 70$
**19.** $14 \bigcirc 23$
**20.** $81 \bigcirc 90$
**21.** $143 \bigcirc 144$
**22.** $4 + 7 \bigcirc 10$
**23.** $5 + 9 \bigcirc 15$
**24.** $7 + 7 \bigcirc 16$
**25.** $9 + 4 \bigcirc 14$
**26.** $7 + 3 \bigcirc 11$
**27.** $3 + 3 \bigcirc 8$

The universal set for each inequality is $\{0, 1, 2, 3, 4, \ldots\}$.
Match each pair of number sentences with its solution set.

**28.** $n > 8$ and $n < 12$
**29.** $n > 6$ and $n < 10$
**30.** $n < 20$ and $n > 16$
**31.** $n < 14$ and $n > 12$

$A = \{13\}$
$B = \{9, 10, 11\}$
$C = \{7, 8, 9\}$
$D = \{17, 18, 19\}$

## Addition and subtraction on the number line




$$(6 + 3) - 3 = 6$$

The top arrow shows the addition $6 + 3 = 9$.
The bottom arrow shows the subtraction $9 - 3 = 6$.
**Addition undoes subtraction.**

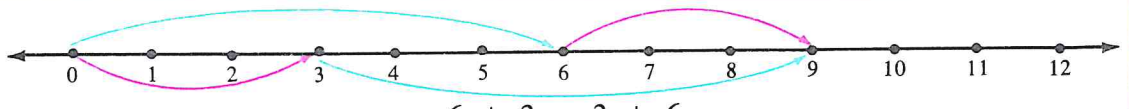
### EXERCISES

Show each pair of equations on a number line.

1. $4 + 6 = 10,\ 10 - 6 = 4$
2. $2 + 9 = 11,\ 11 - 9 = 2$
3. $8 + 4 = 12,\ 12 - 4 = 8$
4. $3 + 7 = 10,\ 10 - 7 = 3$

Show each equation on a number line.

5. $(6 + 4) - 4 = a$
6. $(7 + 5) - 5 = b$
7. $(8 + 1) - 1 = c$
8. $(7 + 2) - 2 = d$
9. $(2 + 6) - 6 = e$
10. $(5 + 3) - 3 = f$




$$6 + 3 = 3 + 6$$

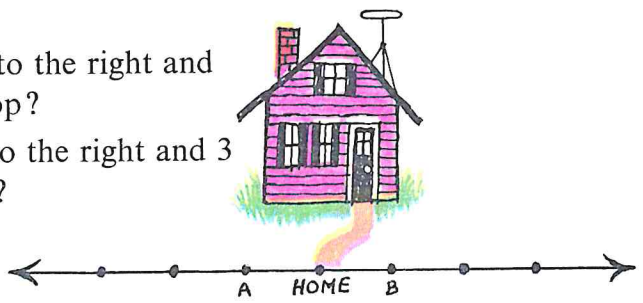
The top arrows show the addition $6 + 3 = 9$.
The bottom arrows show the addition $3 + 6 = 9$.
The order of the addends does not affect the sum.
This is the **Commutative Property of Addition.**

Show each pair of equations on a number line.

11. $4 + 6 = 10,\ 6 + 4 = 10$
12. $3 + 5 = 8,\ 5 + 3 = 8$
13. $2 + 9 = 11,\ 9 + 2 = 11$
14. $3 + 9 = 12,\ 9 + 3 = 12$

Answer the questions.

15. Bob started from home, went 3 to the right and 2 to the left. Where did Bob stop?
16. Jim started from home, went 2 to the right and 3 to the left. Where did Jim stop?
17. Is $3 - 2$ equal to $2 - 3$?
18. Is subtraction commutative?



In each of these equations we are looking for a **sum.** We use addition to name the solution. The solution (sum) is 8.

$3 + 5 = m$ $\quad$ $5 + 3 = m$
$m - 3 = 5$ $\quad$ $m - 5 = 3$

In each of these equations we are looking for an **addend.** We use subtraction to name the solution. The solution (difference) is 5.

$3 + n = 8$ $\quad$ $n + 3 = 8$
$8 - n = 3$ $\quad$ $8 - 3 = n$

When we replace the $m$ by 8 or the $n$ by 5 we get the same four equations. These equations form a **family of facts.**

$3 + 5 = 8$ $\quad$ $5 + 3 = 8$
$8 - 5 = 3$ $\quad$ $8 - 3 = 5$

## EXERCISES

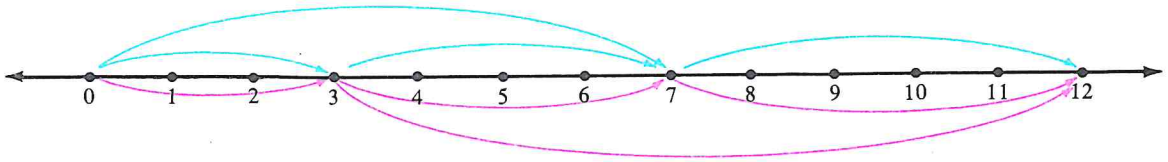
Match the equations that are in the same family of facts.

| | |
|---|---|
| **1.** $4 + 6 = 10$ | **a.** $3 + 9 = 12$ |
| **2.** $8 - 2 = 6$ | **b.** $3 + 7 = 10$ |
| **3.** $12 - 9 = 3$ | **c.** $8 - 4 = 4$ |
| **4.** $4 + 7 = 11$ | **d.** $2 + 6 = 8$ |
| **5.** $10 - 7 = 3$ | **e.** $10 - 4 = 6$ |
| **6.** $4 + 4 = 8$ | **f.** $4 = 11 - 7$ |
| **7.** $17 - 8 = 9$ | **g.** $12 - 6 = 6$ |
| **8.** $7 + 8 = 15$ | **h.** $17 - 9 = 8$ |
| **9.** $6 + 6 = 12$ | **i.** $14 = 9 + 5$ |
| **10.** $5 + 9 = 14$ | **j.** $15 - 7 = 8$ |

Complete each family of facts.

| | | |
|---|---|---|
| **11.** $7 + 9 = 16$ | **17.** $4 + 9 = 13$ | **23.** $2 + 2 = 4$ |
| **12.** $3 + 8 = 11$ | **18.** $6 + 9 = 15$ | **24.** $8 + 8 = 16$ |
| **13.** $8 + 9 = 17$ | **19.** $6 + 8 = 14$ | **25.** $9 + 9 = 18$ |
| **14.** $5 + 4 = 9$ | **20.** $6 - 3 = 3$ | **26.** $14 - 7 = 7$ |
| **15.** $5 + 5 = 10$ | **21.** $9 - 3 = 6$ | **27.** $13 - 5 = 8$ |
| **16.** $5 + 6 = 11$ | **22.** $12 - 3 = 9$ | **28.** $12 - 7 = 5$ |

## Three addends on the number line




The blue arrows show: $(3 + 4) + 5 = 7 + 5 = 12$

The red arrows show: $3 + (4 + 5) = 3 + 9 = 12$

$(3 + 4) + 5 = 3 + (4 + 5)$

> The way the addends are grouped does not affect the sum.
> This is the **Associative Property of Addition.**

### EXERCISES

On a number line, show two ways the addends may be grouped.

**1.** $3 + 4 + 2$
**2.** $6 + 2 + 3$
**3.** $4 + 5 + 1$
**4.** $2 + 3 + 7$
**5.** $6 + 1 + 3$
**6.** $3 + 4 + 5$
**7.** $4 + 2 + 5$
**8.** $2 + 7 + 3$
**9.** $4 + 4 + 0$

Show each equation on a number line.

**10.** $3 + (2 + 1) = (3 + 2) + 1$
**11.** $(6 + 1) + 3 = 6 + (1 + 3)$
**12.** $(5 + 4) + 2 = 5 + (4 + 2)$
**13.** $(8 + 2) + 3 = 8 + (2 + 3)$

Show each pair of equations on a number line.

**14.** $(2 + 4) + 5 = a$ and $2 + (4 + 5) = b$
**15.** $(1 + 5) + 3 = c$ and $1 + (5 + 3) = d$
**16.** $(2 + 3) + 4 = e$ and $2 + (3 + 4) = f$
**17.** $6 + (1 + 5) = g$ and $(6 + 1) + 5 = h$

Complete the sentence.

**18.**

> **The Associative Property of Addition:**
> The way the ___?___ are grouped does not affect the sum.

Show each equation on a number line.

**19.** $(3 + 4) + (2 + 1) = a$
**20.** $(3 + 1) + (4 + 2) = b$
**21.** $(4 + 2) + (3 + 2) = c$
**22.** $(2 + 1) + (5 + 2) = d$
**23.** $(3 + 3) + (2 + 2) = e$
**24.** $(1 + 4) + (3 + 2) = f$

**A**

$$\begin{aligned} 3 + 6 &= 3 + (5 + 1) \\ &= (3 + 5) + 1 \\ &= 8 + 1 \\ &= 9 \end{aligned}$$

**B**

$$\begin{aligned} 6 + 8 &= 6 + (4 + 4) \\ &= (6 + 4) + 4 \\ &= 10 + 4 \\ &= 14 \end{aligned}$$

**C**

$$\begin{aligned} 12 + 4 &= (10 + 2) + 4 \\ &= 10 + (2 + 4) \\ &= 10 + 6 \\ &= 16 \end{aligned}$$

**A** names the sum $3 + 6$ when we know $3 + 5 = 8$.
**B** names the sum $6 + 8$, which is greater than 10.
**C** names the sum when one addend is greater than 10.

## EXERCISES

Copy and complete the equations.

**1.** $\begin{aligned} 4 + 3 &= 4 + (2 + 1) \\ &= (4 + a) + 1 \\ &= b + 1 = c \\ 4 + 3 &= c \end{aligned}$

**2.** $\begin{aligned} 7 + 8 &= 7 + (7 + 1) \\ &= (7 + d) + 1 \\ &= e + 1 = f \\ 7 + 8 &= f \end{aligned}$

**3.** $\begin{aligned} 8 + 9 &= 8 + (8 + 1) \\ &= (8 + g) + 1 \\ &= h + 1 = i \\ 8 + 9 &= i \end{aligned}$

**4.** $\begin{aligned} 7 + 6 &= 7 + (3 + 3) \\ &= (7 + j) + 3 \\ &= k + 3 = l \\ 7 + 6 &= l \end{aligned}$

**5.** $\begin{aligned} 4 + 9 &= 4 + (6 + 3) \\ &= (4 + m) + 3 \\ &= n + 3 = z \\ 4 + 9 &= z \end{aligned}$

**6.** $\begin{aligned} 6 + 9 &= 6 + (4 + 5) \\ &= (6 + p) + 5 \\ &= q + 5 = r \\ 6 + 9 &= r \end{aligned}$

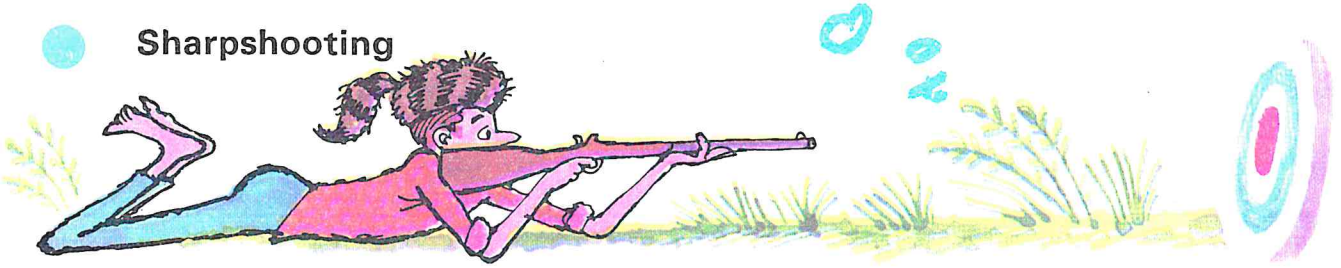
Use the associative property to name the sums.

**7.** $13 + 4 = a$
**8.** $12 + 7 = b$
**9.** $15 + 4 = c$
**10.** $16 + 4 = d$
**11.** $16 + 9 = e$
**12.** $19 + 9 = f$
**13.** $7 + (3 + 6) = g$
**14.** $2 + (8 + 6) = h$
**15.** $9 + (1 + 4) = i$
**16.** $(7 + 2) + 8 = j$
**17.** $(4 + 4) + 6 = k$
**18.** $(9 + 5) + 5 = l$

Use the associative property to show

**19.** $6 + 5 = 7 + 4$
**20.** $7 + 4 = 8 + 3$
**21.** $9 + 9 = 8 + 10$
**22.** $9 + 4 = 10 + 3$
**23.** $8 + 4 = 10 + 2$
**24.** $7 + 4 = 10 + 1$

## Sharpshooting



You need to practice answering questions in mathematics quickly and correctly, just as the sharpshooter practices hitting his target.

See how quickly you can answer the questions in **A**. Then see if you can answer the questions in **B** even more quickly.

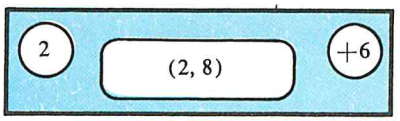
### A

1. What is the sum of 8 and 6?
2. Which is less, $7 + 4$ or $3 + 9$?
3. Bob had 9 marbles and Ben had 6. How many did they have together?
4. What is the place value of the 4 in 4628?
5. If a peach costs 8¢ and an apple costs 7¢, what do they cost together?
6. If it is 6 o'clock now, what will the time be in 8 hours?
7. In this set, name the numeral that is not a name for 6: $\{4 + 2, 6 \times 2, 12 - 6, 18 - 12$, the number 1 greater than 5$\}$.
8. What is the tens' digit in 48,769?
9. Use Roman numerals to complete the following: I am ___?___ years and ___?___ months old.
10. Which is less, $18 - 9$ or $15 - 7$?
11. If you have 4 dollars and a quarter, how many cents do you have in all?

### B

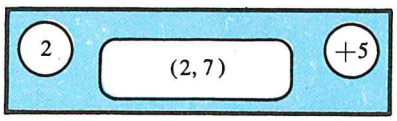
1. How much greater than 9 is 17?
2. Name the sum of 9 and 7.
3. If one addend is 6 and the sum is 14, what is the other addend?
4. Which is greater, $4 + 5$ or $6 + 2$?
5. If you wash 8 cups and 7 saucers, how many dishes do you wash in all?
6. What is the difference in length between 1 foot and 5 inches?
7. If you have a dozen eggs and break 6, how many eggs are left unbroken?
8. There are 14 animals in the field, 5 cows and the rest horses. How many horses are in the field?
9. Use Roman numerals to write names for these numbers: 19, $20 + 5$, 1062.
10. Which is greater, $9 + 8$ or $10 + 6$?
11. If you have 8 dollars and a half-dollar, how many cents do you have in all?

This is a special kind of TV. We use it to tune in **number pairs.**
The *first number* in the pair is tuned in with the number knob. The *rule* for naming the second number is tuned in with the rule knob. The *number pair* shows on the screen.

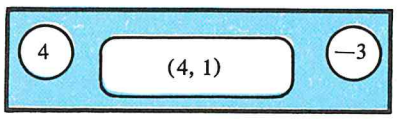



## EXERCISES

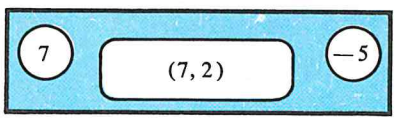
Study each TV and complete the number pairs in the set.



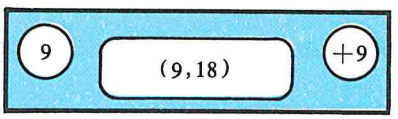

**1.** {(2, 7), (3, 8), (5, *a*), (6, *b*), (7, *c*), (9, *d*), (8, *e*)}




**2.** {(4, 1), (5, 2), (7, *f*), (9, *g*), (10, *h*), (11, *i*), (12, *j*)}




**3.** {(7, 2), (8, *k*), (9, *l*), (6, *m*), (5, *n*), (11, *p*), (12, *q*)}




**4.** {(9, 18), (7, 16), (6, *r*), (5, *s*), (8, *t*), (3, *u*), (1, *v*)}

Use the first pairs in each set to name the rule.
Then complete the number pairs in the set.

**5.** {(4, 7), (6, 9), (7, *a*), (3, *b*), (5, *c*), (2, *d*), (8, *e*), (9, *f*)}
**6.** {(6, 2), (4, 0), (9, 5), (7, *g*), (8, *h*), (9, *i*), (11, *j*), (13, *k*)}
**7.** {(4, 2), (3, 1), (2, 0), (8, 6), (9, *l*), (10, *m*), (6, *n*), (5, *p*)}
**8.** {(2, 2), (3, 3), (4, *q*), (5, *r*), (6, *s*), (7, *t*), (8, *u*), (9, *v*)}
**9.** {(3, 9), (4, 10), (*w*, 11), (*x*, 12), (*y*, 13), (*z*, 14), (*a*, 15), (*b*, 8)}

Pair the sets which use the same rule.

**10.** $A$ = {(2, 6), (3, 7), (4, 8), (5, 9)}
**11.** $B$ = {(15, 9), (8, 2), (12, 6), (14, 8)}
**12.** $C$ = {(0, 6), (3, 9), (5, 11), (7, 13)}
**13.** $D$ = {(9, 6), (3, 0), (4, 1), (6, 3)}

$E$ = {(13, 7), (11, 5), (10, 4), (9, 3)}
$F$ = {(8, 5), (10, 7), (5, 2), (7, 4)}
$G$ = {(6, 10), (7, 11), (8, 12), (9, 13)}
$H$ = {(1, 7), (2, 8), (4, 10), (6, 12)}

## Solving word problems

I had 16 marbles but lost 9. How many do I have now?

To solve the problem, think as follows:

*Step 1* **Ask: What are the sets in the problem?** A set of 16 marbles and a subset of 9 marbles.

*Step 2* **Ask: What is happening to the sets?** The subset of 9 is separated from the set of 16.

*Step 3* **Write your equation.** You know when you separate sets, you subtract numbers (look back at page 28). So you can write the equation $16 - 9 = n$. (Of course you might have written three other equations from this family of facts. The one we have written is probably the easiest.)

*Step 4* **Solve your equation.** You know that $16 - 9 = 7$. So 7 is the solution of the equation.

*Step 5* **Use your solution to answer the question asked in the problem.** How many do I have now? I now have 7 marbles.

### EXERCISES

Use sets and numbers to solve the problems.

1. How many more are in a set of 14 than in a set of 9?
2. A subset of 8 is separated from a set, leaving a subset of 9. How many were in the set at first?
3. If you have 9 apples and your friend has 6 apples, how many do you have together?
4. If you have 6 apples and want to give an apple to each of 15 friends, how many more apples do you need?
5. You gave away 9 apples and have 6 left. How many apples did you have at the beginning?
6. If there are 18 school holidays and you have had 9 of them, how many holidays are left?
7. If it is now 7 o'clock and you start a 4 hour trip, what time will it be when you arrive?

Complete each sentence. Write $<$, $>$, or $=$ to replace the ◯.

1. $8 + 9 \bigcirc 7 + 6$
2. $6 + 6 \bigcirc 7 + 5$
3. $4 + 6 \bigcirc 6 + 3$
4. $9 + 2 \bigcirc 1 + 10$
5. $7 + 8 \bigcirc 6 + 10$
6. $7 + 7 \bigcirc 4 - 3$
7. $3 + 8 \bigcirc 4 + 7$
8. $6 + 3 \bigcirc 18 - 9$
9. $6 + 4 \bigcirc 2 + 9$
10. $0 + 4 \bigcirc 4 - 0$
11. $4 + 2 \bigcirc 12 - 5$
12. $4 + 9 \bigcirc 3 + 10$
13. $3 + 4 \bigcirc 15 - 7$
14. $6 + 9 \bigcirc 5 + 10$
15. $2 + 5 \bigcirc 18 - 9$

Complete the other names for the numbers shown in red.

| | | | | | | |
|---|---|---|---|---|---|---|
| 16. | 9 | $8 + a$ | $3 + b$ | $11 - c$ | $4 + d$ | $17 - e$ |
| 17. | 12 | $6 + f$ | $4 + g$ | $9 + h$ | $3 + i$ | $12 + j$ |
| 18. | 7 | $5 + k$ | $12 - l$ | $16 - m$ | $0 + n$ | $11 - z$ |
| 19. | 15 | $7 + p$ | $6 + q$ | $9 + r$ | $8 + s$ | $10 + t$ |
| 20. | 16 | $8 + u$ | $9 + v$ | $7 + w$ | $6 + x$ | $16 - y$ |

Solve the problems.

21. Sarah and Kathy each had 4 dimes. How many dimes did they have in all?
22. Peter has 6 pencils, Herb has 3 pencils, and Jack has 7 pencils. How many pencils do they have in all?
23. Betty has 9 pieces of candy. If she wants to give a piece of candy to each of her 14 friends, how many more pieces of candy does she need?
24. If Carol breaks 9 of her 12 crayons, how many of her crayons are not broken?
25. Steve had 9 cents. He wanted to buy a 15 cent bag of candy. His mother gave him the extra money. How much money did she give him?
26. Sue had 11 cents. She bought an eraser for 4 cents. How much money does she have left?
27. Alan had 13 cents. He bought a pencil for 6 cents. How much money does he have left?

*28. Carol has 10 crayons and Barbara has 12. If Lois has more crayons than Carol but fewer than Barbara, how many crayons does Lois have?

## One addend less than 10

Solve the equations.

1. $12 + 4 = (10 + 2) + 4 = 10 + (2 + 4) = 10 + 6 = a$
2. $13 + 5 = (10 + 3) + 5 = 10 + (b + c) = 10 + d = e$
3. $14 + 4 = (10 + 4) + 4 = 10 + (f + g) = 10 + h = i$
4. $23 + 4 = (20 + 3) + 4 = 20 + (j + k) = 20 + l = m$
5. $23 + 7 = (20 + 3) + 7 = 20 + (n + z) = 20 + p = q$

Use the number line to name the sums.

6. $23 + 0 = a$
7. $23 + 2 = b$
8. $23 + 3 = c$
9. $23 + 5 = d$
10. $23 + 6 = e$
11. $23 + 7 = f$
12. $23 + 8 = g$
13. $23 + 9 = h$

Name the rule. Then complete the number pairs in the set.

14. $\{(21, 25), (22, 26), (23, a), (24, b), (26, c), (29, d)\}$
15. $\{(34, 37), (35, 38), (36, e), (38, f), (39, g), (40, h)\}$
16. $\{(32, 40), (33, 41), (35, i), (36, j), (38, k), (39, l)\}$
17. $\{(41, 50), (42, 51), (43, m), (44, n), (46, p), (48, q)\}$

Complete the number patterns.

18.

| + | 2 | 3 | 7 | 4 | 8 | 9 |
|---|---|---|---|---|---|---|
| 32 | *a* | *b* | *c* | *d* | *e* | *f* |

19.

| + | 3 | 9 | 2 | 5 | 4 | 6 |
|---|---|---|---|---|---|---|
| 68 | *g* | *h* | *i* | *j* | *k* | *l* |

20.

| + | 1 | 2 | 7 | 6 | 4 | 8 |
|---|---|---|---|---|---|---|
| 29 | *m* | *n* | *p* | *q* | *r* | *s* |

21.

| + | 8 | 5 | 7 | 2 | 6 | 1 |
|---|---|---|---|---|---|---|
| 56 | *t* | *u* | *v* | *w* | *x* | *y* |

22.

| + | 6 | 8 | 9 | 7 | 4 | 0 |
|---|---|---|---|---|---|---|
| 24 | *a* | *b* | *c* | *d* | *e* | *f* |
| 35 | *g* | *h* | *i* | *j* | *k* | *l* |
| 47 | *m* | *n* | *p* | *q* | *r* | *s* |
| 61 | *t* | *u* | *v* | *w* | *x* | *y* |

23.

| + | 23 | 46 | 52 | 87 | 31 | 65 |
|---|---|---|---|---|---|---|
| 4 | *a* | *b* | *c* | *d* | *e* | *f* |
| 7 | *g* | *h* | *i* | *j* | *k* | *l* |
| 5 | *m* | *n* | *p* | *q* | *r* | *s* |
| 9 | *t* | *u* | *v* | *w* | *x* | *y* |

Solve the equations.

1. $16 - 4 = (10 + 6) - 4 = 10 + (a - b) = 10 + c = d$
2. $36 - 5 = (30 + 6) - 5 = 30 + (e - f) = 30 + g = h$
3. $48 - 6 = (40 + 8) - 6 = 40 + (i - j) = 40 + k = l$

Solve the equations.

4. $22 - 4 = (10 + 12) - 4 = 10 + (12 - 4) = 10 + a = b$
5. $32 - 7 = (20 + 12) - 7 = 20 + (12 - 7) = 20 + c = d$
6. $43 - 5 = (30 + 13) - 5 = 30 + (e - f) = 30 + g = h$
7. $42 - 9 = (30 + 12) - 9 = 30 + (i - j) = 30 + k = l$
8. $71 - 6 = (60 + 11) - 6 = 60 + (m - n) = 60 + p = q$
9. $82 - 5 = (70 + 12) - 5 = 70 + (r - s) = 70 + t = u$

Use the number line to name the differences.

10. $53 - 2 = a$
11. $53 - 9 = b$
12. $53 - 3 = c$
13. $53 - 5 = d$
14. $53 - 4 = e$
15. $53 - 6 = f$
16. $53 - 7 = g$
17. $53 - 8 = h$

Name the rule. Then complete the number pairs in the set.

18. $\{(32, 29), (31, 28), (30, a), (29, b), (28, c), (27, d), (26, e), (25, f)\}$
19. $\{(44, 39), (43, 38), (42, h), (41, i), (40, j), (39, k), (38, l), (37, m)\}$
20. $\{(65, 62), (64, 61), (63, n), (59, z), (54, p), (57, q), (58, r), (56, s)\}$
21. $\{(81, 77), (80, 76), (74, t), (78, u), (73, v), (75, w), (70, x), (58, y)\}$

Complete the number patterns.

22.

| + | 2 | 6 | $a$ | $c$ | $e$ |
|---|---|---|---|---|---|
| 34 | 36 | 40 | 39 | 38 | 37 |

23.

| + | 1 | 8 | $u$ | $v$ | $x$ |
|---|---|---|---|---|---|
| 32 | 33 | 40 | 37 | 36 | 38 |

24.

| + | 5 | 6 | $i$ | $n$ | $r$ |
|---|---|---|---|---|---|
| 45 | 50 | 51 | 47 | 48 | 49 |

25.

| + | 2 | 6 | $a$ | $c$ | $e$ |
|---|---|---|---|---|---|
| 57 | 59 | 63 | 58 | 60 | 61 |

## Adding and subtracting tens and ones

Study the methods for naming the sum 43 + 25 and the difference 48 − 25.

| Expanded numerals | Three step | Short form |
|---|---|---|
| $43 = 40 + 3$<br>$+\ 25 = 20 + 5$<br>$60 + 8 = 68$ | 43<br>+ 25<br>8<br>60<br>68 | 43<br>+ 25<br>68 |
| $48 = 40 + 8$<br>$-\ 25 = -\ 20 - 5$<br>$20 + 3 = 23$ | 48<br>− 25<br>3<br>20<br>23 | 48<br>− 25<br>23 |

### EXERCISES

Use expanded numerals to name the sums and differences.

**1.** 42 + 24 **2.** 36 + 22 **3.** 63 + 42 **4.** 65 + 24 **5.** 77 − 42 **6.** 86 − 34 **7.** 24 − 14

Use the 3-step method and then the short method to name the sums and differences.

**8.** 63 + 21 **9.** 77 + 42 **10.** 83 + 36 **11.** 24 + 74 **12.** 75 − 32 **13.** 86 − 23 **14.** 79 − 56

All these methods can be used for adding and subtracting hundreds, tens, and ones. The second method will have four steps.

Use the 4-step method and then the short method to name the sums and differences.

**15.** 421 + 234 **17.** 145 + 423 **19.** 627 + 321 **21.** 486 − 235 **23.** 475 − 314 **25.** 876 − 230

**16.** 452 + 326 **18.** 318 + 451 **20.** 462 + 125 **22.** 752 − 321 **24.** 265 − 153 **26.** 456 − 352

Study the methods for naming the sum 49 + 25 and the difference 43 − 25.

| Expanded numerals | Three step | Short form |
|---|---|---|
| $\begin{array}{rcl} 49 & = & 40 + 9 \\ +\ 25 & = & 20 + 5 \\ \hline & & 60 + 14 = 74 \end{array}$ | $\begin{array}{r} 49 \\ +\ 25 \\ \hline 14 \\ 60 \\ \hline 74 \end{array}$ | $\begin{array}{r} {\scriptstyle 1} \\ 49 \\ +\ 25 \\ \hline 74 \end{array}$ |
| $\begin{array}{rcl} 43 & = & {\scriptstyle 30\ \ 13} \\ & & \not{40} + \not{3} \\ -\ 25 & = & -\ 20 - 5 \\ \hline & & 10 + 8 = 18 \end{array}$ | $\begin{array}{r} {\scriptstyle 3}\,{\scriptstyle 13} \\ \not{43} \\ -\ 25 \\ \hline 8 \\ 10 \\ \hline 18 \end{array}$ | $\begin{array}{r} {\scriptstyle 3}\,{\scriptstyle 13} \\ \not{43} \\ -\ 25 \\ \hline 18 \end{array}$ |

## EXERCISES

Use expanded numerals to name the sums and differences.

**1.** $\begin{array}{r} 47 \\ +\ 29 \\ \hline \end{array}$ **2.** $\begin{array}{r} 36 \\ +\ 49 \\ \hline \end{array}$ **3.** $\begin{array}{r} 58 \\ +\ 26 \\ \hline \end{array}$ **4.** $\begin{array}{r} 92 \\ -\ 76 \\ \hline \end{array}$ **5.** $\begin{array}{r} 34 \\ -\ 18 \\ \hline \end{array}$ **6.** $\begin{array}{r} 83 \\ -\ 67 \\ \hline \end{array}$ **7.** $\begin{array}{r} 64 \\ -\ 39 \\ \hline \end{array}$

Use the 3-step method to name the sums and differences.

**8.** $\begin{array}{r} 29 \\ +\ 45 \\ \hline \end{array}$ **9.** $\begin{array}{r} 37 \\ +\ 44 \\ \hline \end{array}$ **10.** $\begin{array}{r} 69 \\ +\ 12 \\ \hline \end{array}$ **11** $\begin{array}{r} 32 \\ -\ 19 \\ \hline \end{array}$ **12.** $\begin{array}{r} 46 \\ -\ 27 \\ \hline \end{array}$ **13.** $\begin{array}{r} 48 \\ -\ 19 \\ \hline \end{array}$ **14.** $\begin{array}{r} 51 \\ -\ 26 \\ \hline \end{array}$

Use the short method to name the sums and differences.

**15.** $\begin{array}{r} 35 \\ +\ 27 \\ \hline \end{array}$ **19.** $\begin{array}{r} 44 \\ +\ 39 \\ \hline \end{array}$ **23.** $\begin{array}{r} 53 \\ +\ 29 \\ \hline \end{array}$ **27.** $\begin{array}{r} 127 \\ +\ 467 \\ \hline \end{array}$ **31.** $\begin{array}{r} 259 \\ +\ 19 \\ \hline \end{array}$ **35.** $\begin{array}{r} 235 \\ +\ 529 \\ \hline \end{array}$

**16.** $\begin{array}{r} 63 \\ -\ 47 \\ \hline \end{array}$ **20.** $\begin{array}{r} 93 \\ -\ 66 \\ \hline \end{array}$ **24.** $\begin{array}{r} 53 \\ -\ 29 \\ \hline \end{array}$ **28.** $\begin{array}{r} 350 \\ -\ 126 \\ \hline \end{array}$ **32.** $\begin{array}{r} 775 \\ -\ 48 \\ \hline \end{array}$ **36.** $\begin{array}{r} 334 \\ -\ 225 \\ \hline \end{array}$

**17.** $\begin{array}{r} 65 \\ +\ 18 \\ \hline \end{array}$ **21.** $\begin{array}{r} 25 \\ +\ 28 \\ \hline \end{array}$ **25.** $\begin{array}{r} 45 \\ +\ 25 \\ \hline \end{array}$ **29.** $\begin{array}{r} 555 \\ +\ 236 \\ \hline \end{array}$ **33.** $\begin{array}{r} 136 \\ +\ 24 \\ \hline \end{array}$ **37.** $\begin{array}{r} 666 \\ +\ 627 \\ \hline \end{array}$

**18.** $\begin{array}{r} 65 \\ -\ 18 \\ \hline \end{array}$ **22.** $\begin{array}{r} 72 \\ -\ 67 \\ \hline \end{array}$ **26.** $\begin{array}{r} 81 \\ -\ 27 \\ \hline \end{array}$ **30.** $\begin{array}{r} 555 \\ -\ 236 \\ \hline \end{array}$ **34.** $\begin{array}{r} 392 \\ -\ 146 \\ \hline \end{array}$ **38.** $\begin{array}{r} 666 \\ -\ 627 \\ \hline \end{array}$

## Adding hundreds, tens, and ones

Study the two methods for naming the sum 456 + 372.

$$\begin{array}{r} 456 \\ +\ 372 \\ \hline 8 \\ 120 \\ 700 \\ \hline 828 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 456 \\ +\ 372 \\ \hline 828 \end{array}$$

Study the two methods for naming the sum 456 + 378.

$$\begin{array}{r} 456 \\ +\ 378 \\ \hline 14 \\ 120 \\ 700 \\ \hline 834 \end{array} \qquad \begin{array}{r} {\scriptstyle 11} \\ 456 \\ +\ 378 \\ \hline 834 \end{array}$$

### EXERCISES

Use the 4-step method and then the short method to name each sum.

| | | | | | |
|---|---|---|---|---|---|
| **1.** 357 + 234 | **2.** 674 + 319 | **3.** 622 + 269 | **4.** 427 + 348 | **5.** 716 + 178 | **6.** 692 + 175 |

Use the short method to name the sums.

| | | | | | |
|---|---|---|---|---|---|
| **7.** 768 + 217 | **8.** 626 + 435 | **9.** 398 + 461 | **10.** 738 + 283 | **11.** 588 + 396 | **12.** 894 + 173 |

Use the 4-step method or the short method to name the sums.

| | | | | | |
|---|---|---|---|---|---|
| **13.** 465 + 226 | **15.** 162 + 529 | **17.** 721 + 219 | **19.** 657 + 225 | **21.** 846 + 137 | **23.** 755 + 466 |
| **14.** 693 + 264 | **16.** 375 + 148 | **18.** 536 + 487 | **20.** 558 + 346 | **22.** 625 + 296 | **24.** 457 + 888 |

Solve the problems.

**25.** Sam and Jerry collect baseball trading cards. Sam has 182 cards and Jerry has 234 cards. How many cards do they have in all?

**26.** In Hal's school there are 456 children in the fourth grade and 373 in the fifth grade. How many children are in the two grades?

**27.** On Saturday, 259 people went to the baseball game to cheer for the Blue Team and 287 went to cheer for the Red Team. How many people went to the game in all?

Study the equations and the number patterns.

$$300 + 40 + 7 = 347$$
$$200 + 80 + 9 = 289$$
$$300 + 70 + 2 = 372$$
$$800 + 190 + 18 = 1008$$

| | | | |
|---|---|---|---|
| 300 | 40 | 7 | 347 |
| 200 | 80 | 9 | 289 |
| 300 | 70 | 2 | 372 |
| 800 | 190 | 18 | 1008 |

## EXERCISES

Complete the number patterns.

1.

| | | | |
|---|---|---|---|
| 200 | 50 | 6 | 256 |
| 400 | 70 | 3 | 473 |
| 200 | 40 | 9 | 249 |
| *a* | *b* | *c* | *d* |

2.

| | | | |
|---|---|---|---|
| 100 | 70 | 6 | 176 |
| 600 | 80 | 2 | 682 |
| 800 | 40 | 5 | 845 |
| *e* | *f* | *g* | *h* |

3.

| | | | |
|---|---|---|---|
| 700 | 30 | 6 | 736 |
| 300 | 20 | 9 | 329 |
| 400 | 80 | 1 | *i* |
| *j* | *k* | *l* | *m* |

4.

| | | | |
|---|---|---|---|
| 600 | 40 | 5 | 645 |
| 200 | 80 | 7 | 287 |
| 800 | 60 | 3 | 863 |
| *n* | *s* | *p* | *q* |

5.

| | | | |
|---|---|---|---|
| 900 | 40 | *r* | 947 |
| 500 | *s* | 8 | 528 |
| 400 | 60 | *t* | 465 |
| *u* | 120 | 20 | *v* |

6.

| | | | |
|---|---|---|---|
| 600 | 50 | *w* | 651 |
| *x* | 30 | 7 | 237 |
| 800 | 50 | *y* | 854 |
| 1600 | 130 | 12 | *z* |

Complete the number patterns.

7.

| | | | |
|---|---|---|---|
| 16 | 8 | 9 | *a* |
| 13 | 7 | 6 | *b* |
| *c* | *d* | *e* | *f* |

8.

| | | | |
|---|---|---|---|
| 24 | 3 | 8 | *g* |
| 7 | 8 | 18 | *h* |
| *i* | *j* | *k* | *l* |

9.

| | | | |
|---|---|---|---|
| 14 | 7 | 8 | *m* |
| 36 | 4 | 9 | *n* |
| *z* | *p* | *q* | *r* |

## Subtracting hundreds, tens, and ones

Study the methods for naming the differences.

| Expanded numerals | Short form |
|---|---|
| $\begin{array}{rcl} & & {\scriptstyle 30\quad 16} \\ 746 &=& 700 + \cancel{40} + \cancel{6} \\ -\ 129 &=& -\ 100 - 20 - 9 \\ \hline & & 600 + 10 + 7 = 617 \end{array}$ | $\begin{array}{r} {\scriptstyle 3\ 16} \\ 7\ \cancel{4}\ \cancel{6} \\ -\ 1\ 2\ 9 \\ \hline 6\ 1\ 7 \end{array}$ |
| $\begin{array}{rcl} & & {\scriptstyle 600\quad 140} \\ 749 &=& \cancel{700} + \cancel{40} + 9 \\ -\ 156 &=& -\ 100 - 50 - 6 \\ \hline & & 500 + 90 + 3 = 593 \end{array}$ | $\begin{array}{r} {\scriptstyle 6\ 14} \\ \cancel{7}\ \cancel{4}\ 9 \\ -\ 1\ 5\ 6 \\ \hline 5\ 9\ 3 \end{array}$ |

### EXERCISES

Use expanded numerals to name the differences.

| | | | | | |
|---|---|---|---|---|---|
| **1.** 733 − 214 | **2.** 375 − 148 | **3.** 432 − 215 | **4.** 856 − 273 | **5.** 425 − 192 | **6.** 758 − 495 |

Use the short method to name the differences.

| | | | | | |
|---|---|---|---|---|---|
| **7.** 562 − 236 | **9.** 896 − 437 | **11.** 851 − 606 | **13.** 817 − 676 | **15.** 629 − 487 | **17.** 508 − 433 |
| **8.** 564 − 449 | **10.** 950 − 928 | **12.** 909 − 242 | **14.** 778 − 593 | **16.** 741 − 239 | **18.** 707 − 170 |

Solve the problems.

**19.** Sam has 182 baseball trading cards and Jerry has 234 baseball cards. How many more has Jerry?

**20.** The B. Rains School had a Penny Carnival. On the first day 372 people attended, and on the second day 457 attended. How many more people came on the second day?

**21.** How many people came to the Penny Carnival during the two days?

## Subtracting hundreds, tens, and ones

Study the two methods for naming the difference 425 − 269.

| Expanded numerals | | Short form |
|---|---|---|
| $425 = 400 + \overset{10}{\cancel{20}} + \overset{15}{\cancel{5}}$<br>$-\ 269 = -\ 200 - 60 - 9$<br>$6$ | *Step 1:*<br>$25 = 10 + 15$<br>$15 - 9 = 6$ | $\overset{1\ 15}{4\ \cancel{2}\ \cancel{5}}$<br>$-\ 2\ 6\ 9$<br>$6$ |
| $= \overset{300}{\cancel{400}} + \overset{110}{\cancel{20}} + \overset{15}{\cancel{5}}$<br>$= -\ 200 - 60 - 9$<br>$50 + 6$ | *Step 2:*<br>$410 = 300 + 110$<br>$110 - 60 = 50$ | $\overset{3\ 11\ 15}{\cancel{4}\ \cancel{2}\ \cancel{5}}$<br>$-\ 2\ 6\ 9$<br>$5\ 6$ |
| $= \overset{300}{\cancel{400}} + \overset{110}{\cancel{20}} + \overset{15}{\cancel{5}}$<br>$= -\ 200 - 60 - 9$<br>$100 + 50 + 6 = 156$ | *Step 3:*<br>$300 - 200 = 100$ | $\overset{3\ 11\ 15}{\cancel{4}\ \cancel{2}\ \cancel{5}}$<br>$-\ 2\ 6\ 9$<br>$1\ 5\ 6$ |

## EXERCISES

Use expanded numerals to name the differences.

| | | | | | |
|---|---|---|---|---|---|
| **1.** 425 − 266 | **2.** 725 − 147 | **3.** 922 − 575 | **4.** 711 − 299 | **5.** 640 − 376 | **6.** 613 − 584 |

Use the short method to name the differences.

| | | | | | |
|---|---|---|---|---|---|
| **7.** 721 − 455 | **9.** 443 − 289 | **11.** 955 − 487 | **13.** 751 − 354 | **15.** 683 − 595 | **17.** 470 − 184 |
| **8.** 965 − 487 | **10.** 466 − 169 | **12.** 341 − 262 | **14.** 611 − 594 | **16.** 320 − 283 | **18.** 225 − 87 |

Solve the problem.

**19.** Stephen was in charge of the candy booth at the Penny Carnival. He sold 243 lollipops and 156 chocolate bars. How many more lollipops than chocolate bars did he sell?

## Extending addition

Once you know how to rename in adding hundreds, tens, and ones, it is easy to add greater numbers.

Study these methods for naming the sums.

| | |
|---|---|
| 4527<br>+ 3898<br>15<br>110<br>1300<br>7000<br>8425 | 111<br>4527<br>+ 3898<br>8425 |
| 61839<br>+ 58625<br>14<br>50<br>1400<br>9000<br>110000<br>120464 | 11 1<br>61839<br>+ 58625<br>120464 |

EXERCISES

Name the sums.

| | | | | |
|---|---|---|---|---|
| **1.** 6325 + 2458 | **4.** 2894 + 5433 | **7.** 2796 + 3488 | **10.** 89164 + 99453 | **13.** 32989 + 73888 |
| **2.** 3764 + 3816 | **5.** 5588 + 4259 | **8.** 5764 + 8180 | **11.** 22349 + 24972 | **14.** 729839 + 777666 |
| **3.** 1025 + 8198 | **6.** 4781 + 4469 | **9.** 6527 + 9746 | **12.** 72318 + 74695 | **15.** 3548236 + 7983559 |

Solve the problems.

**16.** The two smallest states in the United States of America are Rhode Island and Delaware. The area of Rhode Island is 1214 square miles and the area of Delaware is 2057 square miles. What is the total area of these two states?

**17.** The next three smallest states are Connecticut, Hawaii and New Jersey. The area of Connecticut is 5009 square miles, of Hawaii is 6424 square miles, and of New Jersey is 8219 square miles.

**a.** What is the total area of Connecticut and Hawaii?

**b.** What is the total area of the five smallest states?

You can also use what you know about subtracting hundreds, tens, and ones to subtract greater numbers.

Study the examples.

| | | |
|---|---|---|
| 3 14 11 17<br>4 5 2 7<br>− 2 8 9 8<br>1 6 2 9 | 2 15 11<br>3 6 1 8<br>− 2 7 3 3<br>8 8 5 | 6 11 13 5 12<br>7 2 3 6 2<br>− 3 5 9 1 8<br>3 6 4 4 4 |

## EXERCISES

Name the differences.

| | | | | |
|---|---|---|---|---|
| **1.** 7394 − 1719 | **3.** 8542 − 3491 | **5.** 9756 − 3439 | **7.** 5372 − 2184 | **9.** 36742 − 12818 |
| **2.** 4923 − 1719 | **4.** 5813 − 1438 | **6.** 8215 − 6838 | **8.** 3347 − 1798 | **10.** 25076 − 23988 |

In subtraction, it is sometimes necessary to rename in different ways.

Study the examples.

| | | |
|---|---|---|
| 500 = 490 + 10 | 503 = 490 + 13 | 2003 = 1990 + 13 |
| 4 9 10<br>5 0 0<br>− 1 7 6<br>3 2 4 | 4 9 13<br>5 0 3<br>− 1 7 6<br>3 2 7 | 1 9 9 13<br>2 0 0 3<br>− 1 0 7 6<br>9 2 7 |

Name the differences.

| | | | | |
|---|---|---|---|---|
| **11.** 503 − 408 | **13.** 500 − 367 | **15.** 900 − 275 | **17.** 8002 − 6334 | **19.** 8020 − 6334 |
| **12.** 208 − 99 | **14.** 602 − 523 | **16.** 700 − 692 | **18.** 5000 − 4896 | **20.** 5000 − 4996 |

## Checking subtraction

We know that addition and subtraction are opposites.
We can use this fact to check our subtraction.

Consider the examples.

| | |
|---|---|
| $\begin{array}{r} 43 \\ -\ 17 \\ \hline 26 \end{array}$ $\quad$ $\begin{array}{r} 17 \\ +\ 26 \\ \hline 43 \end{array}$ | $\begin{array}{r} 4398 \\ -\ 1764 \\ \hline 2634 \end{array}$ $\quad$ $\begin{array}{r} 1764 \\ +\ 2634 \\ \hline 4398 \end{array}$ |

We can save time in checking subtraction if we do not have to write the addition. We can "add up" to check the first subtraction like this:

$$\begin{array}{r} 43 \\ -\ 17 \\ \hline 26 \\ {\scriptstyle 1} \end{array} \quad 6 + 7 = 13 \qquad\qquad \begin{array}{r} 43 \\ -\ 17 \\ \hline 26 \\ {\scriptstyle 1} \end{array} \quad 10 + 20 + 10 = 40$$

### EXERCISES

Solve the equations.

1. $19 - 11 = a$ $\quad 8 + 11 = b$
2. $17 - 5 = c$ $\quad 12 + 5 = d$
3. $21 - 18 = e$ $\quad 3 + 18 = f$
4. $23 - 9 = g$ $\quad 14 + 9 = h$
5. $36 - 20 = i$ $\quad 16 + 20 = j$
6. $46 - 29 = k$ $\quad 17 + 29 = l$

Name the missing addends. Check your work by a separate addition.

7. $\begin{array}{r} 43 \\ -\ 29 \\ \hline \end{array}$
8. $\begin{array}{r} 60 \\ -\ 36 \\ \hline \end{array}$
9. $\begin{array}{r} 80 \\ -\ 48 \\ \hline \end{array}$
10. $\begin{array}{r} 90 \\ -\ 61 \\ \hline \end{array}$
11. $\begin{array}{r} 123 \\ -\ 84 \\ \hline \end{array}$
12. $\begin{array}{r} 102 \\ -\ 69 \\ \hline \end{array}$

Name the missing addends. Check your work by adding up.

13. $\begin{array}{r} 64 \\ -\ 28 \\ \hline \end{array}$
14. $\begin{array}{r} 111 \\ -\ 17 \\ \hline \end{array}$
15. $\begin{array}{r} 92 \\ -\ 24 \\ \hline \end{array}$
16. $\begin{array}{r} 109 \\ -\ 61 \\ \hline \end{array}$
17. $\begin{array}{r} 77 \\ -\ 48 \\ \hline \end{array}$
18. $\begin{array}{r} 123 \\ -\ 84 \\ \hline \end{array}$
19. $\begin{array}{r} 152 \\ -\ 83 \\ \hline \end{array}$
20. $\begin{array}{r} 274 \\ -\ 128 \\ \hline \end{array}$
21. $\begin{array}{r} 306 \\ -\ 208 \\ \hline \end{array}$
22. $\begin{array}{r} 961 \\ -\ 247 \\ \hline \end{array}$
23. $\begin{array}{r} 642 \\ -\ 387 \\ \hline \end{array}$
24. $\begin{array}{r} 879 \\ -\ 296 \\ \hline \end{array}$
25. $\begin{array}{r} 1208 \\ -\ 247 \\ \hline \end{array}$
26. $\begin{array}{r} 1029 \\ -\ 642 \\ \hline \end{array}$
27. $\begin{array}{r} 1175 \\ -\ 879 \\ \hline \end{array}$

## Using columns

Every digit has **place value.** In 436, 4 belongs in the hundreds' place, 3 in the tens' place, and 6 in the ones' place.

| h | t | o |
|---|---|---|
| 4 | 3 | 6 |

When we add or subtract, we must keep the digits in their proper places. We use columns to do this. For example, to name the sum 463 + 26 + 3, we set up columns as in **C**, not as in **A** or **B**.

| **A** | **B** | **C** |
|---|---|---|
| 463 | 463 | 463 |
| 26 | 26 | 26 |
| + 3 | + 3 | + 3 |

### EXERCISES

Name the place value of the 5 in

**1.** 52 **2.** 465 **3.** 506 **4.** 15,784 **5.** 53,264

Use columns to name the sums.

**6.** 347 + 26
**7.** 809 + 48
**8.** 47 + 862
**9.** 3595 + 84
**10.** 468 + 29
**11.** 1027 + 65
**12.** 786 + 29
**13.** 972 + 56
**14.** 593 + 207 + 41 + 28
**15.** 806 + 322 + 92 + 55
**16.** 770 + 86 + 66 + 49
**17.** 637 + 518 + 34 + 58

Use columns to name the differences.

**18.** 447 − 25
**19.** 674 − 25
**20.** 268 − 39
**21.** 548 − 21
**22.** 328 − 56
**23.** 444 − 39
**24.** 281 − 76
**25.** 505 − 24
**26.** 691 − 74
**27.** 843 − 61
**28.** 254 − 37
**29.** 152 − 47
**30.** 993 − 56
**31.** 818 − 37
**32.** 465 − 58
**33.** 468 − 39

Name the sums.

**34.**
484
27
+ 3

**35.**
682
96
+ 284

**36.**
729
38
+ 461

**37.**
29
386
+ 204

**38.**
14
28
+ 486

**39.**
354
27
+ 461

Name the sums.

**40.**
23
76
48
+ 52

**41.**
538
25
396
+ 9

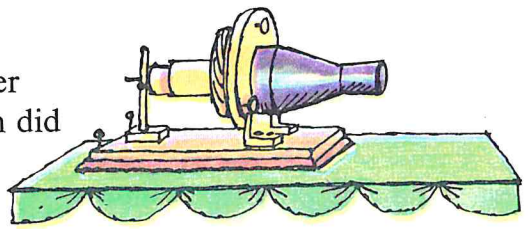
**42.**
756
285
33
+ 584

**43.**
4581
268
7913
+ 509

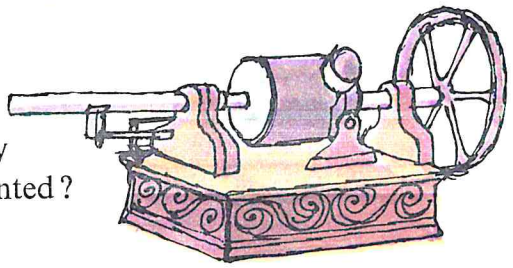
**44.**
9207
540
894
+ 3078

## Famous inventions

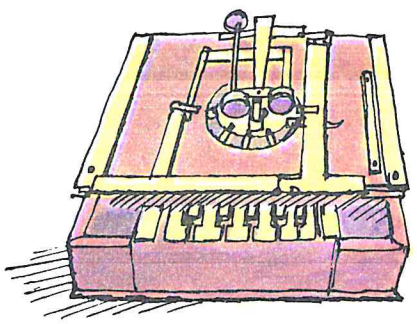
1. Whitney invented the cotton gin in 1793. How many years ago was that?



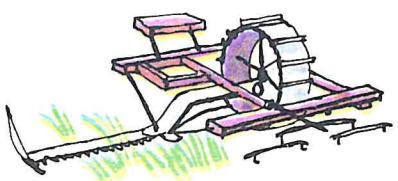
2. Bell invented the telephone 83 years after Whitney invented the cotton gin. When did Bell invent the telephone?



3. Edison invented the phonograph in 1877. Berliner invented records in 1887. How many years after the phonograph were records invented?



4. An adding machine was invented by Pascal in 1642. Two hundred twenty-two years later the typewriter was invented. When was the typewriter invented?

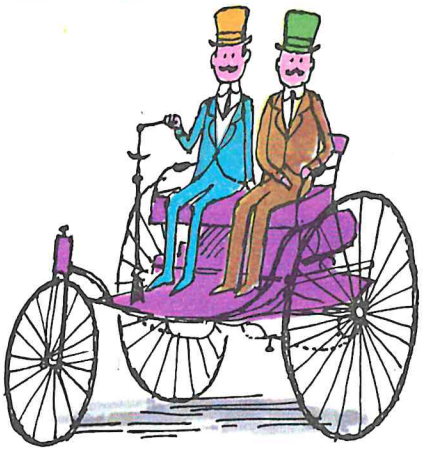


5. The first lawnmower was built in 1868.
   a. How many years ago was that?
   b. How many years was that after Pascal's adding machine had been invented?

6. The balloon was invented in 1783. The first rocket engine was invented 146 years later. When was the rocket engine invented?

1. The first bicycle was invented in 1839 by Macmillan. It had a large back wheel and a small front wheel.

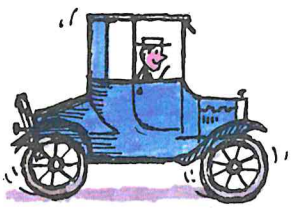
2. Later on, a bicycle with a large front wheel and a small back wheel became popular.



3. The gasoline automobile was invented in 1885 in Germany by Benz. It weighed only 550 pounds, had 3 wheels, and traveled at a speed of 8 miles per hour.

4. One of the oldest gasoline cars with 4 wheels was built in 1887 by Hammel. In 1954 it was still running and could travel at an average speed of 5 miles per hour.

5. One of the most inexpensive cars of all times was the Model-T Ford. In 1923, a person could buy one for about $290.



6. In 1960, a person could buy an automobile for $2,500. The car weighed about 3300 pounds and could travel over 80 miles per hour.

## Coins

These are the common coins in use.

**penny**

**nickel**

**dime**

**quarter**

**half-dollar**

### EXERCISES

Name the total values in cents.

1. 2 dimes and 1 penny
2. 2 quarters and 1 penny
3. 3 nickels and 1 penny
4. 1 half-dollar, 1 quarter and 1 dime

Name the total values in cents.

5.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **pennies** | 3 | 3 | 2 | 1 | | | 4 | 2 | 4 |
| **nickels** | 2 | | 1 | | 1 | | 1 | | 1 |
| **dimes** | 1 | | | 2 | | 2 | | 3 | |
| **quarters** | | 3 | 3 | | 1 | | 2 | | 3 |
| **half-dollars** | 1 | 1 | | 2 | | 3 | 1 | 2 | |
| **total value in cents** | 73 | *a* | *b* | *c* | *d* | *e* | *f* | *g* | *h* |

You can make a total of 37 cents by using 1 quarter, 1 dime, and 2 pennies or by using 3 dimes, 1 nickel, and 2 pennies.

Name two ways of making each total.

6. 12 cents
7. 27 cents
8. 36 cents
9. 14 cents
10. 41 cents
11. 45 cents

(You may show pennies as **p**, nickels as **n**, dimes as **d**, quarters as **qr**.)

Show how to make each total value using the fewest number of coins.

**12.** 27¢ **13.** 17¢ **14.** 33¢ **15.** 42¢ **16.** 62¢ **17.** 47¢

one dollar bill

five dollar bill

## EXERCISES

Name the total values in cents.

**1.**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **penny** | 2 | 4 | | 2 | | 6 | 3 | 8 | 1 |
| **dime** | 2 | 1 | 6 | 2 | 8 | 3 | | 7 | 5 |
| **one dollar** | 2 | 2 | 3 | 1 | 2 | 4 | 4 | 2 | 1 |
| **five dollars** | 1 | 2 | 3 | 1 | 2 | 4 | 2 | 1 | 3 |
| **total value in cents** | 722 | ***a*** | ***b*** | ***c*** | ***d*** | ***e*** | ***f*** | ***g*** | ***h*** |

Name the total values in cents.

**2.**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **penny** | 2 | 3 | 2 | 1 | 4 | 1 | | 2 | 3 |
| **nickel** | 1 | 1 | | 2 | | 1 | | 3 | |
| **dime** | 3 | | 1 | 2 | | 3 | | 2 | 4 |
| **one dollar** | 2 | 3 | 4 | 5 | 6 | 1 | 8 | 9 | 14 |
| **total value in cents** | 237 | ***a*** | ***b*** | ***c*** | ***d*** | ***e*** | ***f*** | ***g*** | ***h*** |

$3.45 is read *three dollars and forty-five cents.*
Use the dollar sign ($) and the point (.) to write each of these.

**3.** 2 dollars and 26 cents
**4.** 3 dollars and 85 cents
**5.** 4 dollars and 16 cents
**6.** 5 dollars and 26 cents
**7.** 4 dollars and 98 cents
**8.** 60 dollars and 17 cents

Name the total values in cents.

**9.** $3.45
**10.** $6.25
**11.** $ .46
**12.** $3.40
**13.** $23.45
**14.** $18.29
**15.** $57.68
**16.** $11.39
**17.** $16.87
**18.** $97.25

## Checking the cost

$7.95 means *seven dollars and ninety-five cents.* The point separates the dollars from the cents.

Sharon said, "The total is fourteen dollars and two cents." She wrote the total as $14.02.

### EXERCISES

Name the sums and differences in cents.

| | | | | | |
|---|---|---|---|---|---|
| **1.** 34¢ + 62¢ | **3.** 87¢ + 53¢ | **5.** 65¢ + 57¢ | **7.** 83¢ − 27¢ | **9.** 36¢ − 17¢ | **11.** 85¢ − 37¢ |
| **2.** 95¢ + 51¢ | **4.** 36¢ + 77¢ | **6.** 72¢ + 46¢ | **8.** 24¢ − 11¢ | **10.** 72¢ − 9¢ | **12.** 55¢ − 27¢ |

If the number of cents is greater than 100, we usually use the dollar sign ($) and the point (.).
The sum in exercise 2 is written $1.46.

**13–16.** Name the sums for exercises 3, 4, 5, and 6 in dollars.

Name the sums and differences in dollars and cents.

| | | | | | |
|---|---|---|---|---|---|
| **17.** $.96 + .83 | **20.** $.38 + .66 | **23.** $2.46 + 3.82 | **26.** $ .38 + 2.49 | **29.** $4.62 − 2.39 | **32.** $8.27 − 1.79 |
| **18.** $.74 + .86 | **21.** $.72 + .70 | **24.** $4.62 + 1.77 | **27.** $4.63 + 2.05 | **30.** $6.52 − 3.19 | **33.** $7.04 − 3.57 |
| **19.** $.92 + .08 | **22.** $.39 + .84 | **25.** $4.28 + .73 | **28.** $9.06 + 2.48 | **31.** $6.20 − 4.70 | **34.** $5.64 − .75 |

These are the amounts on tapes at the supermarket.
Name the total costs.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | $ .17 | **3.** | $ .31 | **5.** | $3.19 | **7.** | $2.45 | **9.** | $3.24 | **11.** | $1.45 |
| | 2.43 | | .31 | | .17 | | 1.19 | | .21 | | 2.36 |
| | .28 | | .27 | | .21 | | .64 | | .19 | | 4.20 |
| | .36 | | .35 | | .27 | | .23 | | .19 | | .28 |
| | .17 | | .27 | | .27 | | .27 | | .37 | | .17 |
| | .21 | | 1.46 | | .27 | | .27 | | 2.24 | | .11 |
| **2.** | $ .23 | **4.** | $2.30 | **6.** | $ .31 | **8.** | $ .29 | **10.** | $ .46 | **12.** | $1.42 |
| | .44 | | 3.50 | | .31 | | .27 | | .36 | | 2.25 |
| | .39 | | 1.10 | | .29 | | 2.25 | | .36 | | .37 |
| | 1.86 | | .14 | | 4.20 | | .37 | | .29 | | .19 |
| | .19 | | .21 | | 3.80 | | .83 | | 1.00 | | .21 |
| | .13 | | .21 | | .25 | | .24 | | .46 | | .17 |

Name the differences.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **13.** | $3.46 | **15.** | $8.52 | **17.** | $12.60 | **19.** | $4.81 | **21.** | $16.34 | **23.** | $10.60 |
| | − 1.26 | | − 4.46 | | − 3.48 | | − 1.90 | | − 6.28 | | − 2.24 |
| **14.** | $4.60 | **16.** | $5.70 | **18.** | $9.63 | **20.** | $3.80 | **22.** | $3.17 | **24.** | $4.00 |
| | − 3.19 | | − 5.45 | | − 2.70 | | − 2.69 | | − 2.26 | | − 3.89 |

There are six toys in the toy shop window, A, B, C, D, E, and F.

A costs $7.50
B costs $3.45
C costs $9.60
D costs $12.30
E costs $8.75
F costs $7.25

**25.** What is the difference in price between the most expensive toy and the least expensive toy?

**26.** How much more
**a.** is A than B? **b.** is D than C?

**27.** Name the total cost of A, B, D, and E.

**28.** Name the total cost of A, B, C, and F.

**29.** Which costs more, A, B, and C together or D, E, and F together?
How much more?

1. In the first hour a train went 63 miles, and in the second hour it went 59 miles.
   a. How many miles did it go in the two hours?
   b. How many fewer miles did it go in the second hour than in the first?

2. If you want to collect 200 shells and have 164 already, how many more do you need to complete your collection?

3. One small herd of cows gave 346 gallons of milk in a week. Another herd gave 580. How many more gallons did the second herd give?

4. Ellen scored 86, 93, 74, 82, and 69 on five tests.
   a. What is the difference between her highest and her lowest scores?
   b. What is the sum of her scores for the five tests?

5. When you are excited, your heart may beat as fast as 150 times a minute. Usually, your heart beats about 70 times a minute.
   a. What is the difference between the two rates?
   b. Can your heart beat more than twice as fast as normal when you are excited?

6. There are 243 passengers on the upper deck of a boat, and 348 on the lower deck.
   a. How many more are there on the lower deck?
   b. How many are there all together?

Copy and complete the equations.

**1.** $17 + 3 = 3 + a$ **2.** $39 + 18 = 18 + b$ **3.** $427 + 64 = 64 + c$

**4.** $4 + 9 = 4 + (d + 3) = (4 + d) + 3 = e + 3 = f$

**5.** $14 + 5 = (10 + g) + 5 = 10 + (g + 5) = 10 + h = i$

**6.** $24 + 5 = (20 + j) + 5 = 20 + (j + 5) = 20 + k = l$

**7.** $23 + 8 = (20 + m) + 8 = 20 + (m + 8) = 20 + n = p$

Copy and complete the examples.

**8.**
$$\begin{array}{r} 63 = 60 + a \\ +\ 29 = 20 + v \\ \hline c + x = e \end{array}$$

**9.**
$$\begin{array}{r} 47 = 40 + s \\ +\ 58 = 50 + x \\ \hline e + i = n \end{array}$$

**10.**
$$\begin{array}{r} 35 = 30 + u \\ +\ 76 = 70 + a \\ \hline p + n = s \end{array}$$

Name the sums and differences.

**11.** $\begin{array}{r} 38 \\ +\ 47 \\ \hline \end{array}$ **13.** $\begin{array}{r} 384 \\ +\ 296 \\ \hline \end{array}$ **15.** $\begin{array}{r} 469 \\ +\ 327 \\ \hline \end{array}$ **17.** $\begin{array}{r} 5472 \\ +\ 3189 \\ \hline \end{array}$ **19.** $\begin{array}{r} 36686 \\ +\ 25296 \\ \hline \end{array}$

**12.** $\begin{array}{r} 47 \\ -\ 29 \\ \hline \end{array}$ **14.** $\begin{array}{r} 485 \\ -\ 329 \\ \hline \end{array}$ **16.** $\begin{array}{r} 925 \\ -\ 814 \\ \hline \end{array}$ **18.** $\begin{array}{r} 6543 \\ -\ 2395 \\ \hline \end{array}$ **20.** $\begin{array}{r} 46896 \\ -\ 32814 \\ \hline \end{array}$

Name the total values in cents.

**21.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **penny** | 2 | 4 | 1 | | 1 | 2 | 2 | 3 |
| **nickel** | 1 | 2 | | 3 | 1 | | 3 | 4 |
| **dime** | 3 | 1 | 3 | 1 | 1 | 1 | 2 | |
| **one dollar** | 2 | 1 | 3 | 4 | 1 | 2 | 3 | 5 |
| **total value in cents** | 237 | *a* | *b* | *c* | *d* | *e* | *f* | *g* |

Name the sums and differences.

**22.** $\begin{array}{r} \$2.46 \\ +\ 1.37 \\ \hline \end{array}$ **24.** $\begin{array}{r} \$6.93 \\ +\ 1.72 \\ \hline \end{array}$ **26.** $\begin{array}{r} \$1.34 \\ .22 \\ .31 \\ .42 \\ \hline \end{array}$ **27.** $\begin{array}{r} \$2.37 \\ .14 \\ .21 \\ .35 \\ .67 \\ 1.23 \\ \hline \end{array}$

**23.** $\begin{array}{r} \$8.52 \\ -\ 4.75 \\ \hline \end{array}$ **25.** $\begin{array}{r} \$3.78 \\ -\ 1.59 \\ \hline \end{array}$

## LOOKING BACK

To add we rename two addends as the sum. To subtract we rename a sum and an addend as the missing addend.

Complete the number chart.

**1.**

| sum | *a* | *b* | *c* | 15 | 7 | 16 | 12 | 12 | 10 | *j* | *k* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| addend | 3 | 6 | 4 | 8 | *e* | 7 | 9 | *h* | *i* | 4 | 9 |
| addend | 6 | 7 | 9 | *d* | 1 | *f* | *g* | 7 | 8 | 8 | 9 |

**The Associative Property:** the way the addends are grouped does not affect the sum.

Solve the equations.

**2.** $3 + (4 + 2) = (3 + 4) + a$ **3.** $(7 + 3) + b = 7 + (3 + 5)$

**4.** $12 + 5 = (10 + 2) + 5 = 10 + (2 + 5) = 10 + 7 = c$

**5.** $24 + 2 = (20 + 4) + 2 = 20 + (d + e) = 20 + f = g$

**The Commutative Property:** the order of the addends does not affect the sum.

Solve the equations.

**6.** $7 + 2 = 2 + a$ **7.** $9 + 4 = b + 9$ **8.** $8 + 0 = 0 + c$

We use place value, the basic facts, and the associative and commutative properties to add and subtract greater numbers.

Name the sums and differences.

**9.** $10 + 80$ **11.** $30 + 20$ **13.** $600 + 100$ **15.** $423 + 271$ **17.** $528 + 236$ **19.** $784 + 637$

**10.** $60 - 20$ **12.** $90 - 30$ **14.** $700 - 500$ **16.** $568 - 235$ **18.** $372 - 127$ **20.** $923 - 665$

Can you add 28 and 34 in your head?

Try this method: $28 + 4 = 32$, $32 + 30 = 62$ or this method: $28 + 30 = 58$, $58 + 4 = 62$

Can you subtract 34 from 62 in your head?

Try this method: $62 - 30 = 32$, $32 - 4 = 28$ or this method: $62 - 4 = 58$, $58 - 30 = 28$

Name the sums and differences in your head.

**1.** $39 + 53$ **2.** $43 + 98$ **3.** $324 + 297$ **4.** $67 - 21$ **5.** $84 - 39$ **6.** $421 - 239$

Here is another method for adding. Study the examples.

| | 3 | 6 | 8 |
|---|---|---|---|
| | 8 | 9 | 5 |
| | 1 1 | 1 5 | 1 3 |
| 1 | 2 | 6 | 3 |

$$\begin{array}{r} 368 \\ +\ 895 \\ \hline 1263 \end{array}$$

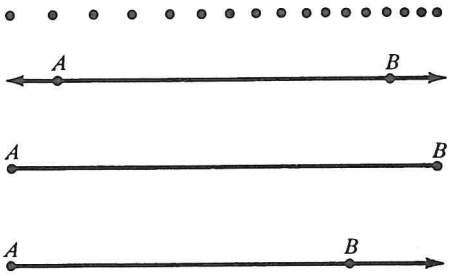
| | 6 | 5 | 9 | 3 |
|---|---|---|---|---|
| | 7 | 4 | 8 | 1 |
| | 1 3 | 0 9 | 1 7 | 0 4 |
| 1 | 4 | 0 | 7 | 4 |

$$\begin{array}{r} 6593 \\ +\ 7481 \\ \hline 14074 \end{array}$$

Use this method to name the sums.

**7.** $384 + 762$ **8.** $983 + 416$ **9.** $6824 + 3986$ **10.** $2954 + 1298$ **11.** $7931 + 2427$ **12.** $7983 + 8793$

## Points, curves, lines, and rays




A **curve** is a set of connected points.

A **line,** like $\overleftrightarrow{AB}$, is straight. It goes on and on in two directions.

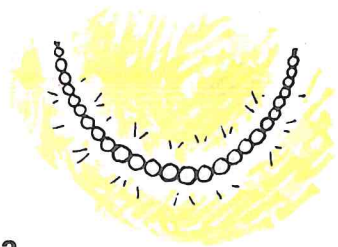
A **line segment,** like $\overline{AB}$, is part of a line. It has two end points, $A$ and $B$.

A **ray,** like $\overrightarrow{AB}$, is part of a line. It has one end point, $A$, and goes on and on in one direction.

### EXERCISES

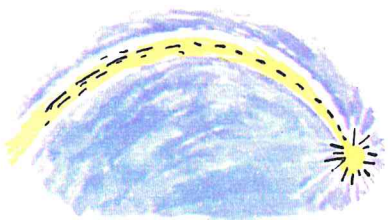
For each picture, write *curve, line, line segment,* or *ray* to show which shape it suggests.

1.

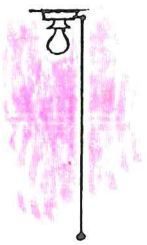


2.

3.

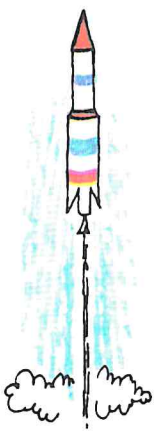


4.



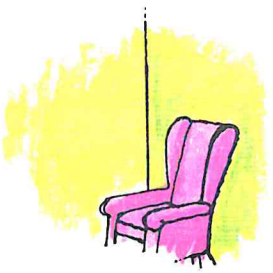
5.

6.

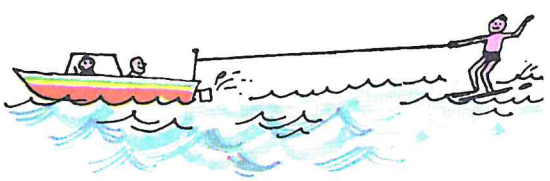


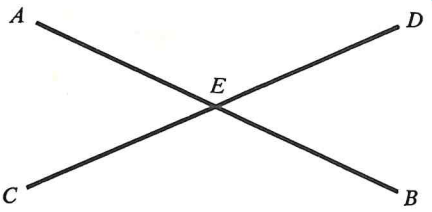
7.

8.



9.



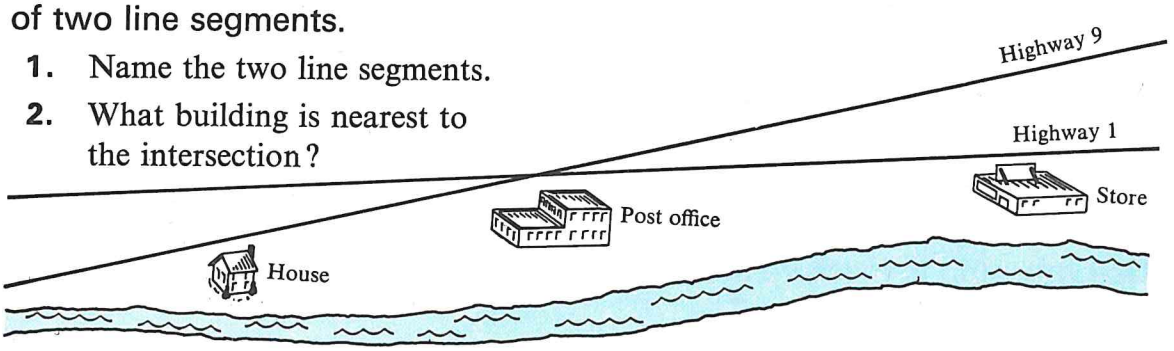



The point $E$ is common to $\overline{AB}$ and $\overline{CD}$.
This point is the **intersection** of $\overline{AB}$ and $\overline{CD}$.

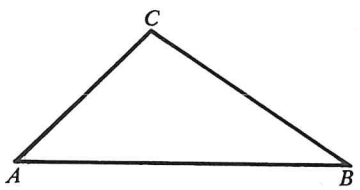
## EXERCISES

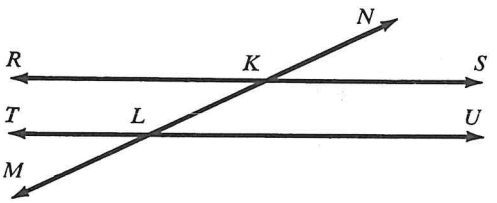
The map suggests the intersection of two line segments.

1. Name the two line segments.
2. What building is nearest to the intersection?




Name the intersection

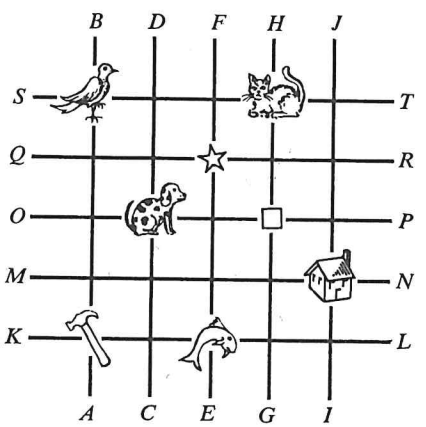



3. of $\overline{AB}$ and $\overline{BC}$
4. of $\overline{AB}$ and $\overline{AC}$
5. of $\overrightarrow{RS}$ and $\overrightarrow{MN}$
6. of $\overrightarrow{TU}$ and $\overrightarrow{MN}$

Name the object shown at the intersection of

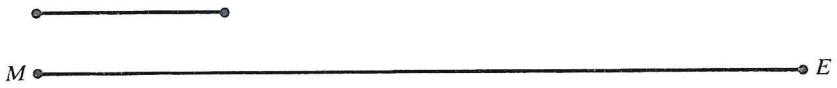
7. $\overline{AB}$ and $\overline{ST}$
8. $\overline{EF}$ and $\overline{QR}$
9. $\overline{IJ}$ and $\overline{MN}$
10. $\overline{AB}$ and $\overline{KL}$
11. $\overline{CD}$ and $\overline{OP}$
12. $\overline{GH}$ and $\overline{ST}$
13. $\overline{EF}$ and $\overline{KL}$
14. $\overline{GH}$ and $\overline{OP}$




Answer the questions.

15. Will two lines always intersect?
16. Can two lines intersect in more than one point?

## Measuring line segments




To measure the **length** of a line segment, we match a unit of length with the line segment.

The **measure** is a number. It tells how many **units** match the line segment.

The measure of $\overline{ME}$ is 4.
$m(\overline{ME}) = 4$

The **length** of $\overline{ME}$ is 4 units.

### EXERCISES

Cut a strip to match the length of this line segment.
This unit is an **inch**.



Use your strip to measure these line segments. For example, write $m(\overline{AB}) = 2$.

1. 


2. 


3. 


4. 

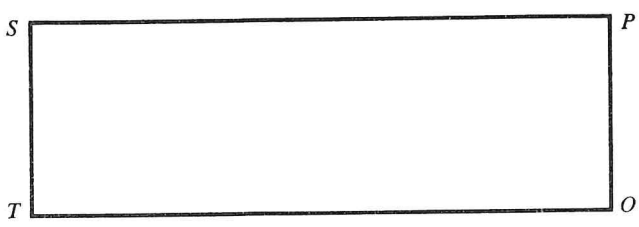

5. 

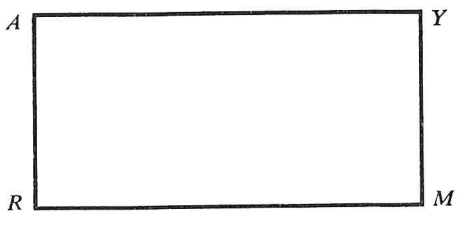

6. 


> Two line segments of the same length are **congruent.** In the exercises above, $\overline{AB}$ and $\overline{EF}$ are congruent. $m(\overline{AB}) = m(\overline{EF}) = 2$
> The length of $\overline{AB}$ and of $\overline{EF}$ is 2 inches.

7. Name two other pairs of congruent line segments in exercises 1 to 6.

Measure each line segment, using your inch strip as the unit.



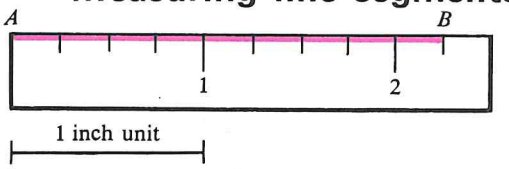

8. a. $m(\overline{ST})$ c. $m(\overline{OP})$
   b. $m(\overline{TO})$ d. $m(\overline{PS})$

9. a. $m(\overline{AR})$ c. $m(\overline{MY})$
   b. $m(\overline{RM})$ d. $m(\overline{YA})$

10. Name the pairs of congruent line segments in exercises 8 and 9.

## Measuring line segments

We use a **ruler** to measure the length of line segments.




The unit of measure is an **inch.** The ruler also shows $\frac{1}{4}$ inch, $\frac{1}{2}$ inch, and $\frac{3}{4}$ inch.

$m(\overline{AB}) = 2\frac{1}{4}$

Since the unit is an inch, the length of $\overline{AB}$ is $2\frac{1}{4}$ inches.

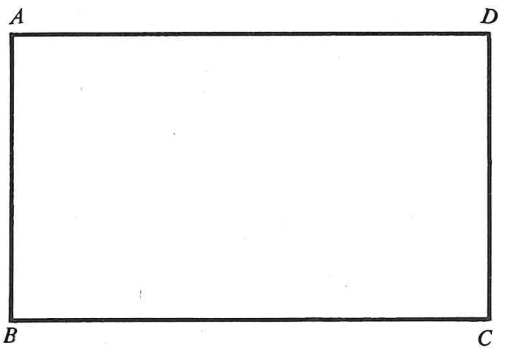
## EXERCISES

With the inch as the unit, use your ruler to draw line segments of these measures.

**1.** $m(\overline{TO}) = 3$ **3.** $m(\overline{RS}) = 7$ **5.** $m(\overline{AB}) = 8\frac{3}{4}$ **7.** $m(\overline{KJ}) = 6\frac{1}{4}$

**2.** $m(\overline{CD}) = 4\frac{1}{2}$ **4.** $m(\overline{LM}) = 5\frac{1}{4}$ **6.** $m(\overline{EF}) = 9$ **8.** $m(\overline{YZ}) = \frac{3}{4}$

Answer the questions for figure *ABCD*.

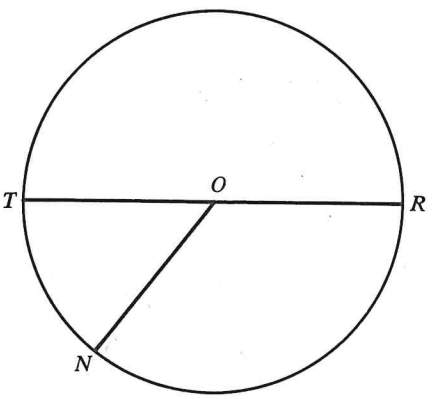



**9.** Measure each line segment in inches.
- **a.** $m(\overline{AB})$
- **b.** $m(\overline{BC})$
- **c.** $m(\overline{CD})$
- **d.** $m(\overline{DA})$

**10.** To get from $A$ to $C$, is it shorter to go by way of $B$ or by way of $D$?

**11.** Name the pairs of congruent line segments in *ABCD*.

Answer the questions for the figure.




**12.** Measure each line segment in inches.
- **a.** $m(\overline{OR})$
- **b.** $m(\overline{ON})$
- **c.** $m(\overline{QT})$
- **d.** $m(\overline{RT})$

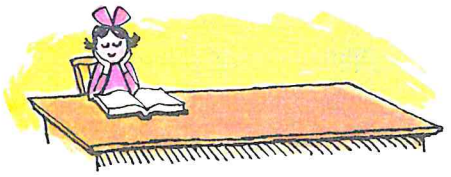
**13.** Which of the line segments are congruent?

Answer the questions.

**14.** Can you measure a line? Why or why not?

**15.** Can you measure a ray? Why or why not?

## Planes

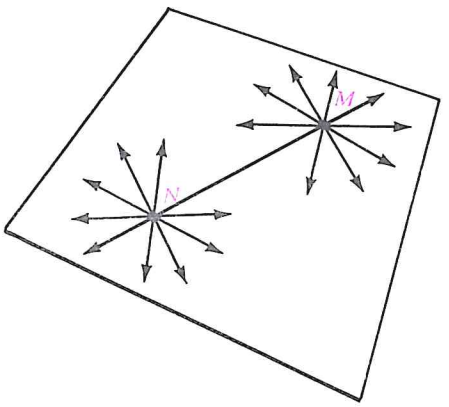


The top of your desk can be thought of as part of a **plane.**

A plane is a flat surface that goes on and on in all directions.

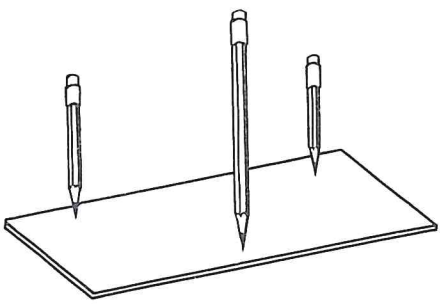
The surface of the back of your hand is not flat, so it cannot be thought of as part of a plane.

### EXERCISES




*M* and *N* are two points in a plane.

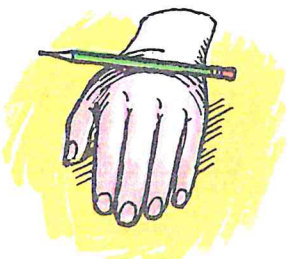
1. How many lines can be drawn through point *M*?
2. How many lines can be drawn through point *N*?
3. How many lines pass through both *M* and *N*?
4. Will all the points in $\overleftrightarrow{MN}$ lie in the plane?



Use a flat piece of cardboard and three pencils as in the diagram to answer the questions.

5. How many planes can pass through 3 points?

*6. Why may a four-legged stool rock while a three-legged stool never will?

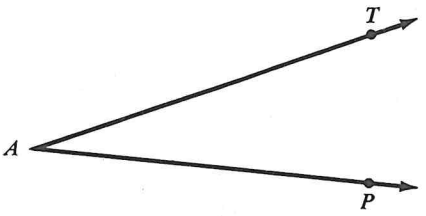


For each statement, write *T* or *F* to show whether it is *true* or *false*.

7. If you place a pencil on a flat surface, each point of the pencil will rest on the surface.
8. If you place a pencil on the back of your hand, each point of the pencil will rest on your hand.

The *union of two rays* with a common end point is called an **angle.**

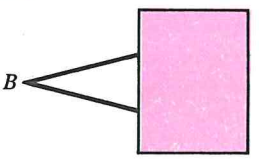
The angle shown here may be named $\angle TAP$ or $\angle PAT$. Point $A$ is the **vertex** of the angle. The rays $\overrightarrow{AT}$ and $\overrightarrow{AP}$ are the **sides** of the angle.

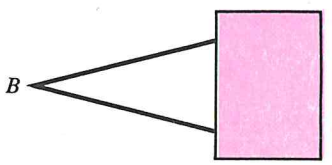



## EXERCISES

### Answer the questions.

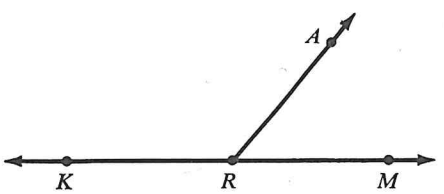
Mary saw the angle with vertex $B$. Part of each ray was covered by a piece of paper. She moved the paper, as shown here.




1. Did Mary see the same rays each time?
2. Did Mary see the same angle each time?

### $\overleftrightarrow{KM}$ is a line.

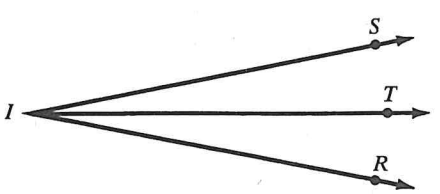



3. Name three rays that have $R$ as the end point.
4. Name two angles.

### Study the slides.

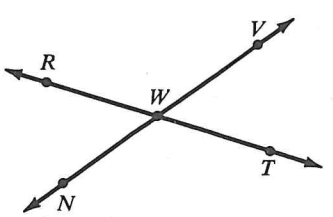
5. Does a slide form an angle with the ground?
6. Which is the faster slide?

### Study the diagram at the right.



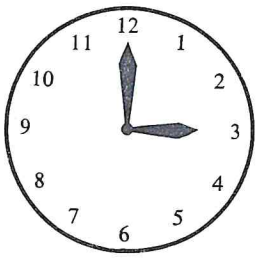

7. Name three rays.
8. Name two angles with a common ray.
9. Name the vertex of $\angle RIS$.

### Study the diagram at the right.

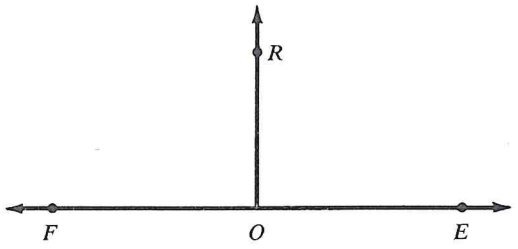



10. Name two lines.
11. Name four rays.
12. Name four angles.

## Right angles




The hands of the clock form a **right angle.**

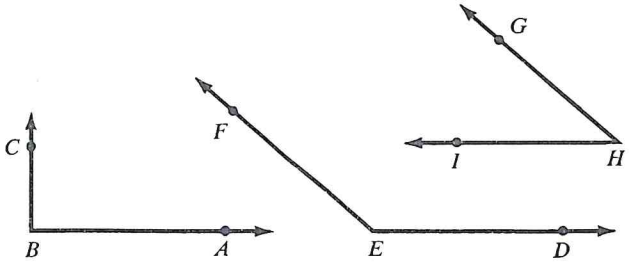


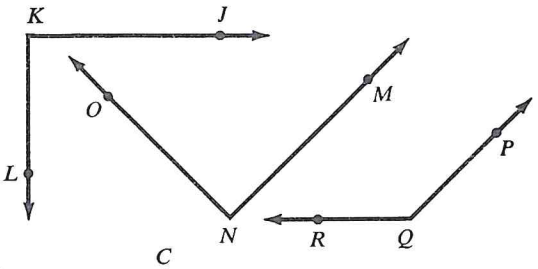

$\overrightarrow{OR}$ forms two right angles with $\overleftrightarrow{FE}$. $\overrightarrow{OR}$ is **perpendicular** to $\overleftrightarrow{FE}$.

## EXERCISES

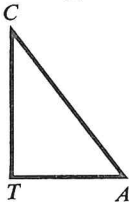
Answer the questions.

1. Is the edge at the left side of this page perpendicular to the edge at the bottom of this page?
2. Is the edge at the left side of this page perpendicular to the edge at the top of this page?
3. Is the edge at the top of this page perpendicular to the edge at the bottom of this page?
4. There are three right angles below. Name them.

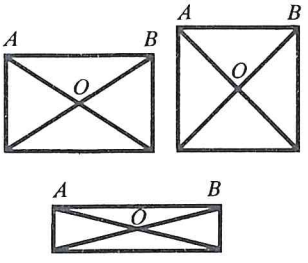



5. Copy this figure. $\angle CTA$ is a right angle. Using the inch as your unit, make $m(\overline{TC}) = 4$ and $m(\overline{TA}) = 3$. Name $m(\overline{CA})$.




6. Betty took two strips of cardboard of the same length and pinned them together at their centers. She could form different angles with them. She drew four line segments to join the ends of the strips as shown here.

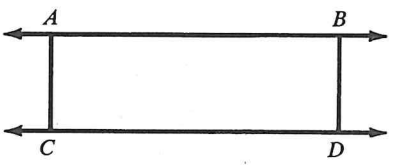



   a. In which figure is $\angle AOB$ a right angle?
   b. What do you notice about that figure?

Two lines in the same plane that never meet to form an angle are **parallel.** The distance between two parallel lines is always the same.

$\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$.

$\overline{AC}$ and $\overline{BD}$ are the same length.




## EXERCISES

Answer the question.

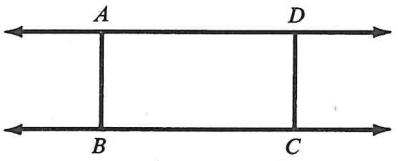
**1.** Mr. Kennison is measuring the width of the path. The path is 4 feet wide at each end. The sides of the path are straight.

Are the sides of the path parallel?

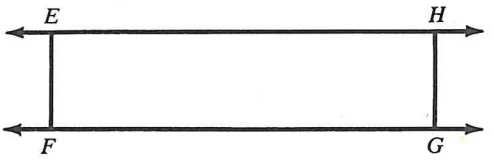
Measure the line segments $\overline{AB}$, $\overline{CD}$, $\overline{EF}$, and $\overline{GH}$.

**2.** Which of these two pairs of line segments are parallel?

**a.**




**b.**




Measure the line segments.

**3.** Which of these two pairs of line segments are parallel?

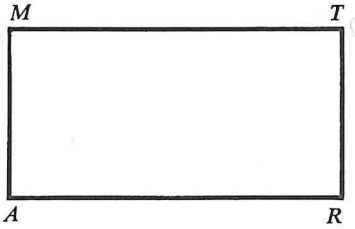
**a.**

**b.**

Follow the directions. Then answer the question.

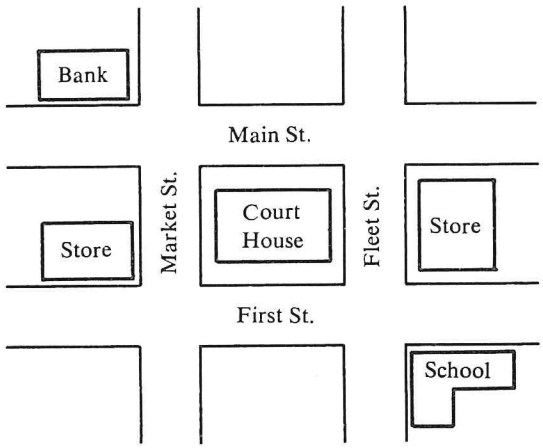
**4.** Draw a line segment and label it $\overline{AR}$. Draw $\angle RAM$ and $\angle ART$ right angles. Make both $\overline{MA}$ and $\overline{TR}$ 2 inches long. Draw $\overline{MT}$.

Will $\overline{MT}$ be parallel to $\overline{AR}$?



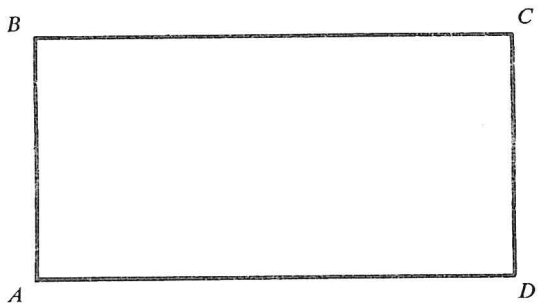
## Parallel lines

Study this street map.

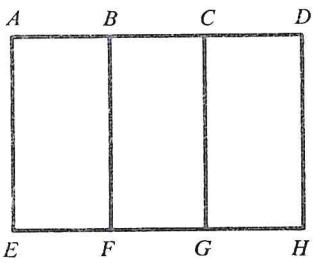



1. Name a street
   a. parallel to Main Street.
   b. perpendicular to Main Street.
2. Name the streets which intersect by the school.
3. a. Name two streets which are perpendicular to First Street.
   b. Are they parallel to each other?

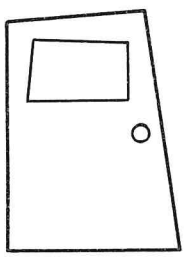
Use the diagrams at the right.




4. $\overline{AB}$ is congruent to ___?___.
5. $\overline{BC}$ is congruent to ___?___.
6. $\overline{AB}$ is perpendicular to ___?___.
7. $\overline{BC}$ is perpendicular to ___?___.
8. $\overline{AB}$ is parallel to ___?___.
9. $\overline{BC}$ is parallel to ___?___.

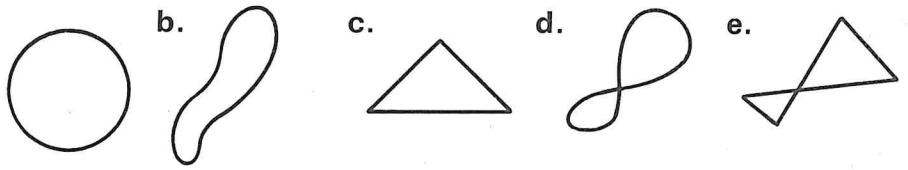



10. Measure each line segment in inches.
    a. m($\overline{AB}$) f. m($\overline{BD}$) k. m($\overline{EG}$)
    b. m($\overline{AC}$) g. m($\overline{BF}$) l. m($\overline{EH}$)
    c. m($\overline{AD}$) h. m($\overline{CD}$) m. m($\overline{FG}$)
    d. m($\overline{AE}$) i. m($\overline{CG}$) n. m($\overline{FH}$)
    e. m($\overline{BC}$) j. m($\overline{EF}$) o. m($\overline{GH}$)
11. List the congruent line segments.
12. Name a line segment parallel to $\overline{AD}$.
13. Name three line segments parallel to $\overline{AE}$.



14. What is unusual about the door at the right?
15. Name the capital letters of the alphabet that have parallel or perpendicular line segments. For example, F has two perpendicular line segments and two parallel line segments.

a. b. c. d. e.

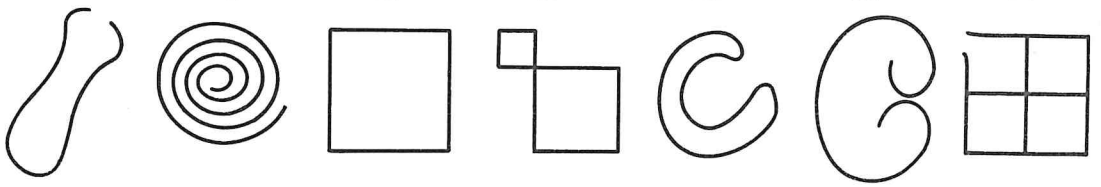


These are all **closed curves.** **a**, **b**, and **c** are **simple closed curves.** **d** and **e** are closed curves, but they are not simple closed curves.

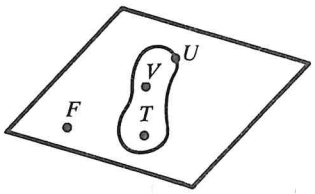
## EXERCISES

For each figure, write *SCC* if it is a *simple closed curve*, *CC* if it is a *closed curve* but not simple, or *C* if it is a *curve* but not closed.

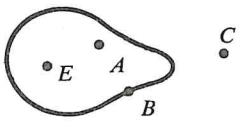
1. 2. 3. 4. 5. 6. 7.



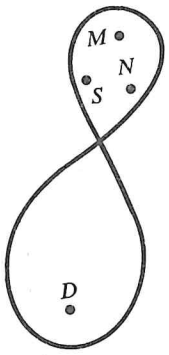
*U* is a point **on** the simple closed curve. *F* is a point **outside** the simple closed curve. *T* and *V* are points **inside** the simple closed curve.




Answer the questions.



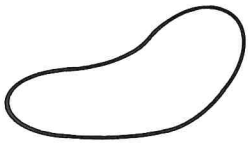

8. Can you join *A* to *E* without intersecting the closed curve?
9. Can you join *A* to *C* without intersecting the closed curve?
10. This figure is a closed curve but not a simple closed curve. Is it always possible to join two points inside a closed curve without intersecting the curve?



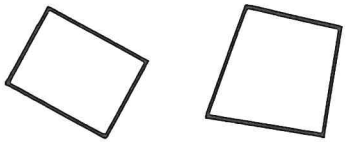

Complete the sentences with *can* or *cannot*.

11. Two points inside a simple closed curve ___?___ be joined without intersecting the curve.
12. A point inside a simple closed curve and a point outside the curve ___?___ be joined without intersecting the curve.

## Special simple closed curves



**Simple closed curves**

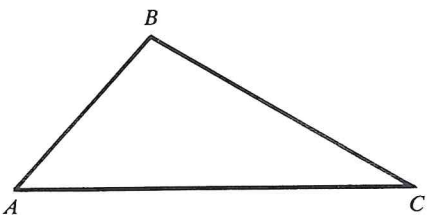


**Polygons**

A **polygon** is a simple closed curve formed by the union of line segments. The line segments that form a polygon are called its **sides.** A polygon has 3 or more sides. The corner points of a polygon are called **vertices.**

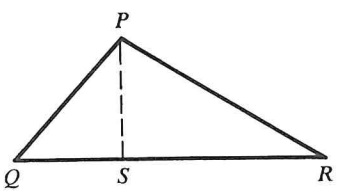
### EXERCISES

Use the figures to help you.



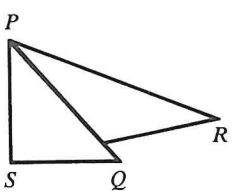

1. How many sides does a triangle have?
2. How many angles are in a triangle?
3. How many vertices are in a triangle?
4. What does the *tri* mean in the word *triangle*?

Draw a triangle on a piece of paper and cut it out. Fold the paper so that the fold passes through the vertex at $P$ and part of the bottom side falls along the other part of the bottom side, as the drawing suggests.




5. What kind of angle is $\angle PSR$?

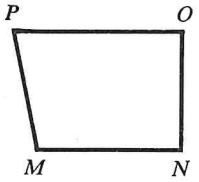
In folding the triangular piece of paper, you have formed two triangles with the same kind of angle.



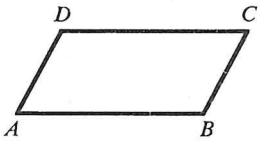

6. What kind of angles are $\angle PSR$ and $\angle PSQ$?
7. What do you call a triangle with this kind of angle?

Draw two line segments to form a right angle. Form a triangle with this angle by making one line segment 3 inches long and the other line segment 4 inches long. Connect the ends of the line segments.

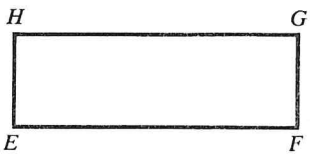
8. What kind of triangle have you drawn?
9. What is the length of the third side of the triangle?



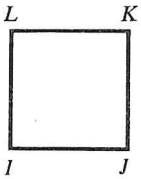

**Quadrilateral**
*MNOP*




**Parallelogram**
*ABCD*




**Rectangle**
*EFGH*

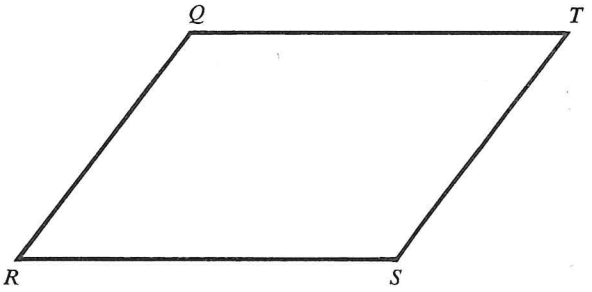



**Square**
*IJKL*

These polygons are all **quadrilaterals** because they all have four sides. Some quadrilaterals have special names. A quadrilateral, like all polygons, can be labeled by the vertices. For example, the rectangle can be named *EFGH*.

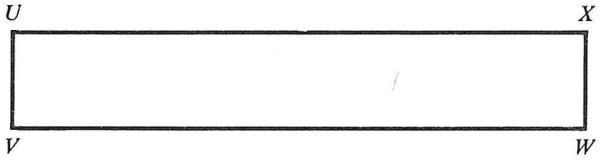
## EXERCISES

*QRST* is a parallelogram.




1. Name the parallel sides.
2. Can you see why this is called a **parallelogram?**
3. Measure each side in inches.
   - **a.** m($\overline{QR}$)
   - **b.** m($\overline{ST}$)
   - **c.** m($\overline{RS}$)
   - **d.** m($\overline{TQ}$)
4. Name the congruent sides.

*UVWX* is a rectangle.




5. Measure each side in inches.
   - **a.** m($\overline{UV}$)
   - **b.** m($\overline{WX}$)
   - **c.** m($\overline{VW}$)
   - **d.** m($\overline{XU}$)
6. Name the congruent sides.
7. How many right angles does a rectangle have?

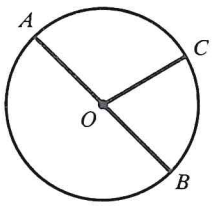
The **distance around** a polygon is called its **perimeter.** The perimeter is found by adding the lengths of all the sides of the polygon.

8. Name the perimeter of parallelogram *QRST*.
9. Name the perimeter of rectangle *UVWX*.

## Circles

A **circle** is a simple closed curve with all points the same distance from the **center** $O$.

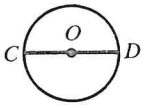
Any line segment with one end point at the center and the other on the circle is a **radius.**
Any line segment that goes through the center and has end points on the circle is a **diameter.**

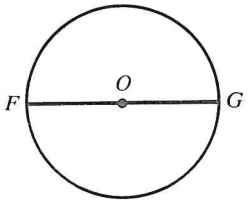


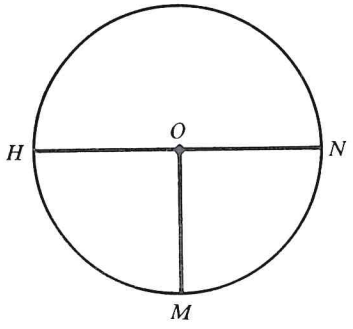

$\overline{OC}$ is a radius and $\overline{AB}$ is a diameter of the circle.

### EXERCISES

Measure each radius and diameter in inches.




1. m($\overline{CO}$)
2. m($\overline{OD}$)
3. m($\overline{CD}$)
4. m($\overline{FO}$)
5. m($\overline{OG}$)
6. m($\overline{FG}$)
7. m($\overline{HO}$)
8. m($\overline{ON}$)
9. m($\overline{OM}$)
10. m($\overline{HN}$)

Complete the sentences.

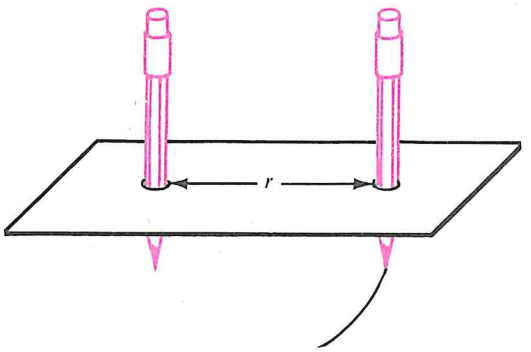
11. The length of a diameter of a circle is ___?___ times the length of a radius of a circle.
12. If you tie a weight to the end of a string and whirl the string around, the path the weight follows is a ___?___.

> The part of a plane inside a closed curve is a **region.** The region inside a triangle is a *triangular region;* the region inside a square is a *square region;* and the region inside a circle is a *circular region.*

13. Describe the shape of the region over which a cow can wander if it is tied to a post with a 20 foot rope.

You can draw a circle with a strip of cardboard and two pencils. Make two holes in the cardboard and put a pencil in each hole. Keep one pencil fixed and move the other all the way around to draw a circle.

The *distance between* the pencils is **the radius** of the circle.

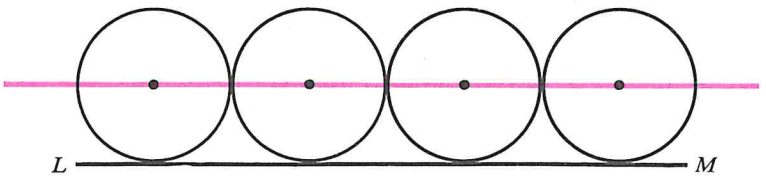



## EXERCISES

Draw a circle with radius

**1.** 1 inch **2.** $1\frac{1}{2}$ inches **3.** 2 inches **4.** $2\frac{1}{2}$ inches

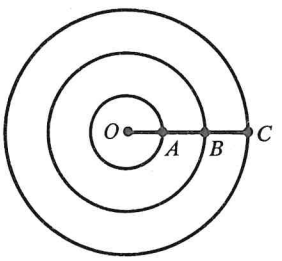
Copy this figure.




**5.** What do the circles have in common?

**6.** Do you think that the line segment through the centers is parallel to $\overline{LM}$? Why?

Copy this figure.




**7.** What do the circles have in common?

Copy this drawing. *A* and *B* are the centers of the circles.

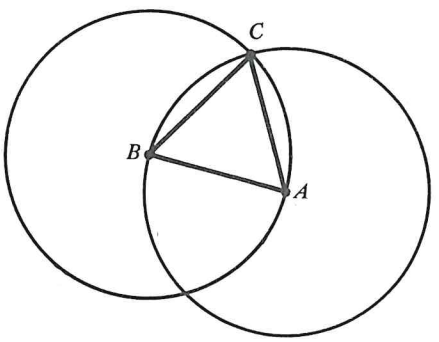
**8.** Is $\overline{AB}$ a radius of each circle?

**9.** Measure each line segment in inches.

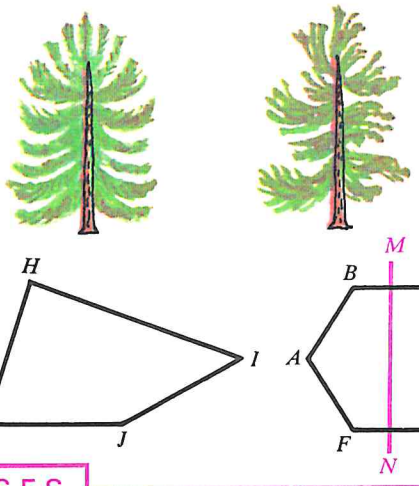
**a.** m($\overline{AB}$) **c.** m($\overline{BC}$)

**b.** m($\overline{CA}$)

**10.** Are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ congruent line segments?



## Symmetry



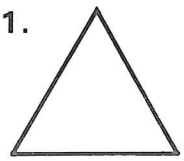

Both the tree on the left and polygon *ABCDEF* are **symmetrical** figures.

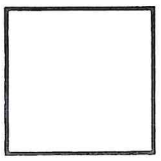
The tree is symmetrical about the tree trunk. Polygon *ABCDEF* is symmetrical about $\overline{MN}$.

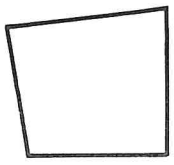
The other tree and polygon *GHIJ* are not symmetrical.

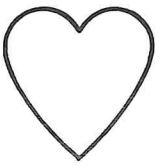
### EXERCISES

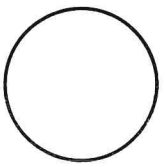
For each figure, write *S* or *NS* to show whether it is *symmetrical* or *not symmetrical*.

1. 

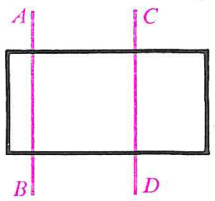
2. 

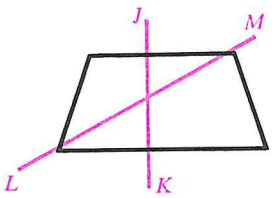
3. 

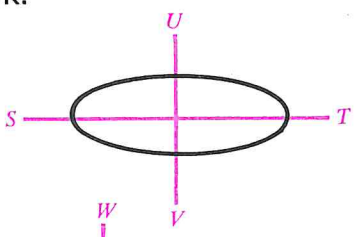
4. 

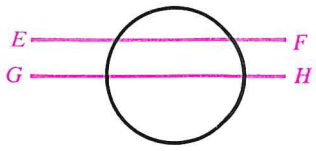
5. 

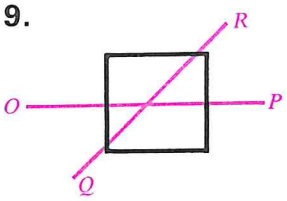
Name the line segment about which each figure is symmetrical. Sometimes *both* line segments will work for the figure and sometimes *neither* will work.

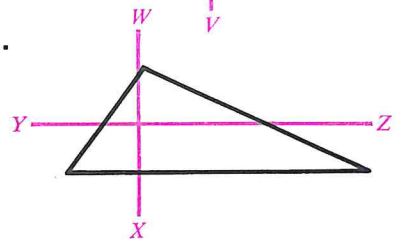
6. 


8. 


10. 


7. 


9. 


11. 


Draw a circle and draw any diameter of it.

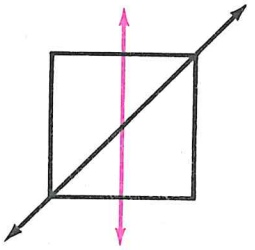
12. Is the circle symmetrical about this diameter?

13. If you drew another diameter, would the circle be symmetrical about this also?

Discuss the question.

14. Are there any symmetrical shapes you can see in the classroom?

Cut out a square region from a piece of paper and fold it along the lines shown in the figure.

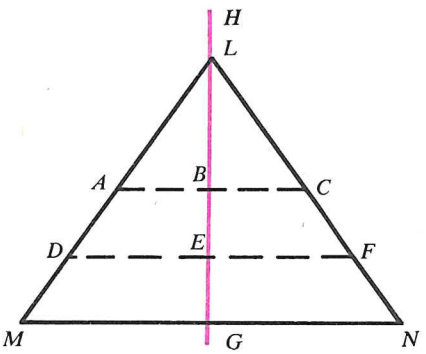


1. Is the square symmetrical about each fold?
2. Can you find other fold lines about which the square is symmetrical?

Cut out a rectangular region from a piece of paper.

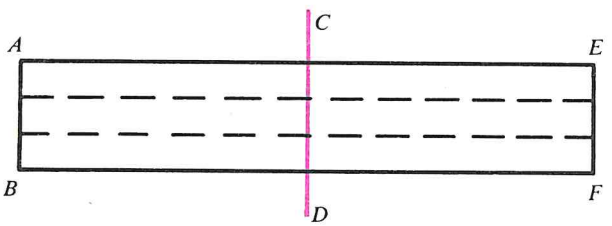
3. Find some fold lines about which the rectangle is symmetrical.

Triangle *LMN* is symmetrical about the line segment $\overline{GH}$. $\overline{AC}$ and $\overline{DF}$ are perpendicular to $\overline{HG}$.



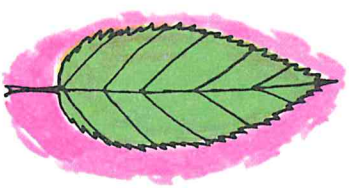

4. Measure each line segment in inches.
   a. m($\overline{AB}$)  c. m($\overline{BC}$)
   b. m($\overline{DE}$)  d. m($\overline{EF}$)
5. Is $A$ the same distance from $\overline{GH}$ as $C$?
6. Is $D$ the same distance from $\overline{GH}$ as $F$?

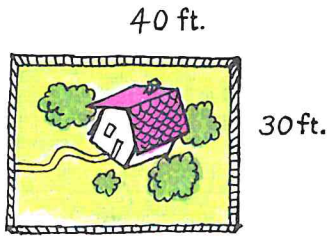
*ABEF* is a rectangle.



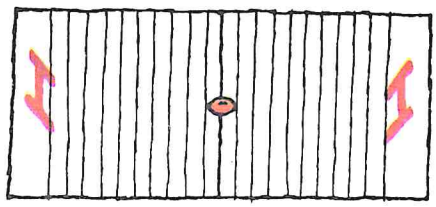

7. Can the points on $\overline{AB}$ be matched with the points on $\overline{EF}$ so the points in each pair are the same distance from $\overline{CD}$?

Here are some pictures of symmetrical objects from nature. Can you name other symmetrical objects?

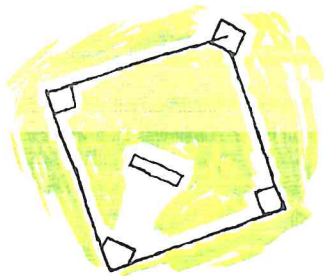





1. Mr. Smith has 120 feet of wire for a fence. His rectangular yard is 40 feet long and 30 feet wide. Does he have enough fencing for the yard?



2. The Warhawks play football on a field that is 300 feet long and 120 feet wide. Name the perimeter of the field.



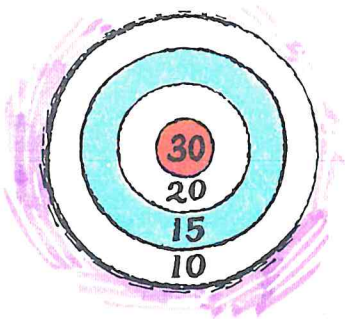
3. It is 90 feet from one base to the next on a baseball diamond. If a boy hits a home run, how far does he have to run around the bases?

4. **a.** John threw three darts that landed in the center of the dart board. How many points did he score?

   ***b.** If Sally scored 55 points with three darts, where did her darts land?

   ***c.** If Abe scored 40 points with three darts, where did his darts land?

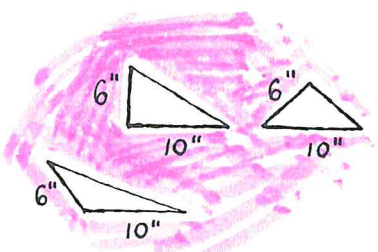
   There are two answers each in parts **b** and **c**.




***5.** A triangle has one side 6 inches long and another side 10 inches long.

   **a.** Could the third side be 3 inches long? Why or why not?

   **b.** Could the third side be 18 inches long? Why or why not?



## Sharpshooting

See how quickly you can answer the questions in **A**. Then see if you can answer the questions in **B** even more quickly.

**A**

1. Name the sum of 29 and 7.
2. If one addend is 26 and the sum is 35, what is the other addend?
3. $43 + 6 + 8 + 9 + 5 = n$
4. If you have a 60 foot rope and use 29 feet, how many feet of rope do you have left?
5. How many years will it be until you are 21 years old?
6. There are 34 in June's class and 29 in Jerry's. How many more are there in June's class?
7. Two girls together picked 43 boxes of fruit. If one girl picked 25, how many did the other pick?
8. Name the sum of 46 and 54.
9. The sides of a triangle are 7 inches, 9 inches, and 11 inches long. What is the perimeter?

**B**

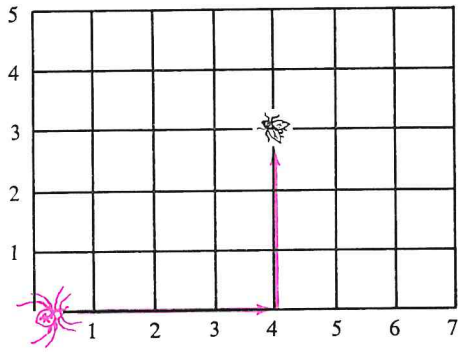
1. Name the difference between 23 and 4.
2. If one addend is 6 and the other is 48, what is the sum?
3. If you had 30 questions to answer and have answered 18, how many must you still answer?
4. $19 + 8 + 7 + 6 + 5 + 8 = n$
5. Mary has 34 beads and Joan has 69. How many more has Joan?
6. The sun shone for 7 hours on Monday, 8 hours on Tuesday, and 11 hours on Wednesday. How many hours was that in all?
7. If you have 23 pictures and are given 18 more, how many do you have now?
8. How many days from today until the end of the month?
9. What is the cardinal number of the union of $A$ = {letters in the alphabet} and $B$ = {the digits}?

## Using more than one number line

The spider said, "If I go 4 tiles to the right and up 3, I will catch the fly."

He could write his moves as the **number pair** (4, 3).

The spider's path may be shown on a diagram like this, using more than one number line.

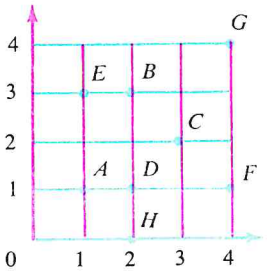



## EXERCISES

Study the diagram at the right.

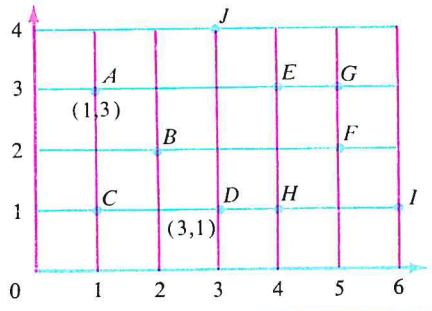
Use a number pair to show the move you must make to get from the point $O$ to each of these points.

**1.** $A$ **2.** $B$ **3.** $C$ **4.** $D$ **5.** $E$ **6.** $F$ **7.** $G$ **8.** $H$




We name the point $D$ by the number pair (3, 1). This means we moved 3 to the right and up 1 from the point $O$. Point $A$ is named by the number pair (1, 3).

The **order of the numbers** in a number pair is important.




Use a number pair to name each point on the diagram above.

**9.** $A$ **10.** $B$ **11.** $C$ **12.** $D$ **13.** $E$ **14.** $F$ **15.** $G$ **16.** $H$ **17.** $I$ **18.** $J$

Use the diagram at the top of the page to answer this question about the spider and the fly.

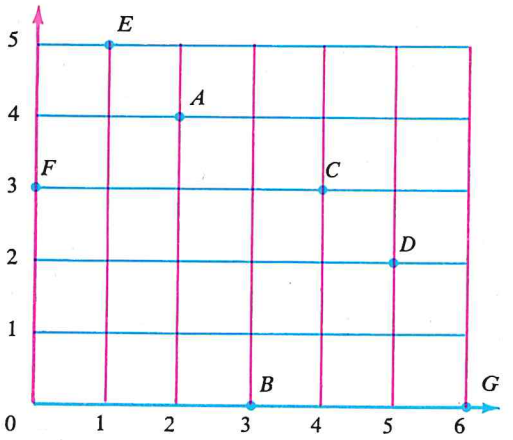
**19.** Before the spider left point $O$, the fly moved 2 tiles to the right and down 2. What number pair shows the moves the spider will *now* have to make to catch the fly?

## Using more than one number line

We can use two sets of number lines to locate points on a plane. A plane like this is called a **number plane.**

*A* may be labeled by the number pair (2, 4). This means *A* is at the intersection of the red 2 line and the blue 4 line.

*B* may be labeled by the number pair (3, 0).



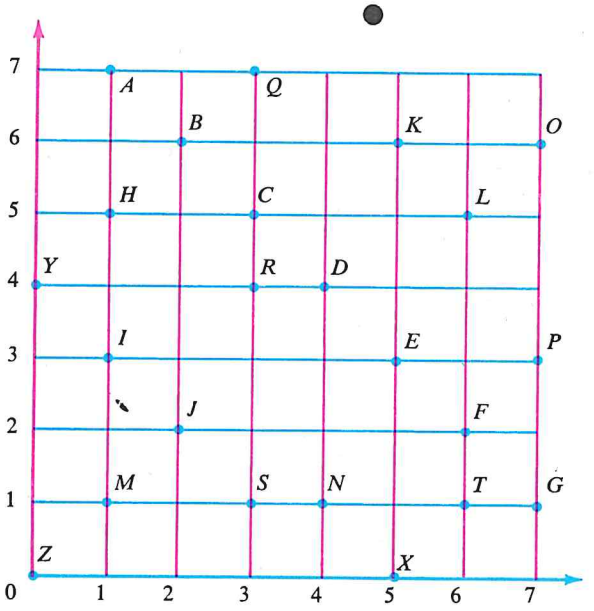

## EXERCISES

Use a number pair to name each point on the number plane above.

**1.** *C* **2.** *D* **3.** *E* **4.** *F* **5.** *G*

Name the letter for each point on the number plane at the right.

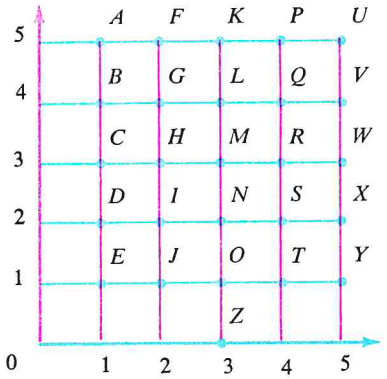
**6.** (2, 2)
**7.** (2, 6)
**8.** (4, 1)
**9.** (6, 2)
**10.** (7, 3)
**11.** (3, 5)
**12.** (3, 1)
**13.** (6, 1)
**14.** (5, 6)
**15.** (4, 4)




Name the number pair for each point.

**16.** *I*
**17.** *N*
**18.** *L*
**19.** *Q*
**20.** *D*
**21.** *C*
**22.** *A*
**23.** *P*
**24.** *G*
**25.** *B*
**26.** *E*
**27.** *J*
**28.** *X*
**29.** *Y*
**30.** *Z*
**31.** *F*

## Spelling with number pairs




We can use the number plane tc play a spelling game. A number pair is used in place of a letter.

For example, the number pairs (1, 4), (3, 1), (5, 1), (4, 2) spell BOYS.

### EXERCISES

Use the number plane to name the words.

1. (5, 4), (1, 5), (1, 3), (1, 5), (4, 1), (2, 2), (3, 1), (3, 2)
2. (2, 5), (5, 5), (3, 2), (3, 2), (5, 1), (2, 5), (1, 5), (1, 3), (1, 1)
3. (2, 4), (2, 2), (4, 3), (3, 4), (4, 2)
4. (1, 3), (1, 5), (4, 1)
5. (3, 0), (1, 1), (1, 4), (4, 3), (1, 5)
6. (4, 1), (3, 1), (5, 1)
7. (4, 4), (5, 5), (1, 1), (1, 1), (3, 2)
8. (3, 5), (2, 2), (3, 2), (2, 4)
9. (2, 1), (1, 5), (3, 3)
10. (4, 2), (3, 1), (4, 3), (4, 3), (5, 1)
11. (4, 1), (3, 1), (1, 1), (4, 2)
12. (4, 2), (1, 5), (3, 2), (1, 2), (5, 1)
13. (3, 4), (1, 5), (3, 2), (1, 2)
14. (2, 5), (1, 5), (1, 3), (4, 1), (4, 2)

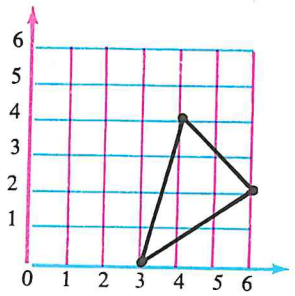
Name the word that is left after you cross out the letters named by the number pairs.

15. OCTIRACMLE (3, 1) (4, 1) (1, 5) (3, 3)
16. WPLHSAJNE (5, 3) (2, 3) (4, 2) (2, 1)
17. BQLIDNGE (1, 4) (4, 4) (1, 2) (2, 4)
18. PVOIKLNTY (5, 4) (3, 5) (3, 4) (5, 1)

Write each of the words as a set of number pairs.

19. MOTHER
20. BASEBALL
21. BUNNY
22. TELEVISION
23. JUMP ROPE
24. TOBOGGAN
25. ZOO
26. DOG
27. NOSE

28. Write your name as a set of number pairs.
29. Make up words with number pairs and present them to the rest of the class.




## Figures on a number plane

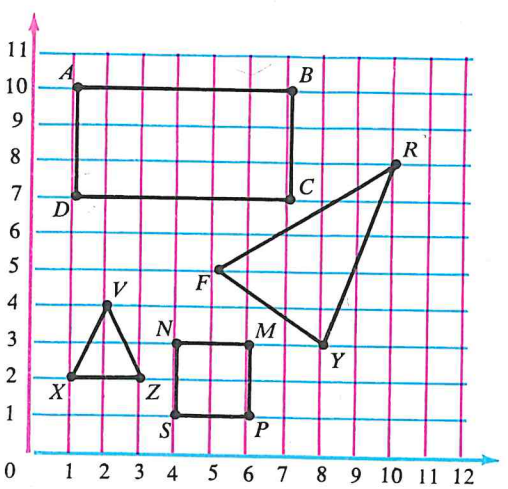
We can show polygons and other figures on a number plane. By joining the points (3, 0), (4, 4) and (6, 2), we have drawn a triangle.

## EXERCISES

Draw a number plane and number each way from 0 to 10.
Use the number plane to answer the questions.

1. **a.** Complete the pattern of number pairs:
   (0, 0), (1, 1), (2, 2), (3, *a*), (4, *b*), (5, *c*), (6, *d*)
   **b.** Use these number pairs to locate points on your number plane.
   **c.** Join the points and name the figure you have drawn.
2. **a.** Completé the pattern of number pairs:
   (0, 0), (1, 2), (2, 4), (3, *a*), (4, *b*), (5, *c*)
   **b.** Use these number pairs to locate points on your number plane.
   **c.** Join the points and name the figure you have drawn.
3. **a.** Join the points (7, 1), (7, 5), (9, 5), and (9, 1).
   **b.** Name the figure you have drawn.
4. **a.** Join the points (0, 6), (2, 10), and (3, 7).
   **b.** Name the figure you have drawn.

Study the number plane.

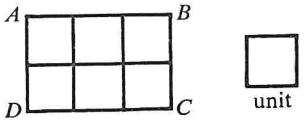



5. Name the number pair for each point:
   **a.** $D$ **b.** $R$ **c.** $P$ **d.** $Y$ **e.** $V$
6. Is $\overline{AB}$ parallel to $\overline{DC}$?
7. Is $\overline{AB}$ parallel to the blue number lines?
8. Do you think $\overline{VX}$ is congruent to $\overline{VZ}$?
9. Name the square and the measure of its perimeter.
10. Name four line segments that are parallel to the red number lines.

## Area

*ABCD* is a simple closed curve. Every simple closed curve forms a **region.**

We *measure* a region by comparing it with a square unit, such as the square inch. The **area** of a region is the number of square units in the region.

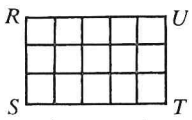



m(*ABCD*) = 6

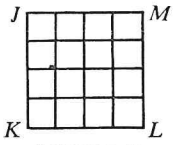
The area of *ABCD* is 6 square units.

### EXERCISES

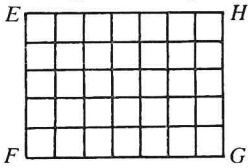
Name the number of squares in each rectangular region.



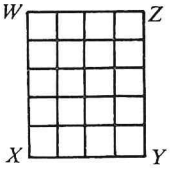

1. m(*RSTU*)




2. m(*JKLM*)



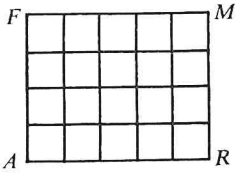

3. m(*EFGH*)

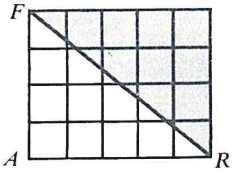



4. m(*WXYZ*)

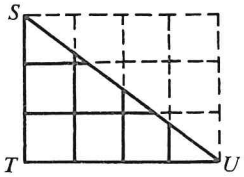
Complete the sentences.

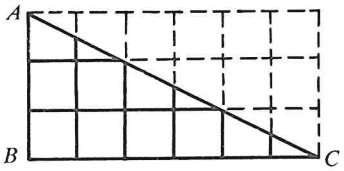
5. The area of rectangular region *RSTU* is ___?___ square units.
6. The area of square region *JKLM* is ___?___ square units.
7. The area of rectangular region *EFGH* is ___?___ square units.
8. The area of rectangular region *WXYZ* is ___?___ square units.

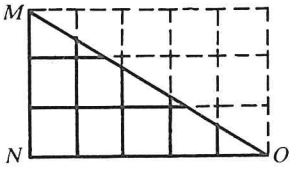


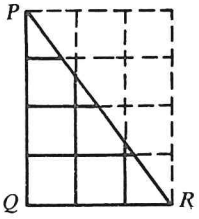




9. The area of rectangular region *FARM* is 30 square units, so the area of triangular region *FAR* is ___?___ square units.



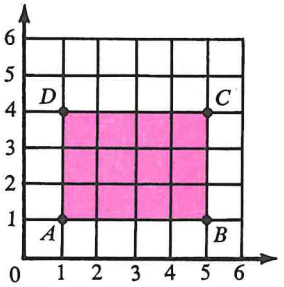




10. The area of triangular region *STU* is ___?___ square units.
11. The area of triangular region *ABC* is ___?___ square units.
12. The area of triangular region *MNO* is ___?___ square units.
13. The area of triangular region *PQR* is ___?___ square units.

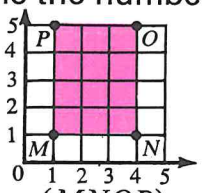
The number plane can help us name the area of regions.

$m(ABCD) = 12$
The area of $ABCD$ is 12 square units.

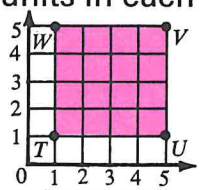



## EXERCISES

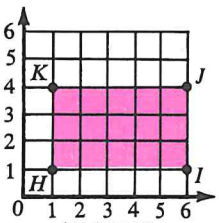
Name the number of square units in each region.




1. $m(MNOP)$

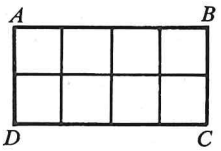


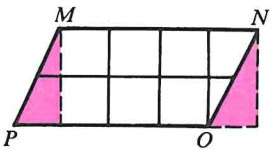

2. $m(TUVW)$




3. $m(HIJK)$

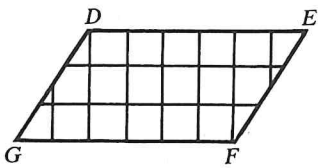
Study the figures. Then complete the sentence.



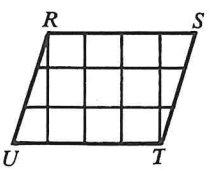




4. The area of rectangular region $ABCD$ is 8 square units, so the area inside parallelogram $MNOP$ is ___?___ square units.

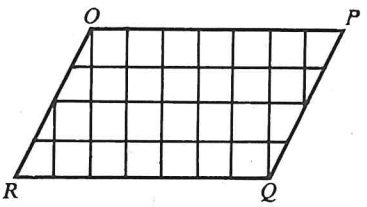
Name the number of square units in each region.




5. $m(DEFG)$




6. $m(RSTU)$



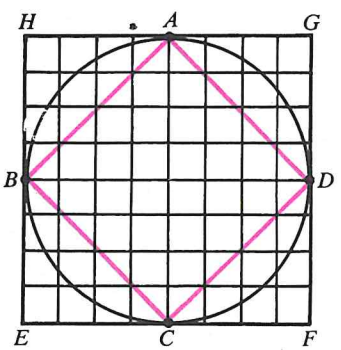

7. $m(OPQR)$

Use the small square as the unit of measure.

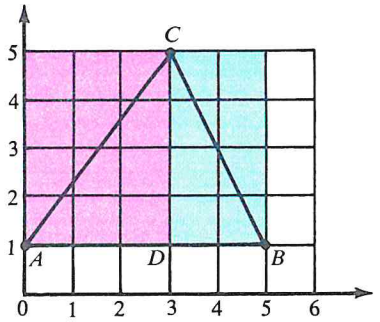
8. Name the area of $EFGH$.

9. Name the area of $ABCD$.

10. Estimate the area of the circular region.



## Area




The area of the red region is 12 square units, so the area inside triangle $ADC$ is 6 square units.

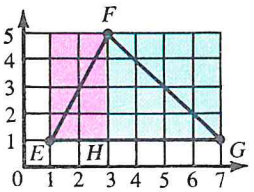
The area of the blue region is 8 square units, so the area inside triangle $DBC$ is 4 square units.

The area of triangular region $ABC$ is $6 + 4 = 10$ square units.

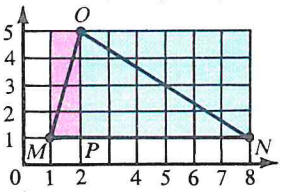
### EXERCISES

Name the area of each triangular region.

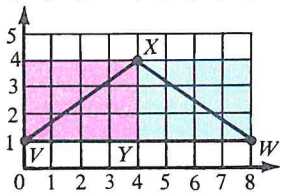
1. $EHF$: ___?___ square units.
2. $FGH$: ___?___ square units.
3. $EFG$: ___?___ square units.




4. $OMP$: ___?___ square units.
5. $NOP$: ___?___ square units.
6. $MNO$: ___?___ square units.

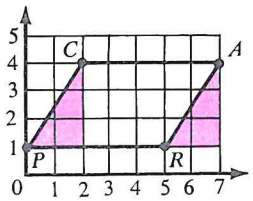



7. $VYX$: ___?___ square units.
8. $WXY$: ___?___ square units.
9. $VWX$: ___?___ square units.

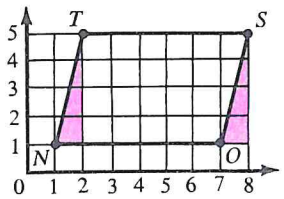



Name the area of each region.

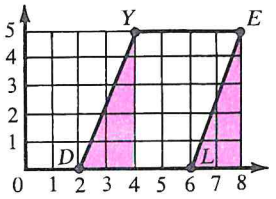
10.



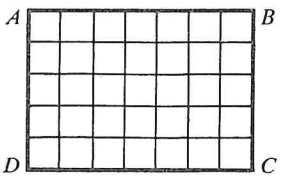

11.



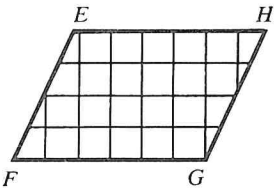

12.



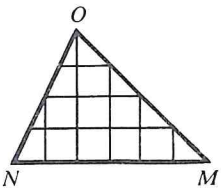

13.



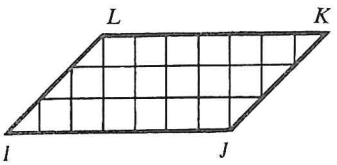

14.




15.




16.



List the members of each set.

1. {the first four letters of the alphabet}
2. {the whole numbers between 12 and 19}
3. {the names of the months that begin with *M*}
4. {the animals in the world with seven eyes}

List the members of the sets.

5. $A \cup B$ when $A = \{2, 4, 6\}$ and $B = \{1, 3, 5\}$
6. $C \cup D$ when $C = \{a, i, u\}$ and $D = \{b, o, n, e\}$

Use exercises 5 and 6 to name the cardinal numbers for

7. $n(A)$
8. $n(B)$
9. $n(C)$
10. $n(D)$
11. $n(A \cup B)$
12. $n(C \cup D)$

Write a compact numeral for

13. $300 + 40 + 6$
14. $4000 + 800 + 70 + 6$
15. $(300 + 40 + 6)$ thousands, $(800 + 70 + 6)$ ones
16. three thousand, nine hundred twenty-six
17. five hundred forty-three thousand, seven hundred eighteen

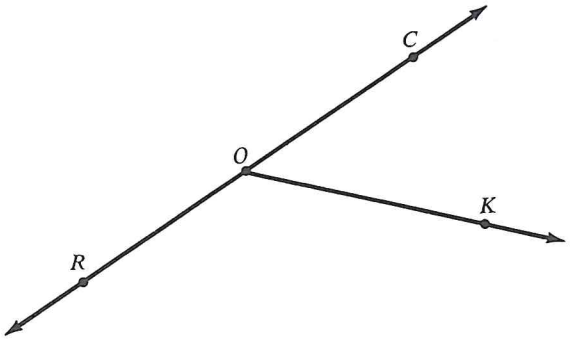
Solve the equations.

| | | | | |
|---|---|---|---|---|
| 18. | $4 + 5 = a$ | $14 + 5 = b$ | $4 + 15 = c$ | $24 + 45 = d$ |
| 19. | $5 + 3 = e$ | $15 + 3 = f$ | $5 + 13 = g$ | $15 + 63 = h$ |
| 20. | $16 - 8 = i$ | $26 - 8 = j$ | $36 - 8 = k$ | $46 - 8 = l$ |
| 21. | $12 - 7 = m$ | $22 - 7 = n$ | $32 - 17 = p$ | $52 - 27 = q$ |

Name the sums and differences.

| | | | | | |
|---|---|---|---|---|---|
| 22. $34 + 26$ | 25. $46 - 29$ | 28. $428 + 346$ | 31. $382 + 268$ | 34. $437 - 219$ | 37. $736 - 548$ |
| 23. $27 + 89$ | 26. $64 - 26$ | 29. $468 + 259$ | 32. $493 + 477$ | 35. $283 - 128$ | 38. $821 - 376$ |
| 24. $37 + 58$ | 27. $55 - 19$ | 30. $293 + 248$ | 33. $677 + 345$ | 36. $562 - 367$ | 39. $941 - 464$ |

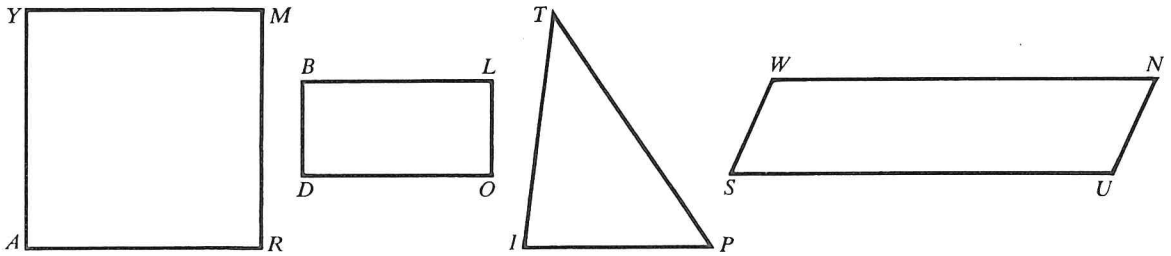
## CHECKING UP




Study the figure.

1. Name the line.
2. Name three rays.
3. Name three line segments.
4. Name two angles.

Use the vertices to name each polygon.




5. triangle ___?___
6. parallelogram ___?___
7. rectangle ___?___
8. square ___?___

Measure the sides of each polygon in inches.

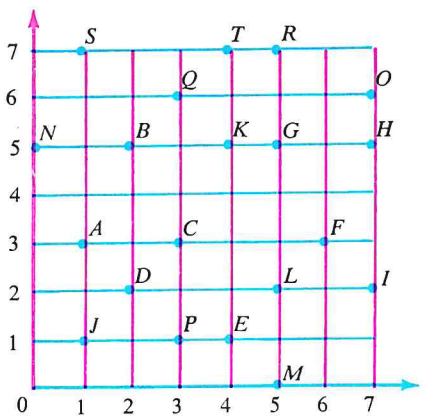
9. $m(\overline{AR})$
10. $m(\overline{YM})$
11. $m(\overline{YA})$
12. $m(\overline{MR})$
13. $m(\overline{TI})$
14. $m(\overline{TP})$
15. $m(\overline{IP})$
16. $m(\overline{BD})$
17. $m(\overline{BL})$
18. $m(\overline{LO})$
19. $m(\overline{DO})$
20. $m(\overline{WS})$
21. $m(\overline{WN})$
22. $m(\overline{NU})$
23. $m(\overline{SU})$

**24–27.** Name four pairs of congruent line segments in the polygons above.

**28–31.** Name the perimeter of each polygon above.

For this number plane, write the letters for these points.

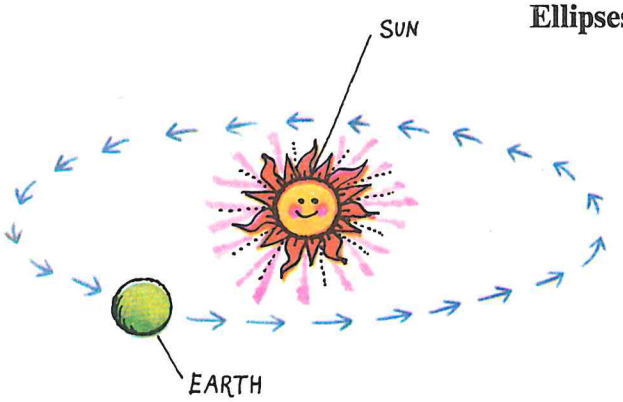
32. (1, 3)
33. (2, 5)
34. (4, 5)
35. (7, 2)
36. (6, 3)
37. (0, 5)




Name the number pairs for these points.

38. $J$
39. $P$
40. $F$
41. $H$
42. $B$
43. $R$

## Ellipses



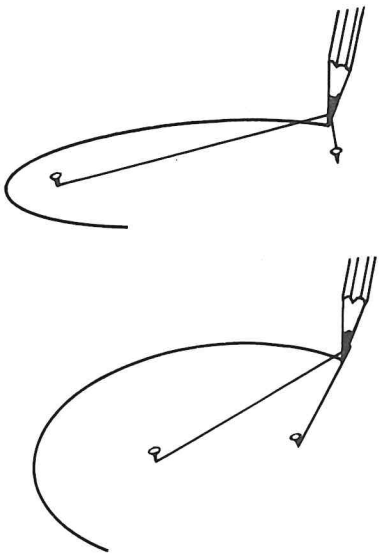

The earth travels around the sun in an orbit shaped like an **ellipse.**

The simple closed curve shown in blue is an **ellipse.** It is like an egg in shape.

Answer the questions.

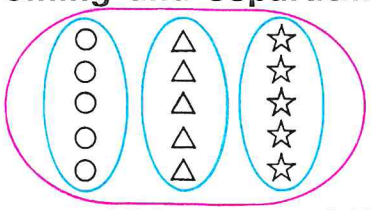
1. Why would we get a rough ride in that car?

2. To draw an ellipse, fasten each end of a piece of string to a piece of paper. Then run a pencil along the string, keeping the string tight, as in our picture.



3. The size and shape of an ellipse will change as you change the length of the string or move the ends of the string.
   What is the effect of moving the ends of the string
   **a.** closer together?
   **b.** farther apart?

## Joining and separating equivalent sets

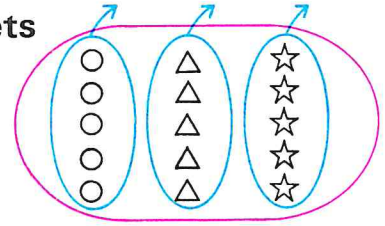


3 sets of 5 form a set of 15.

$5 + 5 + 5 = 15$

3 times 5 equals 15.

$3 \times 5 = 15$



A set of 15 forms 3 sets of 5.

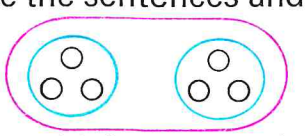
$15 - 5 - 5 - 5 = 0$

15 divided by 5 equals 3.

$15 \div 5 = 3$

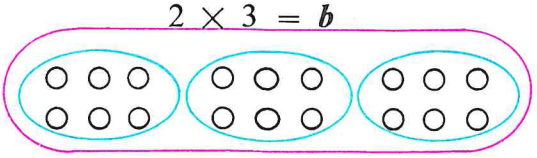
## EXERCISES

Complete the sentences and solve the equations.



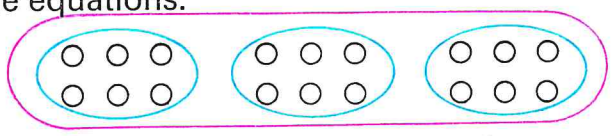
**1.** 2 sets of 3 form a set of $a$.

$2 \times 3 = b$



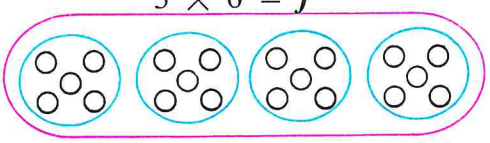
**2.** A set of 18 forms 3 sets of $c$.

$18 \div 3 = d$



**3.** 3 sets of 6 form a set of $e$.

$3 \times 6 = f$



**4.** A set of 20 forms $g$ sets of 5.

$20 \div 5 = h$

**5.** 3 sets of 4 form a set of $a$.

**6.** 4 sets of 4 form a set of $b$.

**7.** 6 sets of 4 form a set of $c$.

**8.** 8 sets of 4 form a set of $d$.

**9.** A set of 20 forms $e$ sets of 4.

**10.** A set of 16 forms $f$ sets of 4.

**11.** A set of 28 forms $g$ sets of 4.

**12.** A set of 24 forms $h$ sets of 4.

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 |
| 29 | 30 | 31 | 32 |

$4 + 4 + 4 + 4 + 4 + 4 = 24$

6 addends of 4 = a sum of 24

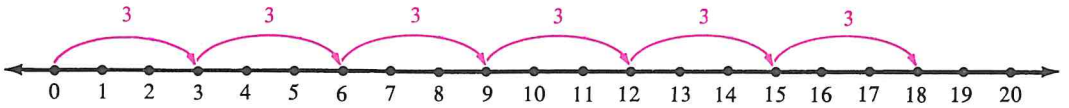
$6 \times 4 = 24$

factor × factor = product

The factor 6 tells how many addends.
The factor 4 is the same as each addend.
The product 24 is the same as the sum.

## EXERCISES

Use the number line to complete the sentences and solve the equations.




1. The sum of 2 addends of 3 is ***a***, so $2 \times 3 = b$.
2. The sum of 3 addends of 3 is ***c***, so $3 \times 3 = d$.
3. The sum of 4 addends of 3 is ***e***, so $4 \times 3 = f$.
4. The sum of 5 addends of 3 is ***g***, so $5 \times 3 = h$.
5. The sum of 6 addends of 3 is ***i***, so $6 \times 3 = j$.

Name the sums.

6.

| number of addends | 3 | 4 | 2 | 6 | 5 | 3 | 4 | 7 | 6 | 2 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| each addend | 4 | 6 | 6 | 3 | 4 | 5 | 4 | 3 | 5 | 4 | 2 |
| sum | 12 | *a* | *b* | *c* | *d* | *e* | *f* | *g* | *h* | *i* | *j* |

Name the products or missing factors.

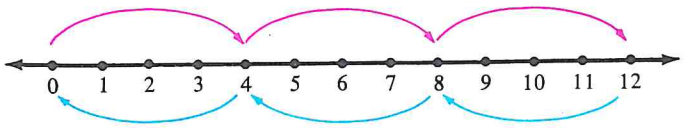
7.

| factor | 3 | 4 | 2 | 5 | 6 | 2 | 5 | 3 | 4 | 6 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| factor | 2 | 3 | 5 | 4 | 2 | 6 | 5 | *g* | 2 | *i* | *j* |
| product | 6 | *a* | *b* | *c* | *d* | *e* | *f* | 12 | *h* | 12 | 15 |

8.

| factor | 5 | 6 | 7 | 8 | 9 | 3 | 4 | 2 | 6 | 8 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| factor | 6 | *a* | *b* | *c* | *d* | *e* | *f* | *g* | *h* | *i* | *j* |
| product | 30 | 18 | 49 | 24 | 27 | 21 | 16 | 18 | 36 | 32 | 28 |

## Multiplication and division on the number line




The red arrows show: $4 + 4 + 4 = 12$, $3 \times 4 = 12$

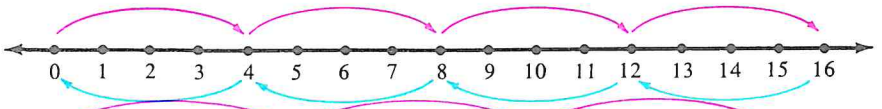
The blue arrows show: $12 - 4 - 4 - 4 = 0$, $12 \div 4 = 3$

> Division and multiplication are opposites.

### EXERCISES

Use $\times$ and $\div$ to write equations for the multiplications and divisions shown on the number lines.

**1.**




**2.**

**3.**

**4.**

Use $\times$ and $\div$ to rewrite the equations.

**5.** $2 + 2 + 2 + 2 = 8$

**6.** $4 + 4 + 4 = 12$

**7.** $6 + 6 + 6 = 18$

**8.** $5 + 5 + 5 + 5 = 20$

**9.** $8 + 8 + 8 = 24$

**10.** $7 + 7 + 7 = 21$

**11.** $10 - (5 \times 2) = 0$

**12.** $14 - (7 \times 2) = 0$

**13.** $9 - (3 \times 3) = 0$

Solve the equations.

**14.** $5 \times 2 = a$ $\quad 10 \div 2 = b$ $\quad 2 \times 5 = c$ $\quad 10 \div 5 = d$

**15.** $3 \times 6 = e$ $\quad 18 \div 3 = f$ $\quad 6 \times 3 = g$ $\quad 18 \div 6 = h$

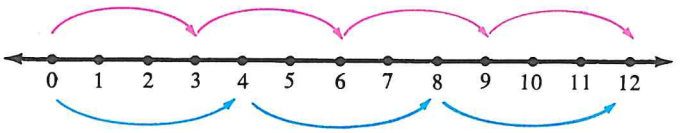
**16.** $5 \times 4 = i$ $\quad 20 \div 4 = j$ $\quad 4 \times 5 = k$ $\quad 20 \div 5 = l$

**17.** $3 \times 7 = m$ $\quad 21 \div 7 = n$ $\quad 7 \times 3 = p$ $\quad 21 \div 3 = q$

Complete the sentence.

**18.** Division is the opposite of ___?___.

Study the number line.




The red arrows show: $3 + 3 + 3 + 3 = 12$, $4 \times 3 = 12$

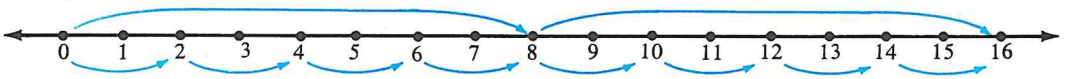
The blue arrows show: $4 + 4 + 4 = 12$, $3 \times 4 = 12$

The order of the factors does not affect the product: $4 \times 3 = 3 \times 4$. This is the **Commutative Property of Multiplication.**

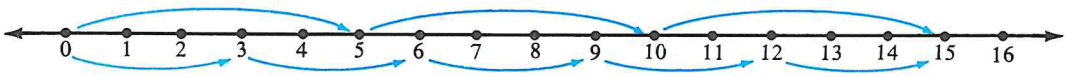
## EXERCISES

Write an equation to show how the commutative property of multiplication is shown on each of these number lines.

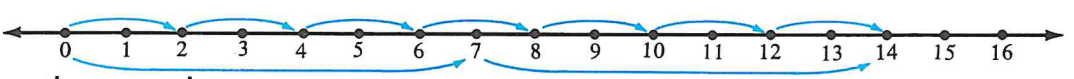
**1.**




**2.**




**3.**




Solve the equations.

**4.** $4 \times 2 = a$ and $2 \times 4 = b$, so $4 \times 2 = c \times 4$

**5.** $3 \times 6 = d$ and $6 \times 3 = e$, so $6 \times 3 = f \times 6$

**6.** $5 \times 2 = g$ and $2 \times 5 = h$, so $5 \times 2 = i \times 5$

**7.** $4 \times 5 = j$ and $5 \times 4 = k$, so $4 \times 5 = l \times 4$

Solve the problem.

**8.** Barry has 12 oranges.

**a.** If he separates the oranges into 6 piles each with the same number, how many will there be in each pile?

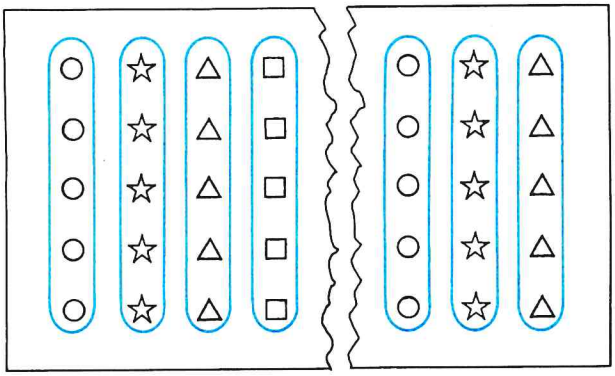
**b.** If he separates the oranges into 2 piles each with the same number, how many will there be in each pile?

## The common factor property

Study the sets and the equations.

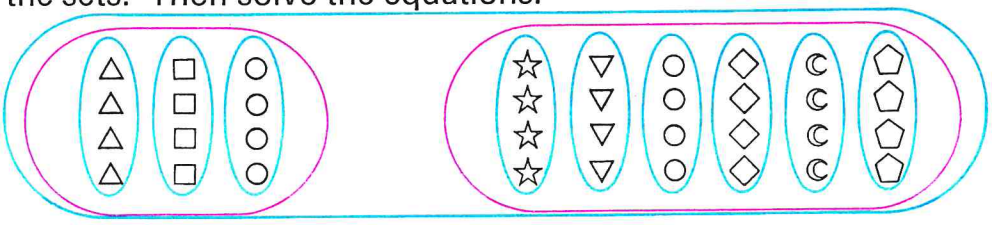
$$\begin{aligned} n(A) &= 4 \times 5 = 20 \\ n(B) &= 3 \times 5 = 15 \\ \hline n(A \cup B) &= 7 \times 5 = 35 \end{aligned}$$

$$\begin{array}{ccccc} (4 \times 5) & + & (3 \times 5) & = & (4 + 3) \times 5 \\ 20 & + & 15 & = & 35 \end{array}$$

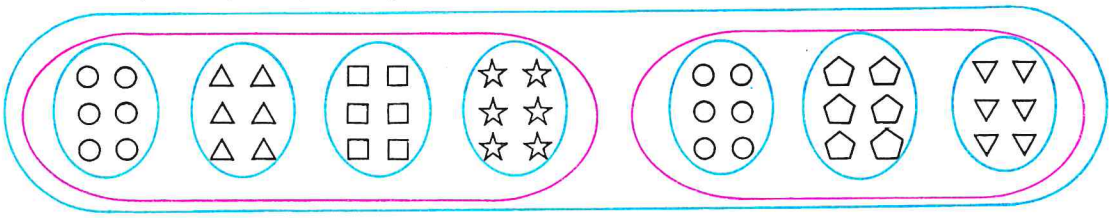


EXERCISES

Study the sets. Then solve the equations.



1. $(3 \times 4) + (6 \times 4) = a + b = c$ $\quad (3 + 6) \times 4 = d \times 4 = e$



2. $(4 \times 6) + (3 \times 6) = f + g = h$ $\quad (4 + 3) \times 6 = i \times 6 = j$

Copy and complete the equations.

3. $(2 \times 5) + (6 \times 5) = a + b = c$ $\quad (2 + 6) \times 5 = d \times 5 = e$
4. $(6 \times 3) + (2 \times 3) = f + g = h$ $\quad (6 + 2) \times 3 = i \times 3 = j$
5. $(4 \times 7) + (4 \times 2) = k + l = m$ $\quad 4 \times (7 + 2) = 4 \times n = p$
6. $(8 \times 1) + (8 \times 2) = q + r = s$ $\quad 8 \times (1 + 2) = 8 \times t = u$
7. $(7 \times 4) + (7 \times 3) = v + w = x$ $\quad 7 \times (4 + 3) = 7 \times y = z$

Solve the problem.

8. Mary sells pencils for 4 cents each. She sold 3 pencils on Monday and 2 pencils on Tuesday. Show two ways for Mary to find the total amount of money she collected.

Study the equations.

**A**

$(6 + 6 + 6) + (6 + 6) = 18 + 12$
$= 30$

$(3 \times 6) + (2 \times 6) = 18 + 12$
$= 30$

**B**

$(6 + 6 + 6) + (6 + 6) = 6 + 6 + 6 + 6 + 6$
$= 5 \times 6$
$= 30$

$(3 \times 6) + (2 \times 6) = (3 + 2) \times 6$
$= 5 \times 6$
$= 30$

The **common addend** is 6.

The **common factor** is 6.

## EXERCISES

Solve the equations.

1. $(2 + 2 + 2) + (2 + 2) = 5 \times 2 = a$
   so $(3 \times 2) + (2 \times 2) = b \times 2 = c$

2. $(3 + 3) + (3 + 3 + 3 + 3) = 6 \times 3 = d$
   so $(2 \times 3) + (4 \times 3) = e \times 3 = f$

3. $(7 + 7) + (7 + 7 + 7) = 5 \times 7 = g$
   so $(2 \times 7) + (3 \times 7) = h \times 7 = i$

4. $6 + (6 + 6 + 6) = j \times 6 = k$
   so $(1 \times 6) + (3 \times 6) = l \times 6 = m$

5. $(8 + 8 + 8 + 8) + 8 = r \times 8 = s$
   so $(4 \times 8) + (1 \times 8) = t \times 8 = u$

6. $(5 + 5 + 5 + 5 + 5) + (5 + 5 + 5) = n \times 5 = z$
   so $(5 \times 5) + (3 \times 5) = p \times 5 = q$

7. $3 \times 2 = a$ and $4 \times 2 = b$
   so $7 \times 2 = c$

8. $5 \times 3 = d$ and $4 \times 3 = e$
   so $9 \times 3 = f$

9. $3 \times 4 = g$ and $4 \times 4 = h$
   so $7 \times 4 = i$

10. $5 \times 2 = j$ and $4 \times 2 = k$
    so $9 \times 2 = l$

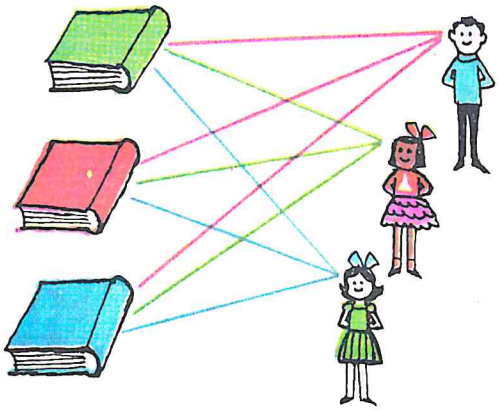
11. $5 \times 6 = m$ and $2 \times 6 = n$
    so $7 \times 6 = p$

Complete the sentences.

12. To name the sum $(4 \times 3) + (5 \times 3)$, we may first name the products $4 \times 3 = 12$ and $5 \times 3 = 15$ and then name the sum ___?___, or we may first name the sum $4 + 5 = 9$ and then name the product ___?___.
    This is an example of the **Common Factor Property** or the **Distributive Property.**

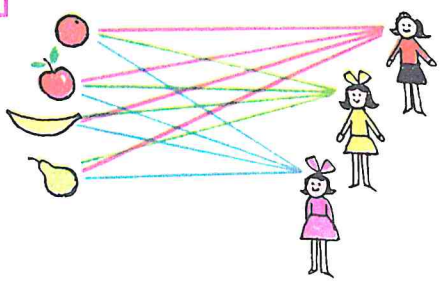
## Pairing members of sets

Donna, Mary Lou, and Garry each have a green math book, a red social studies book, and a blue science book.



Every member of set $A$ is **paired** with every member of set $B$. There are $3 \times 3$, that is, 9 **pairings.**

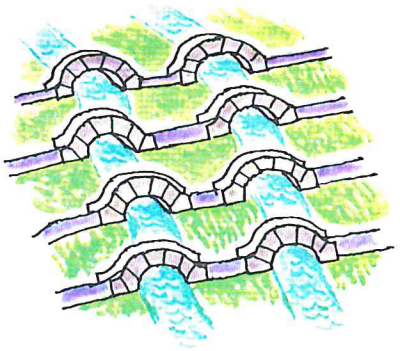
### EXERCISES



Name the number of pairings.

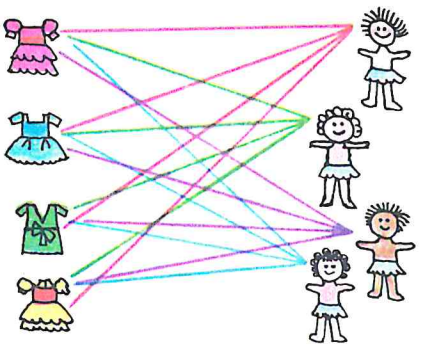
1. Helen, Sharon, and Milly each ate an orange, an apple, a banana, and a pear. How many pieces of fruit were eaten in all?

2. Sam, Jack, and Mark each bought a kite, a paper plane, and a basketball. How many toys did they buy in all?

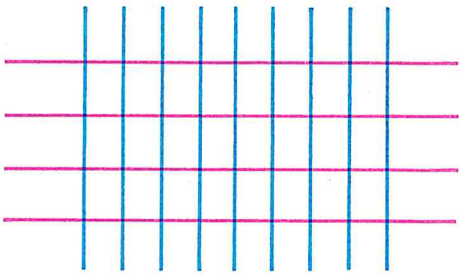


3. There are 2 rivers and 4 roads with bridges over each river. How many bridges are there in all?

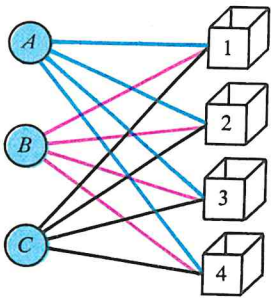
4. There are 3 pictures and 5 picture frames of the same size. How many different ways can the 3 pictures be framed?



5. Every doll can wear each of the dresses. How many different ways can the 4 dolls be dressed?



1. How many points are there where the blue line segments and the red line segments intersect?




2. Each ball fits in each of the boxes. How many pairings can we make of a ball in a box?

3. There are 4 children in June's group and 4 children in Ellen's group. Every girl in June's group is friendly with every girl in Ellen's group. How many pairs of friends are there between the two groups?

4. Each doll fits in each of the carriages. How many pairings can we make of a doll with a carriage?

5. There are 6 children at the party and 6 party hats. How many pairings can we make of a child with a hat?

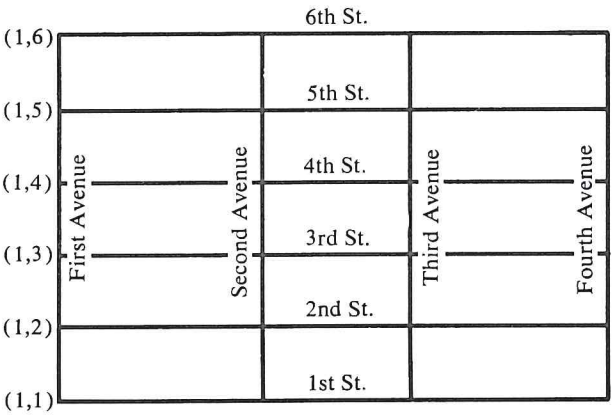
6. There are two balls for the children at the party. If each child plays with each ball, how many pairings of a child and a ball will there be?

## Pairing members of sets

Study the diagram.

Every avenue meets every street.

We can label the points where First Avenue meets the streets with the pairs (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), and (1, 6).




### EXERCISES

Use the diagram to help you.

1. List the pairs to name the points where Second Avenue meets the streets.
2. How many pairs will be needed to label all the points where the Avenues and the Streets meet?

List all the different pairs formed by pairing every member of the first set with every member of the second set.

3. $A = \{\bigcirc, \square, \triangle\}$

   $B = \{1, 2, 3, 4, 5\}$

4. $R = \{a, b, c, d\}$

   $S = \{l, m, n, o\}$

5. $E = \{\mathbb{C}, \star\}$

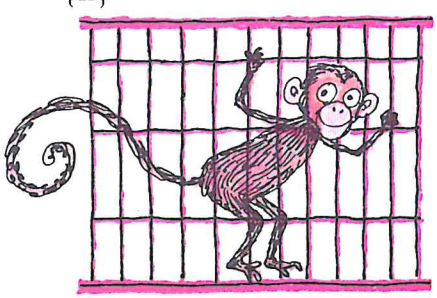
   $F = \{i, j, k, l, m\}$

6. $W = \{I, II, III, IV, V\}$

   $X = \{h\}$

Solve the problem.

7. Grace, Edna, Fred, and Tony each made a visit to the monkeys' cage, to the lions' cage, and to the bears' cage. How many pairs like (Grace, monkeys' cage) would you write to show all the visits?

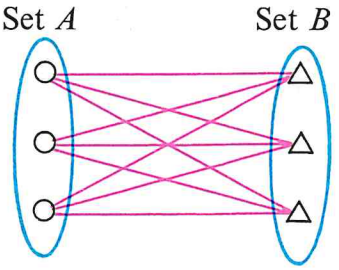


## How many pairings?

$n(A) = 3 \qquad n(B) = 3$

The number of pairings of objects in $A$ with objects in $B$ is $3 \times 3$.

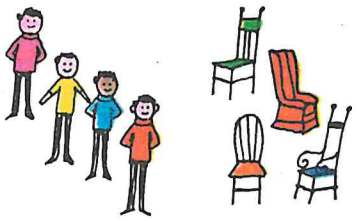
$n(A) \times n(B) = 3 \times 3 = 9$




## EXERCISES

Name the cardinal number of each set and the number of pairings, if every member of the first set is paired with every member of the second set.

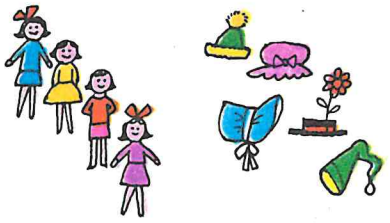
**1.**



$n(A) = a$ $\qquad$ $n(B) = b$

Number of pairings $= c$

**2.**



$n(R) = d$ $\qquad$ $n(S) = e$

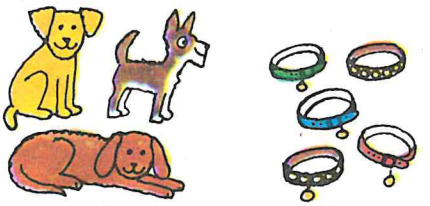
Number of pairings $= f$

**3.**

$n(E) = g$ $\qquad$ $n(F) = h$

Number of pairings $= i$

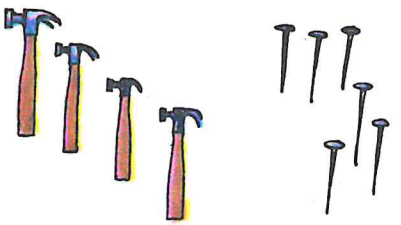
**4.**



$n(X) = j$ $\qquad$ $n(Y) = k$

Number of pairings $= l$

**5.**



$n(M) = m$ $\qquad$ $n(N) = n$

Number of pairings $= p$

**6.**

$n(G) = q$ $\qquad$ $n(H) = r$

Number of pairings $= s$

## Basic multiplication facts

Multiplications and divisions with both factors less than ten are **basic facts.** We call them basic because we use them in all multiplications and divisions. There are 36 **basic facts** to memorize.

### EXERCISES

Copy and complete the number patterns.

**1.**

| × | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 4 | 6 | 8 | *a* | *c* | *e* | *n* | *z* |

**2.**

| × | 3 | 1 | 4 | 2 | 5 | 8 | 6 | 9 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 12 | 4 | 16 | 8 | *r* | *s* | *u* | *v* | *z* |

**3.**

| × | 2 | 4 | 1 | 3 | 5 | 7 | 9 | 6 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 12 | 24 | 6 | 18 | *a* | *c* | *e* | *n* | *z* |

**4.**

| × | 4 | 3 | 2 | 1 | 5 | 9 | 8 | 7 | 6 |
|---|---|---|---|---|---|---|---|---|---|
| 8 | *r* | *s* | *u* | *v* | *x* | *z* | *a* | *c* | *e* |

Solve the equations and complete the sentences.

**5.** $2 \times 0 = a$ **7.** $4 \times 0 = e$ **9.** $7 \times 0 = r$ **11.** $0 \times 9 = u$

**6.** $0 \div 2 = c$ **8.** $0 \div 4 = n$ **10.** $0 \div 7 = s$ **12.** $0 \div 9 = v$

**13.** When 0 is one factor, the product will be ___?___.

**14.** When 0 is the product, one factor will be ___?___.

**15.** $8 \times 1 = a$ **17.** $6 \times 1 = e$ **19.** $1 \times 5 = r$ **21.** $1 \times 2 = u$

**16.** $8 \div 8 = c$ **18.** $6 \div 6 = n$ **20.** $5 \div 1 = s$ **22.** $2 \div 1 = v$

**23.** When 1 is one factor, the product will be the same as the other ___?___.

**24.** When one factor and the product are the same, the other factor will be ___?___.

Copy and complete the number patterns.

**1.**

| × | 7 | 4 | 1 | 8 | 5 | 2 | 9 | 6 | 3 |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 21 | *a* | *c* | *e* | *i* | *n* | *r* | *s* | *u* |

**2.**

| × | 3 | 6 | 9 | 2 | 5 | 8 | 1 | 4 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| 7 | 21 | *v* | *x* | *z* | *a* | *c* | *e* | *i* | *n* |

**3.**

| × | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|---|---|---|---|
| 9 | 81 | *r* | *s* | *u* | *v* | *x* | *z* | *a* | *c* |

**4.**

| × | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 5 | *e* | *i* | *n* | *r* | *s* | *u* | *v* | *x* | *z* |

Solve the equations.

**5.** $2 \times 3 = a$
$3 \times 2 = b$
$2 \times 3 = c \times 2$

**6.** $9 \times 3 = d$
$3 \times 9 = e$
$9 \times 3 = f \times 9$

**7.** $6 \times 4 = g$
$4 \times 6 = h$
$6 \times 4 = 4 \times i$

Complete the sentence.

**8.** **The Commutative Property of Multiplication:** the order of the factors does not affect the ___?___.

The numbers in the chart below are products from the basic facts.

| 4 | 6 | 8 | 9 | 10 | 12 | 14 | 15 | 16 | 18 |
|---|---|---|---|---|---|---|---|---|---|
| 20 | 21 | 24 | 25 | 27 | 28 | 30 | 32 | 35 | 36 |
| 40 | 42 | 45 | 48 | 54 | 56 | 63 | 64 | 72 | 81 |

**9.** Rewrite the chart, replacing each product by its factors. For example, replace 14 with $7 \times 2$, and replace 12 with $6 \times 2$ and $4 \times 3$.

**10.** How many numbers in the chart have more than one pair of factors?

## Families of facts

These equations show that the order of the factors does not affect the product. (This is the commutative property.)

$$4 \times 5 = 20$$
$$\textbf{factor} \times \textbf{factor} = \textbf{product}$$
$$5 \times 4 = 20$$

The equations of the same color show that division is the opposite of multiplication.

$$20 \div 5 = 4$$
$$\textbf{product} \div \textbf{factor} = \textbf{factor}$$
$$20 \div 4 = 5$$

> The four equations form a **family of facts** with 4 and 5 as the factors and 20 as the product.

### EXERCISES

Solve the equations to complete each family of facts.

| | | | | |
|---|---|---|---|---|
| **1.** | $3 \times 7 = 21$ | $a \times 3 = 21$ | $21 \div 3 = b$ | $c \div 7 = 3$ |
| **2.** | $5 \times 3 = d$ | $3 \times 5 = 15$ | $e \div 5 = 3$ | $15 \div f = 5$ |
| **3.** | $4 \times 8 = 32$ | $g \times 4 = 32$ | $32 \div h = 8$ | $i \div 8 = 4$ |
| **4.** | $9 \times 5 = j$ | $5 \times 9 = 45$ | $45 \div 5 = k$ | $l \div 9 = 5$ |
| **5.** | $2 \times m = 12$ | $6 \times 2 = 12$ | $n \div 2 = 6$ | $12 \div p = 2$ |
| **6.** | $4 \times 3 = q$ | $3 \times 4 = 12$ | $12 \div r = 4$ | $12 \div 4 = s$ |

Write three other equations to complete each family of facts.

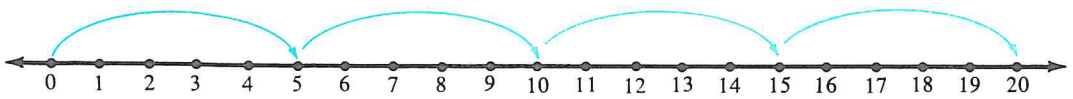
**7.** $5 \times 6 = 30$ **8.** $4 \times 7 = 28$ **9.** $3 \times 8 = 24$ **10.** $3 \times 9 = 27$

Write one other equation to complete each family of facts.

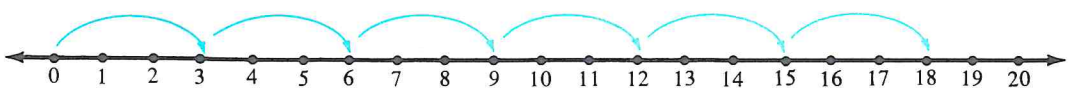
**11.** $2 \times 2 = 4$ **12.** $3 \times 3 = 9$ **13.** $6 \times 6 = 36$ **14.** $9 \times 9 = 81$

Write an equation for the multiplication shown on each number line. Then write the other equations to complete each family of facts.

**15.**




**16.**



Study the equations and the number pattern.

$6 \times 5 = 30$, $3 \times 5 = 15$, so $9 \times 5 = 45$

| × | 6 | 3 | 9 |
|---|---|---|---|
| 5 | 30 | 15 | 45 |

## EXERCISES

Solve the equations.

1. $6 \times 5 = a$, $2 \times 5 = b$, so $8 \times 5 = c$ and $5 \times 8 = d$
2. $5 \times 6 = e$, $2 \times 6 = f$, so $7 \times 6 = g$ and $6 \times 7 = h$
3. $6 \times 6 = i$, $2 \times 6 = j$, so $8 \times 6 = k$ and $6 \times 8 = l$
4. $6 \times 7 = m$, $3 \times 7 = n$, so $9 \times 7 = g$ and $7 \times 9 = p$

Solve the equations.

5. $6 \times 5 = 30$
   $3 \times 5 = 15$
   $9 \times 5 = e$

7. $6 \times 5 = k$
   $1 \times 5 = n$
   $7 \times 5 = p$

9. $y \times 6 = 30$
   $1 \times 6 = w$
   $6 \times 6 = x$

11. $d \times 8 = 40$
    $4 \times 8 = e$
    $9 \times 8 = g$

6. $4 \times 7 = f$
   $3 \times 7 = i$
   $7 \times 7 = 49$

8. $5 \times 7 = r$
   $4 \times 7 = s$
   $9 \times 7 = u$

10. $3 \times 8 = z$
    $a \times 8 = 40$
    $8 \times 8 = c$

12. $h \times 9 = 45$
    $i \times 9 = 36$
    $j \times 9 = 81$

Name the products.

13. $3 + 2 = 5$
    $\times 3 \quad \times 3$
    $9 + 6 = e$

15. $4 + 3 = 7$
    $\times 4 \quad \times 4$
    $q + r = s$

17. $6 + 2 = 8$
    $\times 3 \quad \times 3$
    $a + c = e$

14. $2 + 3 = 5$
    $\times 4 \quad \times 4$
    $8 + i = j$

16. $6 + 3 = 9$
    $\times 3 \quad \times 3$
    $u + v = w$

18. $7 + 2 = 9$
    $\times 4 \quad \times 4$
    $g + i = j$

Complete the number patterns.

19.

| × | 3 | 2 | 5 |
|---|---|---|---|
| 5 | *a* | *c* | *e* |

21.

| × | 4 | 2 | 6 |
|---|---|---|---|
| 6 | *x* | *z* | *a* |

23.

| × | 8 | 1 | 9 |
|---|---|---|---|
| 5 | *u* | *v* | *x* |

20.

| × | 3 | 2 | 5 |
|---|---|---|---|
| 6 | *i* | *n* | *p* |

22.

| × | 4 | 3 | 7 |
|---|---|---|---|
| 5 | *c* | *e* | *i* |

24.

| × | 7 | 2 | 9 |
|---|---|---|---|
| 6 | *z* | *a* | *e* |

## Division and multiplication as opposites

$3 \times 8 = n$
$n \div 8 = 3$

$n$ is a product, so we use multiplication to solve these equations.

$3 \times n = 24$
$24 \div 3 = n$

$n$ is a factor, so we use division to solve these equations.

### EXERCISES

Solve the equations.

1. $3 \times 7 = 21$ so $21 \div 7 = a$
2. $8 \times 5 = 40$ so $40 \div 8 = b$
3. $6 \times 9 = 54$ so $54 \div 6 = c$
4. $2 \times 9 = d$ so $e \div 9 = 2$
5. $7 \times 7 = f$ so $g \div 7 = 7$
6. $4 \times h = 32$ so $32 \div 4 = i$
7. $9 \times 8 = j$ so $k \div 8 = 9$
8. $8 \times 7 = l$ so $m \div 8 = 7$
9. $5 \times 5 = n$ so $p \div 5 = 5$
10. $2 \times 6 = q$ so $r \div 2 = 6$
11. $7 \times 8 = s$ so $t \div 8 = 7$
12. $9 \times 1 = u$ so $v \div 1 = 9$
13. $3 \times 4 = w$ so $12 \div 4 = x$
14. $6 \times 6 = y$ so $36 \div z = 6$

Complete the sentences.

15. Division undoes ___?___.
16. Division is the ___?___ of multiplication.

Name the products.

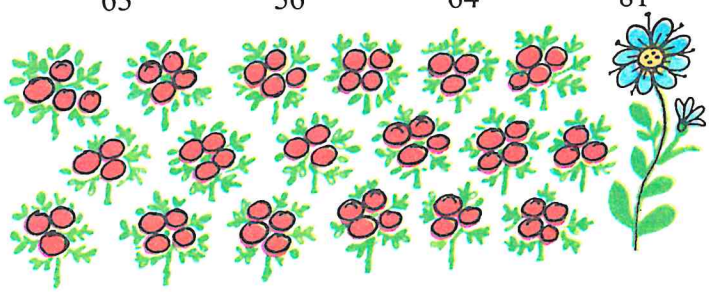
17. $7 \times 6$
18. $6 \times 8$
19. $3 \times 8$
20. $4 \times 7$
21. $9 \times 6$
22. $9 \times 5$
23. $7 \times 5$

Name the missing factors.

24. $6 \times a = 18$
25. $4 \times c = 20$
26. $6 \times e = 30$
27. $9 \times z = 63$
28. $k \times 6 = 36$
29. $l \times 8 = 64$
30. $n \times 9 = 81$

Solve the problem.

31. If there are 18 tomato plants in Mr. Johnston's garden and each row has 6 plants, how many rows are there?



Compare the equations and the working forms.

| Multiplication | Division |
|---|---|
| $3 \times 4 = 12$ $\quad \begin{array}{r} 4 \\ \times\ 3 \\ \hline 12 \end{array}$ | $12 \div 4 = 3$ $\quad 4\overline{)12}\ 3$ |

## EXERCISES

Solve the equations.

1. $3 \times 4 = 12$, so $12 \div 4 = a$
2. $6 \times 7 = 42$, so $42 \div 7 = b$
3. $6 \times 8 = 48$, so $48 \div 6 = c$
4. $9 \times 9 = 81$, so $81 \div 9 = d$
5. $5 \times 6 = 30$, so $30 \div 6 = e$
6. $7 \times 8 = 56$, so $56 \div 8 = f$
7. $9 \times 7 = 63$, so $63 \div 7 = g$
8. $3 \times 7 = 21$, so $21 \div 3 = h$

Name the products and the missing factors.

| | | | | | |
|---|---|---|---|---|---|
| 9. $\begin{array}{r} 9 \\ \times\ 6 \\ \hline \end{array}$ | 11. $\begin{array}{r} 8 \\ \times\ 7 \\ \hline \end{array}$ | 13. $\begin{array}{r} 7 \\ \times\ 6 \\ \hline \end{array}$ | 15. $\begin{array}{r} 4 \\ \times\ 9 \\ \hline \end{array}$ | 17. $\begin{array}{r} 9 \\ \times\ a \\ \hline 18 \end{array}$ | 19. $\begin{array}{r} 4 \\ \times\ i \\ \hline 28 \end{array}$ |
| 10. $\begin{array}{r} 7 \\ \times\ b \\ \hline 49 \end{array}$ | 12. $\begin{array}{r} 8 \\ \times\ c \\ \hline 72 \end{array}$ | 14. $\begin{array}{r} 5 \\ \times\ 9 \\ \hline \end{array}$ | 16. $\begin{array}{r} 8 \\ \times\ 6 \\ \hline \end{array}$ | 18. $\begin{array}{r} 9 \\ \times\ e \\ \hline 36 \end{array}$ | 20. $\begin{array}{r} 7 \\ \times\ n \\ \hline 42 \end{array}$ |

Name the products.

| | | | | | |
|---|---|---|---|---|---|
| 21. $9\overline{)a}\ 6$ | 23. $7\overline{)e}\ 3$ | 25. $4\overline{)i}\ 4$ | 27. $8\overline{)m}\ 4$ | 29. $5\overline{)p}\ 9$ | 31. $7\overline{)r}\ 2$ |
| 22. $6\overline{)c}\ 8$ | 24. $3\overline{)g}\ 4$ | 26. $2\overline{)j}\ 9$ | 28. $9\overline{)n}\ 5$ | 30. $8\overline{)q}\ 9$ | 32. $7\overline{)s}\ 6$ |

Name the missing factors.

| | | | | | |
|---|---|---|---|---|---|
| 33. $4\overline{)28}$ | 35. $6\overline{)36}$ | 37. $9\overline{)81}$ | 39. $8\overline{)32}$ | 41. $5\overline{)30}$ | 43. $7\overline{)42}$ |
| 34. $7\overline{)63}$ | 36. $9\overline{)27}$ | 38. $8\overline{)64}$ | 40. $6\overline{)54}$ | 42. $9\overline{)36}$ | 44. $7\overline{)56}$ |

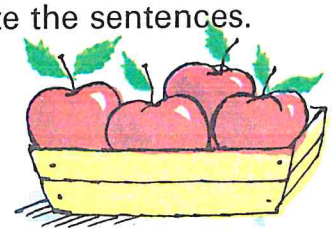
## Matching sets

John put 4 bananas on each plate, that is, 4 **per** plate. If he filled 6 plates with bananas, he had 6 sets of 4, so John had 24 bananas.

EXERCISES

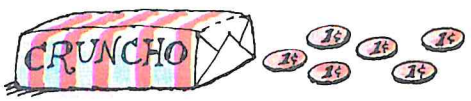
Complete the sentences.

**1.**



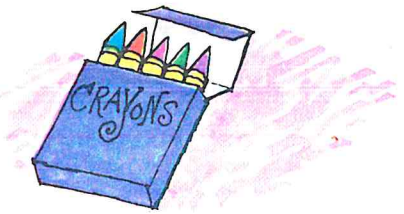
If there are 4 apples *per* box,
there are ***a*** apples in 2 boxes,
***b*** apples in 3 boxes,
and ***c*** apples in 4 boxes.

**2.**




If 1 candy bar costs 6¢,
2 candy bars cost ***g***¢,
5 candy bars cost ***h***¢,
and 7 candy bars cost ***i***¢.

**3.**




If there are 5 crayons *per* box,
there are ***d*** crayons in 2 boxes,
***e*** crayons in 4 boxes,
and ***f*** crayons in 6 boxes.

**4.**

If 1 pencil costs 3¢,
3 pencils cost ***j***¢,
6 pencils cost ***k***¢,
and 9 pencils cost ***l***¢.

**5.** There are 8 pints in 1 gallon, so there are ***a*** pints in 2 gallons, ***b*** pints in 8 gallons, and ***c*** pints in 5 gallons.

**6.** There are 7 days in 1 week, so there are ***e*** days in 3 weeks, ***f*** days in 6 weeks, ***g*** days in 5 weeks, and ***h*** days in 7 weeks.

**7.** If there are 9 students *per* row, there are ***i*** students in 2 rows.

**8.** If there are 6 doughnuts *per* box, there are ***j*** doughnuts in 3 boxes and ***k*** doughnuts in 4 boxes.

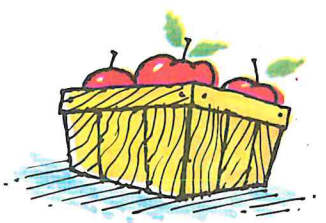
**9.** If there are 4 quarts *per* gallon, there are ***l*** quarts in 6 gallons.

**10.** If there are 5 school days *per* week, there are ***m*** school days in 7 weeks and ***n*** school days in 8 weeks.

**11.** If there are 8 marbles *per* bag, there are ***p*** marbles in 5 bags.

Ellen has 48 flowers to plant in 6 rows. She thinks, "I must separate the 48 flowers into 6 equivalent sets. $6 \times 8 = 48$, so I can form 6 sets of 8." Ellen will plant 8 flowers **per** row to put the 48 flowers in 6 rows.

1. There are 30 desks in 5 rows. If there are the same number of desks in each row, how many desks are there per row?



2. There are 45 apples in 9 boxes. If there are the same number of apples in each box, how many apples are there per box?

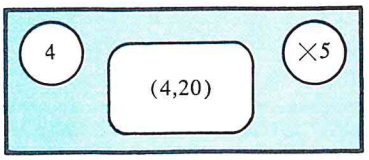
3. There are 54 books on 6 shelves. If there are the same number of books on each shelf, how many books are there per shelf?

4. Tom had 49 baseball cards to trade with 7 friends. If he gives each friend the same number of cards, how many cards does he give per friend?

5. Bobby and his friends walked 9 miles in 3 hours. If they walked the same number of miles each hour, how many miles per hour did they walk?

6. Sally and Betty walked 12 miles in 4 hours. If they walked the same number of miles each hour, how many miles per hour did they walk?

## Number pairs

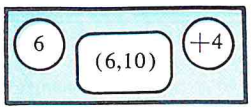



Tom turned the number knob to 4 and tuned in the rule *multiply by 5*.

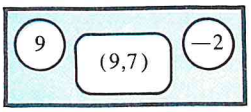
The screen showed the number pair (4, 20).

### EXERCISES

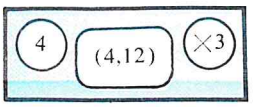
Use the function TV to name the rule. Then complete the number pairs in each set.

1. 

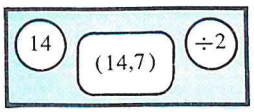
{(6, 10), (8, *a*), (10, *b*), (12, *c*), (16, *d*), (18, *e*), (14, *f*), (20, *g*)}

2. 

{(9, 7), (11, *a*), (13, *b*), (17, *c*), (19, *d*), (5, *e*), (15, *f*), (7, *g*)}

3. 

{(4, 12), (6, *a*), (9, *b*), (8, *c*), (9, *d*), (3, *e*), (2, *f*), (7, *g*)}

4. 

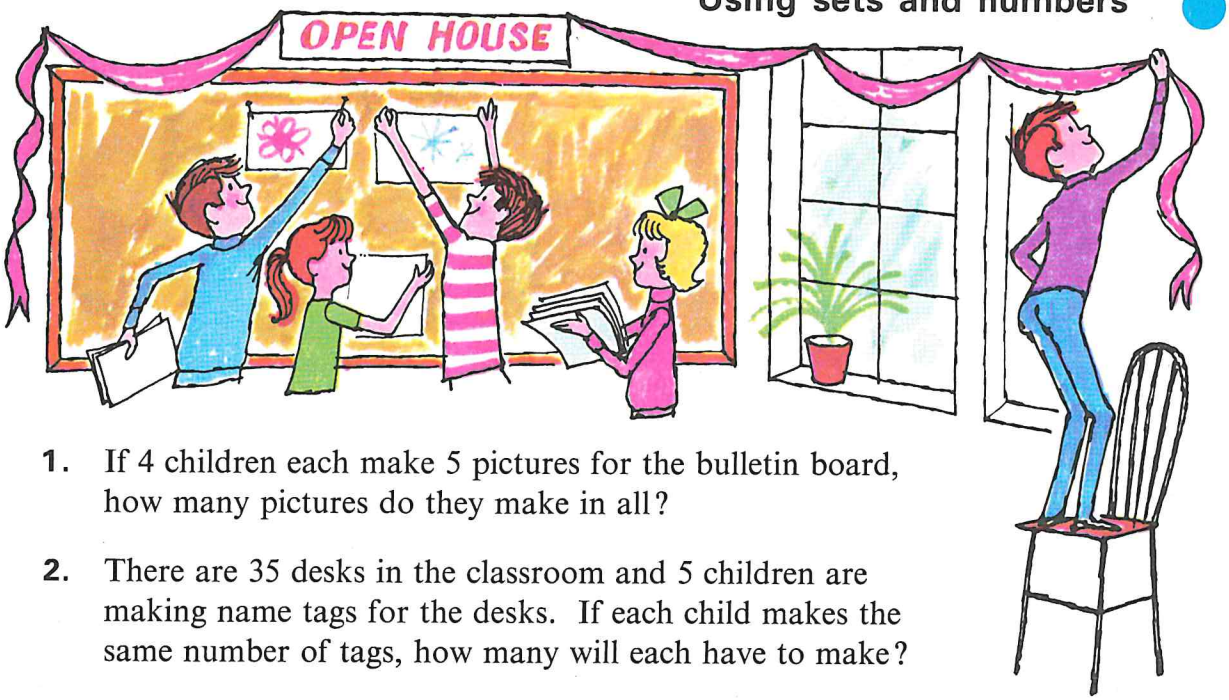
{(14, 7), (12, *a*), (18, *b*), (4, *c*), (10, *d*), (6, *e*), (16, *f*), (8, *g*)}

Name the addition or subtraction rule. Then complete the number pairs in each set.

5. {(36, 39), (37, 40), (40, *a*), (42, *b*), (45, *c*), (48, *d*), (49, *e*)}
6. {(72, 78), (75, 81), (77, *a*), (80, *b*), (82, *c*), (84, *d*), (86, *e*)}
7. {55, 48), (52, 45), (50, *a*), (49, *b*), (43, *c*), (37, *d*), (24, *e*)}

Name the multiplication or division rule. Then complete the number pairs in each set.

8. {(1, 5), (3, 15), (7, 35), (6, *a*), (9, *b*), (4, *c*), (2, *d*)}
9. {(12, 3), (8, 2), (4, 1), (20, *a*), (16, *b*), (24, *c*), (28, *d*)}
10. {(36, 6), (12, 2), (18, 3), (6, *a*), (42, *b*), (24, *c*), (48, *d*)}
11. {(3, 3), (6, 6), (5, 5), (7, *a*), (8, *b*), (2, *c*), (9, *d*)}
12. {(5, 0), (4, 0), (6, 0), (8, *a*), (7, *b*), (9, *c*), (3, *d*)}
13. {(7, 70), (9, 90), (11, 110), (20, *a*), (36, *b*), (78, *c*), (2, *d*)}
14. {(90, 9), (120, 12), (50, *a*), (270, *b*), (390, *c*), (1000, *d*)}




1. If 4 children each make 5 pictures for the bulletin board, how many pictures do they make in all?

2. There are 35 desks in the classroom and 5 children are making name tags for the desks. If each child makes the same number of tags, how many will each have to make?

3. If there are 8 windows with 8 panes per window, how many panes are there in all?

4. If each pane in exercise 3 is to have 2 pictures pasted on it, how many pictures will be needed for each window?

5. If there are 8 classrooms, and 9 children in each classroom will make reports for the parents, how many children will make reports in all?

6. If 48 students in all will be ushers for Parents' Night and the same number of ushers are needed for each of the 8 classrooms, how many ushers will there be per classroom?

7. If Philip is preparing a book display and wants to put 9 books on each desk, how many desks will be used to hold 90 books?

8. If 6 children make reports and each report takes 12 minutes, how long will all the reports take?

9. If the school parking lot can hold 100 cars, and each car can hold 6 people, how many people can come in cars to Parents' Night and park in the parking lot?

## Sharpshooting

Answer part A as quickly as you can. Then see if you can answer part B even more quickly.

### A

1. Name the product of 9 and 8.
2. $48 + 7 + 9 + 8 + 6 = n$
3. $8 + 8 + 8 + 8 + 8 + 8 = n$
4. If the product is 35 and one factor is 7, what is the other factor?
5. What is the cost of 7 sweaters at \$9 each?
6. Name the whole number which is greater than 19 and less than 21.
7. How many pints are in 7 gallons?
8. How many vertices are in 9 rectangles?
9. What is the total value of the 7 in 73,465?
10. $7 \times 98{,}672 = 690{,}704$, so $690{,}704 \div 7 = n$
11. What change do you get from \$1.00 if you spend 83¢?
12. What is the difference between 101 and 89?

### B

1. $8 \times 7 = n$
2. $34 - 16 = n$
3. $72 \div n = 8$
4. Name the product which has 4 as one factor and 7 as the other factor.
5. If 8 sweaters each cost the same and together they cost \$64, what is the cost of 1 sweater?
6. $4289 + 6349 = n$
7. What is the greatest number you can name with a four digit numeral?
8. How much less than 91 is 19?
9. How many days are there in 9 weeks?
10. $(8 \times 7) - (4 \times 4) = n$
11. If $A = \{a, b, c, d\}$ and $B = \{b, d, e, f\}$, how many objects are in $A \cup B$?
12. What is the cost of 9 muffins at 8¢ each?

### Be Careful!

There are 17 birds on the branch of a tree. A man shoots one bird. How many birds are left on the branch?

$$\begin{array}{r} 10 \\ 10 \\ 10 \\ 10 \\ +\ 10 \\ \hline 50 \end{array} \qquad 5 \times 10 = 5 \text{ tens} = 50$$

$$\begin{array}{r} 100 \\ 100 \\ 100 \\ 100 \\ +\ 100 \\ \hline 500 \end{array} \qquad 5 \times 100 = 5 \text{ hundreds} = 500$$

## EXERCISES

Name the products.

1. $7 \times 10 = a$
2. $2 \times 10 = b$
3. $8 \times 10 = c$
4. $19 \times 10 = d$
5. $43 \times 10 = e$
6. $83 \times 10 = f$
7. $3 \times 100 = g$
8. $9 \times 100 = h$
9. $14 \times 100 = i$
10. $18 \times 100 = j$
11. $45 \times 100 = k$
12. $38 \times 100 = l$

Name the missing factors.

13. $40 = a \times 10$
14. $50 = b \times 10$
15. $90 = c \times 10$
16. $130 = d \times 10$
17. $420 = e \times 10$
18. $760 = f \times 10$
19. $100 = g \times 10$
20. $100 = h \times 100$
21. $800 = i \times 100$
22. $1200 = j \times 100$
23. $2700 = k \times 100$
24. $1000 = l \times 100$

### The Associative Property of Multiplication

$2 \times (2 \times 4) = 2 \times 8 = 16$ $\qquad$ $(2 \times 2) \times 4 = 4 \times 4 = 16$

$$2 \times (2 \times 4) = (2 \times 2) \times 4$$

The equations show an example of the associative property.

The way the factors are grouped does not affect the product.

Solve the equations.

25. $(3 \times 2) \times 2 = a \times 2 = b$
    $3 \times (2 \times 2) = 3 \times c = d$
26. $(2 \times 2) \times 5 = e \times 5 = f$
    $2 \times (2 \times 5) = 2 \times g = h$
27. $3 \times (2 \times 5) = 3 \times i = j$
    $(3 \times 2) \times 5 = k \times 5 = l$
28. $4 \times (2 \times 5) = u \times n = p$
    $(4 \times 2) \times 5 = q \times r = s$

## Using the associative property

$$\begin{aligned} 3 \times 20 &= 3 \times (2 \times 10) \\ &= (3 \times 2) \times 10 \\ &= 60 \end{aligned}$$

$$\begin{aligned} 3 \times 200 &= 3 \times (2 \times 100) \\ &= (3 \times 2) \times 100 \\ &= 600 \end{aligned}$$

These equations show how the associative property is used in multiplying with tens and hundreds.

### EXERCISES

Copy and complete the equations.

1. $2 \times 30 = 2 \times (3 \times 10) = (2 \times a) \times 10 = b \times 10 = c$
2. $3 \times 40 = 3 \times (4 \times 10) = (3 \times d) \times 10 = e \times 10 = f$
3. $4 \times 60 = 4 \times (6 \times 10) = (4 \times g) \times 10 = h \times 10 = i$
4. $4 \times 200 = 4 \times (2 \times 100) = (4 \times j) \times 100 = k \times 100 = l$
5. $6 \times 300 = 6 \times (3 \times 100) = (6 \times m) \times 100 = n \times 100 = p$
6. $5 \times 700 = 5 \times (7 \times 100) = (5 \times q) \times 100 = r \times 100 = s$

Name the products.

| | | | | | | |
|---|---|---|---|---|---|---|
| 7. $10 \times 3$ | 9. $40 \times 3$ | 11. $40 \times 8$ | 13. $80 \times 5$ | 15. $400 \times 3$ | 17. $600 \times 8$ | 19. $900 \times 7$ |
| 8. $30 \times 3$ | 10. $60 \times 6$ | 12. $70 \times 4$ | 14. $300 \times 3$ | 16. $400 \times 2$ | 18. $700 \times 6$ | 20. $800 \times 5$ |

Name the missing factors.

| | | |
|---|---|---|
| 21. $3 \times a = 60$ | 26. $2 \times f = 160$ | 31. $3 \times k = 900$ |
| 22. $4 \times b = 80$ | 27. $3 \times g = 180$ | 32. $2 \times l = 800$ |
| 23. $2 \times c = 80$ | 28. $4 \times h = 120$ | 33. $6 \times m = 1800$ |
| 24. $3 \times d = 90$ | 29. $2 \times i = 400$ | 34. $7 \times n = 2100$ |
| 25. $4 \times e = 280$ | 30. $3 \times j = 600$ | 35. $8 \times z = 2400$ |

Complete the number chart.

36.

| **factor** | 6 | 7 | 8 | 7 | 8 | 6 | 200 | 9 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| **factor** | 10 | 40 | 70 | *d* | *e* | 200 | 7 | 400 | *i* |
| **product** | *a* | *b* | *c* | 490 | 240 | *f* | *g* | *h* | 2100 |

## Multiplying numbers greater than 10

Study the methods for naming the product $4 \times 23$.

| Expanded numerals | Number patterns |
|---|---|
| $\begin{array}{r} 23 \\ \times\ 4 \\ \hline \end{array} = \begin{array}{r} 20 + 3 \\ \times\ 4 \\ \hline 80 + 12 = 92 \end{array}$ | $\begin{array}{l} 4 \times 20 = 80 \\ 4 \times\ 3 = 12 \\ \hline 4 \times 23 = 92 \end{array}$ |

### EXERCISES

Use expanded numerals to name the products.

**1.** $\begin{array}{r} 16 \\ \times\ 4 \\ \hline \end{array}$ **2.** $\begin{array}{r} 13 \\ \times\ 5 \\ \hline \end{array}$ **3.** $\begin{array}{r} 17 \\ \times\ 8 \\ \hline \end{array}$ **4.** $\begin{array}{r} 18 \\ \times\ 5 \\ \hline \end{array}$ **5.** $\begin{array}{r} 19 \\ \times\ 6 \\ \hline \end{array}$ **6.** $\begin{array}{r} 35 \\ \times\ 4 \\ \hline \end{array}$ **7.** $\begin{array}{r} 26 \\ \times\ 3 \\ \hline \end{array}$

Complete the number patterns.

**8.** $\begin{array}{l} 4 \times 10 = a \\ 4 \times\ 6 = c \\ \hline 4 \times 16 = e \end{array}$ **10.** $\begin{array}{l} 5 \times 10 = k \\ 5 \times\ 7 = m \\ \hline 5 \times 17 = n \end{array}$ **12.** $\begin{array}{l} 6 \times 10 = v \\ 6 \times\ 4 = w \\ \hline 6 \times 14 = x \end{array}$ **14.** $\begin{array}{l} 7 \times 10 = f \\ 7 \times\ 3 = i \\ \hline 7 \times 13 = j \end{array}$

**9.** $\begin{array}{l} 3 \times 20 = f \\ 3 \times\ 4 = i \\ \hline 3 \times 24 = j \end{array}$ **11.** $\begin{array}{l} 6 \times 30 = r \\ 6 \times\ 2 = s \\ \hline 6 \times 32 = u \end{array}$ **13.** $\begin{array}{l} 8 \times 20 = a \\ 8 \times\ 6 = c \\ \hline 8 \times 26 = e \end{array}$ **15.** $\begin{array}{l} 8 \times 30 = k \\ 8 \times\ 9 = m \\ \hline 8 \times 39 = n \end{array}$

Use number patterns to name the products.

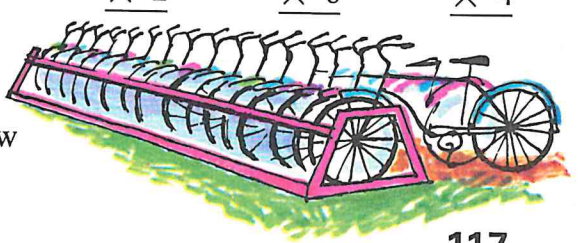
**16.** $\begin{array}{r} 15 \\ \times\ 3 \\ \hline \end{array}$ **19.** $\begin{array}{r} 23 \\ \times\ 8 \\ \hline \end{array}$ **22.** $\begin{array}{r} 18 \\ \times\ 2 \\ \hline \end{array}$ **25.** $\begin{array}{r} 13 \\ \times\ 7 \\ \hline \end{array}$ **28.** $\begin{array}{r} 18 \\ \times\ 6 \\ \hline \end{array}$ **31.** $\begin{array}{r} 17 \\ \times\ 3 \\ \hline \end{array}$ **34.** $\begin{array}{r} 19 \\ \times\ 7 \\ \hline \end{array}$

**17.** $\begin{array}{r} 13 \\ \times\ 4 \\ \hline \end{array}$ **20.** $\begin{array}{r} 18 \\ \times\ 3 \\ \hline \end{array}$ **23.** $\begin{array}{r} 13 \\ \times\ 9 \\ \hline \end{array}$ **26.** $\begin{array}{r} 14 \\ \times\ 8 \\ \hline \end{array}$ **29.** $\begin{array}{r} 16 \\ \times\ 5 \\ \hline \end{array}$ **32.** $\begin{array}{r} 17 \\ \times\ 4 \\ \hline \end{array}$ **35.** $\begin{array}{r} 21 \\ \times\ 5 \\ \hline \end{array}$

**18.** $\begin{array}{r} 22 \\ \times\ 9 \\ \hline \end{array}$ **21.** $\begin{array}{r} 12 \\ \times\ 7 \\ \hline \end{array}$ **24.** $\begin{array}{r} 36 \\ \times\ 5 \\ \hline \end{array}$ **27.** $\begin{array}{r} 17 \\ \times\ 6 \\ \hline \end{array}$ **30.** $\begin{array}{r} 38 \\ \times\ 2 \\ \hline \end{array}$ **33.** $\begin{array}{r} 19 \\ \times\ 8 \\ \hline \end{array}$ **36.** $\begin{array}{r} 29 \\ \times\ 4 \\ \hline \end{array}$

Solve the problem.

**37.** If there are 16 bicycles in each rack, how many bicycles are there in 4 racks?



## Preparing for the short method

> On the next page you will learn a short method for multiplying. The 3-step method will help you understand the short method.

Study the example.

$$\begin{array}{r} 28 \\ \times\ 6 \\ \hline 48 \\ 120 \\ \hline 168 \end{array}$$

$$\begin{aligned} 6 \times\ 8 &= 48 \\ 6 \times 20 &= 120 \\ \hline 6 \times 28 &= 168 \end{aligned}$$

Use the 3-step method to name the products.

| | | | | | |
|---|---|---|---|---|---|
| **1.** $\begin{array}{r} 12 \\ \times\ 4 \\ \hline \end{array}$ | **3.** $\begin{array}{r} 42 \\ \times\ 3 \\ \hline \end{array}$ | **5.** $\begin{array}{r} 58 \\ \times\ 7 \\ \hline \end{array}$ | **7.** $\begin{array}{r} 17 \\ \times\ 4 \\ \hline \end{array}$ | **9.** $\begin{array}{r} 47 \\ \times\ 4 \\ \hline \end{array}$ | **11.** $\begin{array}{r} 85 \\ \times\ 6 \\ \hline \end{array}$ |
| **2.** $\begin{array}{r} 31 \\ \times\ 6 \\ \hline \end{array}$ | **4.** $\begin{array}{r} 72 \\ \times\ 8 \\ \hline \end{array}$ | **6.** $\begin{array}{r} 72 \\ \times\ 9 \\ \hline \end{array}$ | **8.** $\begin{array}{r} 38 \\ \times\ 6 \\ \hline \end{array}$ | **10.** $\begin{array}{r} 73 \\ \times\ 9 \\ \hline \end{array}$ | **12.** $\begin{array}{r} 97 \\ \times\ 9 \\ \hline \end{array}$ |

> When you use the short method, you multiply and then add, as here: $(4 \times 6) + 3 = 24 + 3 = 27$.

Solve the equations.

| | | |
|---|---|---|
| **13.** $(8 \times 4) + 5 = a$ | **17.** $(7 \times 4) + 3 = e$ | **21.** $(8 \times 7) + 4 = i$ |
| **14.** $(6 \times 3) + 2 = b$ | **18.** $(7 \times 6) + 5 = f$ | **22.** $(8 \times 4) + 3 = j$ |
| **15.** $(7 \times 3) + 2 = c$ | **19.** $(8 \times 6) + 7 = g$ | **23.** $(4 \times 2) + 3 = k$ |
| **16.** $(8 \times 3) + 2 = d$ | **20.** $(8 \times 9) + 7 = h$ | **24.** $(7 \times 7) + 5 = l$ |

Name the ones' digit in the compact numeral for each of these.

| | | |
|---|---|---|
| **25.** $(4 \times 7) + 3$ | **30.** $(6 \times 8) + 5$ | **35.** $(8 \times 3) + 6$ |
| **26.** $(4 \times 2) + 2$ | **31.** $(9 \times 7) + 6$ | **36.** $(7 \times 7) + 2$ |
| **27.** $(8 \times 8) + 9$ | **32.** $(4 \times 9) + 8$ | **37.** $(9 \times 9) + 3$ |
| **28.** $(6 \times 6) + 7$ | **33.** $(4 \times 5) + 6$ | **38.** $(5 \times 7) + 2$ |
| **29.** $(6 \times 9) + 5$ | **34.** $(5 \times 5) + 9$ | **39.** $(6 \times 7) + 9$ |

**40–54.** Name the tens' digit in the compact numeral for exercises 25 to 39.

## Multiplying with numbers greater than 10

Study the methods for naming the product $4 \times 23$.

| Three step | | Short form | |
|---|---|---|---|
| 23 | | 1 | |
| $\times$ 4 | | 23 | $4 \times 3 = 12$ |
| 12 | $4 \times 3 = 12$ | $\times$ 4 | Write 2 in the ones' place. |
| 80 | $4 \times 20 = 80$ | 92 | $4 \times 2 = 8, \quad 8 + 1 = 9$ |
| 92 | $80 + 12 = 92$ | | Write 9 in the tens' place. |

### EXERCISES

Use the 3-step method to name the products.

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** 16 $\times$ 8 | **3.** 18 $\times$ 3 | **5.** 24 $\times$ 7 | **7.** 64 $\times$ 9 | **9.** 33 $\times$ 3 | **11.** 43 $\times$ 5 | **13.** 38 $\times$ 2 |
| **2.** 13 $\times$ 4 | **4.** 64 $\times$ 3 | **6.** 59 $\times$ 7 | **8.** 22 $\times$ 4 | **10.** 26 $\times$ 6 | **12.** 18 $\times$ 7 | **14.** 24 $\times$ 9 |

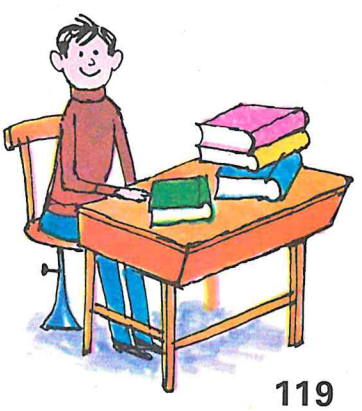
Use the short method to name the products.

| | | | | | | |
|---|---|---|---|---|---|---|
| **15.** 47 $\times$ 3 | **18.** 52 $\times$ 6 | **21.** 94 $\times$ 3 | **24.** 37 $\times$ 2 | **27.** 46 $\times$ 6 | **30.** 68 $\times$ 3 | **33.** 93 $\times$ 5 |
| **16.** 23 $\times$ 8 | **19.** 63 $\times$ 4 | **22.** 87 $\times$ 5 | **25.** 76 $\times$ 4 | **28.** 27 $\times$ 5 | **31.** 85 $\times$ 4 | **34.** 53 $\times$ 8 |
| **17.** 64 $\times$ 9 | **20.** 36 $\times$ 7 | **23.** 47 $\times$ 8 | **26.** 62 $\times$ 9 | **29.** 38 $\times$ 5 | **32.** 57 $\times$ 9 | **35.** 49 $\times$ 6 |

Solve the problems.

**36.** Jennie's teacher asked her to give 6 sheets of paper to each of the 32 students in the class. How many sheets of paper did Jennie give out?

**37.** There are 29 students in Ed's class. Each student has 4 books in his desk. How many books are there in all of the desks?



## Multiplying numbers greater than 100

Study the methods for naming the product $3 \times 228$.

| Expanded numerals | Four step |
|---|---|
| $228 = 200 + 20 + 8$<br>$\times 3 \qquad\qquad \times 3$<br>$600 + 60 + 24 = 684$ | $\begin{array}{r} 228 \\ \times\ 3 \\ \hline 24 \\ 60 \\ 600 \\ \hline 684 \end{array}$<br>$3 \times 8 = 24$<br>$3 \times 20 = 60$<br>$3 \times 200 = 600$<br>$3 \times 228 = 684$ |

### EXERCISES

Use expanded numerals to name the products.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | $\begin{array}{r} 317 \\ \times\ 2 \\ \hline \end{array}$ | **3.** | $\begin{array}{r} 162 \\ \times\ 4 \\ \hline \end{array}$ | **5.** | $\begin{array}{r} 281 \\ \times\ 7 \\ \hline \end{array}$ | **7.** | $\begin{array}{r} 461 \\ \times\ 8 \\ \hline \end{array}$ | **9.** | $\begin{array}{r} 516 \\ \times\ 6 \\ \hline \end{array}$ | **11.** | $\begin{array}{r} 731 \\ \times\ 5 \\ \hline \end{array}$ |
| **2.** | $\begin{array}{r} 314 \\ \times\ 6 \\ \hline \end{array}$ | **4.** | $\begin{array}{r} 541 \\ \times\ 8 \\ \hline \end{array}$ | **6.** | $\begin{array}{r} 366 \\ \times\ 4 \\ \hline \end{array}$ | **8.** | $\begin{array}{r} 281 \\ \times\ 9 \\ \hline \end{array}$ | **10.** | $\begin{array}{r} 719 \\ \times\ 8 \\ \hline \end{array}$ | **12.** | $\begin{array}{r} 889 \\ \times\ 7 \\ \hline \end{array}$ |

Use the 4-step method to name the products.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **13.** | $\begin{array}{r} 215 \\ \times\ 4 \\ \hline \end{array}$ | **16.** | $\begin{array}{r} 361 \\ \times\ 7 \\ \hline \end{array}$ | **19.** | $\begin{array}{r} 813 \\ \times\ 5 \\ \hline \end{array}$ | **22.** | $\begin{array}{r} 608 \\ \times\ 8 \\ \hline \end{array}$ | **25.** | $\begin{array}{r} 732 \\ \times\ 9 \\ \hline \end{array}$ | **28.** | $\begin{array}{r} 921 \\ \times\ 6 \\ \hline \end{array}$ |
| **14.** | $\begin{array}{r} 641 \\ \times\ 7 \\ \hline \end{array}$ | **17.** | $\begin{array}{r} 461 \\ \times\ 7 \\ \hline \end{array}$ | **20.** | $\begin{array}{r} 406 \\ \times\ 7 \\ \hline \end{array}$ | **23.** | $\begin{array}{r} 604 \\ \times\ 7 \\ \hline \end{array}$ | **26.** | $\begin{array}{r} 921 \\ \times\ 8 \\ \hline \end{array}$ | **29.** | $\begin{array}{r} 921 \\ \times\ 9 \\ \hline \end{array}$ |
| **15.** | $\begin{array}{r} 546 \\ \times\ 6 \\ \hline \end{array}$ | **18.** | $\begin{array}{r} 350 \\ \times\ 5 \\ \hline \end{array}$ | **21.** | $\begin{array}{r} 436 \\ \times\ 8 \\ \hline \end{array}$ | **24.** | $\begin{array}{r} 705 \\ \times\ 9 \\ \hline \end{array}$ | **27.** | $\begin{array}{r} 939 \\ \times\ 4 \\ \hline \end{array}$ | **30.** | $\begin{array}{r} 286 \\ \times\ 6 \\ \hline \end{array}$ |

Complete the number patterns.

**31.**

| $\times$ | 4 |
|---|---|
| 300 | $a$ |
| 20 | $b$ |
| 4 | $c$ |
| 324 | $d$ |

**32.**

| $\times$ | 6 |
|---|---|
| 100 | $e$ |
| 40 | $f$ |
| 6 | $g$ |
| 146 | $h$ |

**33.**

| $\times$ | 8 |
|---|---|
| 200 | $i$ |
| 70 | $j$ |
| 8 | $k$ |
| 278 | $l$ |

**34.**

| $\times$ | 7 |
|---|---|
| 300 | $m$ |
| 40 | $n$ |
| 2 | $p$ |
| $q$ | $r$ |

## Multiplying numbers greater than 100

Study the methods for naming the product $4 \times 386$.

| Four step | Short form |
|---|---|
| 386<br>× 4<br>24<br>320<br>1200<br>1544 | 32<br>38 6<br>× 4<br>1544<br>$4 \times 6 = 24$. Write 4 in the ones' place.<br>$4 \times 80 = 320$, $320 + 20 = 340$. Write 4 in the tens' place.<br>$4 \times 300 = 1200$, $1200 + 300 = 1500$. Write 5 in the hundreds' place and 1 in the thousands' place. |

### EXERCISES

Use the 4-step method to name the products.

| | | | | | |
|---|---|---|---|---|---|
| **1.** 234 × 2 | **2.** 432 × 3 | **3.** 362 × 3 | **4.** 416 × 3 | **5.** 428 × 3 | **6.** 427 × 4 |

Use the short method to name the products.

| | | | | | |
|---|---|---|---|---|---|
| **7.** 620 × 6 | **9.** 380 × 4 | **11.** 209 × 8 | **13.** 407 × 6 | **15.** 623 × 9 | **17.** 384 × 7 |
| **8.** 385 × 7 | **10.** 624 × 6 | **12.** 397 × 3 | **14.** 486 × 4 | **16.** 783 × 5 | **18.** 639 × 4 |

Solve the problems.

**19.** If there are 257 pages in each history book, how many pages are there in 6 history books?

**20.** There are 364 cars at the drive-in movie. If there are 4 people in each car, how many people are there in all?

**21.** If there are 156 candies in each bag, how many candies are there in 9 bags?

**22.** There are 480 sheets of paper in a ream. How many sheets of paper are there in 7 reams?

## Two-stage problems

A set of 19 combined with a set of 18 forms a set of $n$.
To solve this problem, you write the equation

$$19 + 18 = n.$$

Three sets of 4 combined with two sets of 5 form a set of $p$.
To solve this problem, you write the equation

$$(3 \times 4) + (2 \times 5) = p.$$

To solve this equation, you must think about **two** combinations of sets, three sets of 4 and two sets of 5. So you must *first* rename $(3 \times 4)$ and $(2 \times 5)$. Then you can add. Problems like this are called **two-stage problems.**

### EXERCISES

Ken had 4 bags each containing 5 apples. Elizabeth had 3 bags each containing 6 apples. How many apples did they have in all?

To solve the problem, think as follows:

*Step 1* **Ask: What are the sets?** 4 sets of 5 and 3 sets of 6.

*Step 2* **Ask: What is happening to the sets?** They are being joined.

*Step 3* **Write the equation.** The numbers are $(4 \times 5)$ and $(3 \times 6)$. Our equation is $(4 \times 5) + (3 \times 6) = n$.

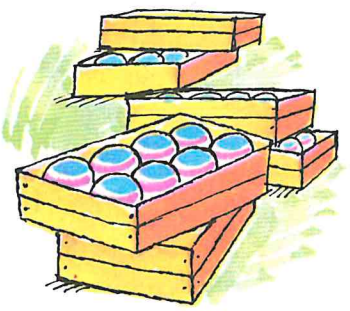
*Step 4* **Solve the equation.** $4 \times 5 = 20$ and $3 \times 6 = 18$, so $20 + 18 = 38$.

*Step 5* **Use the number in the problem.** They had 38 apples in all.

Solve the problems.

1. Yesterday Bob traveled for 3 hours at 40 miles an hour. Today he traveled for 4 hours at 40 miles an hour. How far did he travel in the two days?
2. Monica bought 5 dozen chocolate cookies, and Sue bought 3 dozen sugar cookies. If there are 12 cookies in a dozen, how many cookies did the girls buy in all?

1. Thirty balls are put into boxes. Each box can hold 8 balls. How many full boxes are there? How many balls are left over?

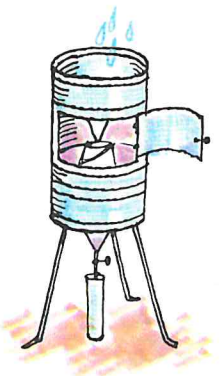


2. If there are 4 boxes each containing 5 balls and 6 boxes each containing 6 balls, how many balls are there in all?

3. Today we traveled for 4 hours at the rate of 45 miles an hour. Yesterday we traveled for 3 hours at the rate of 50 miles an hour. How much farther did we travel today than yesterday?

4. The men on Mr. Wells' fishing boat caught 73 pounds of fish on Monday and 86 pounds of fish on Tuesday. The men on Mr. Rust's boat caught 64 pounds of fish on Monday and 97 pounds of fish on Tuesday. Which boat took more fish? How much more?

5. Helen's scores on three tests were 67, 84, and 73. Ann's scores on the same tests were 71, 81, and 75. How much more than Helen did Ann score in all?



6. Last year it rained on 161 days. The year before that it rained on 147 days. Which year had more rainy days? How many more?

7. Which costs more, 3 rulers at 9 cents each or 4 erasers at 7 cents each? How much more?

8. In one week, Danny's dog ate 3 cans of food costing 29¢ each and his cat ate 4 cans of food costing 19¢ each. Which pet cost more to feed? How much more?

9. At the pet store, Danny bought a scratching post for $1.59 and 3 rubber bones for 89¢ each. How much did he spend in all?

## CHECKING UP

Complete the number pattern.

1.

| factor | 2 | 3 | 4 | 5 | 7 | 6 | 8 | 5 | 3 | 4 | 6 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| factor | 7 | 6 | 4 | 3 | 4 | 6 | 5 | $h$ | $i$ | $j$ | $k$ | $l$ |
| product | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | 45 | 21 | 20 | 48 | 63 |

Name the products and missing factors.

**2.** $7 \times 5 = a$

**3.** $5 \times b = 45$

**4.** $c \times 7 = 56$

**5.** $2 \times 8 = d$

**6.** $3 \times e = 12$

**7.** $2 \times 5 = f$

**8.** $9 \times g = 54$

**9.** $h \times 6 = 48$

**10.** $4 \times 7 = i$

**11.** $j \times 4 = 20$

**12.** $6 \times k = 36$

**13.** $9 \times 9 = l$

Solve the equations.

**14.**
$$\begin{array}{r} 3 \times 5 = a \\ 4 \times 5 = b \\ \hline 7 \times 5 = c \end{array}$$

**15.**
$$\begin{array}{r} 6 \times 4 = d \\ e \times 4 = 8 \\ \hline 8 \times 4 = f \end{array}$$

**16.**
$$\begin{array}{r} 7 \times g = 63 \\ 2 \times 9 = 18 \\ \hline h \times 9 = i \end{array}$$

Name the products.

**17.** $13 \times 5$

**18.** $47 \times 2$

**19.** $38 \times 3$

**20.** $57 \times 5$

**21.** $75 \times 6$

**22.** $46 \times 4$

**23.** $21 \times 9$

**24.** $32 \times 8$

**25.** $313 \times 3$

**26.** $423 \times 5$

**27.** $516 \times 5$

**28.** $742 \times 3$

**29.** $621 \times 7$

**30.** $317 \times 8$

**31.** $391 \times 3$

**32.** $152 \times 4$

**33.** $179 \times 2$

**34.** $254 \times 3$

**35.** $627 \times 4$

**36.** $628 \times 8$

**37.** $407 \times 6$

**38.** $523 \times 7$

**39.** $721 \times 8$

**40.** $852 \times 6$

**41.** $958 \times 7$

Write four equations for each set of two factors and a product.

**42.** {8, 7, 56}

**43.** {9, 7, 63}

**44.** {9, 5, 45}

**45.** {7, 6, 42}

**46.** {8, 9, 72}

**47.** {9, 6, 54}

To multiply, we rename two factors as a product. To divide, we rename a product and one factor as the other factor.

Write the numerals to complete the number pattern.

**1.**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **factor** | 3 | 6 | 7 | 4 | *e* | *f* | *g* | *h* | 8 | 3 | 7 | 6 |
| **factor** | 4 | 6 | 8 | 2 | 6 | 7 | 8 | 5 | *i* | *j* | *k* | *l* |
| **product** | *a* | *b* | *c* | *d* | 48 | 21 | 48 | 35 | 64 | 27 | 49 | 54 |

Multiplication and division are opposites.

Solve the equations.

**2.** $3 \times 6 = a$ so $18 \div 6 = b$

**3.** $4 \times 8 = c$ so $32 \div 8 = d$

**4.** $6 \times 9 = e$ so $54 \div 9 = f$

**5.** $6 \times 7 = g$ so $42 \div 6 = h$

Multiplication is commutative: the order of the factors does not affect the product.

Solve the equations.

**6.** $4 \times 3 = a \times 4$

**7.** $7 \times 2 = 2 \times b$

**8.** $3 \times 8 = 8 \times c$

There are 36 multiplication-division facts to be learned. They form families of number facts.

Write three other equations for each family of facts.

**9.** $3 \times 8 = 24$

**10.** $72 \div 9 = 8$

**11.** $6 \times 7 = 42$

The **Common Factor Rule:** to name the sum $(4 \times 3) + (5 \times 3)$ we may either add 12 and 15 or multiply $(4 + 5) \times 3$.

Copy and complete the equations.

**12.** $(5 \times 3) + (4 \times 3) = (5 + a) \times 3 = b \times 3 = c$

**13.** $4 \times (9 + 7) = (4 \times 9) + (4 \times d) = 36 + e = f$

## LOOKING WAY BACK

Set $A = \{2, 3, 5, 7, 11, 13, 17, 19\}$. Set $B = \{1, 3, 4, 7, 11, 18\}$.

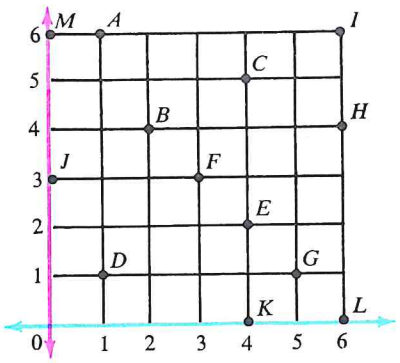
1. List the members of the union of set $A$ and set $B$.
2. Name the cardinal numbers of these sets.
   a. $n(A)$ b. $n(B)$ c. $n(A \cup B)$

Name a compact numeral for

3. four thousand, seven hundred fifty-eight
4. twenty-three thousand, nine hundred four
5. five million, eighty-six thousand, sixty-one
6. nine million, six hundred five thousand, seven

Use the number plane to name the number pair for each point.

7. $D$ 9. $H$ 11. $K$
8. $C$ 10. $F$ 12. $I$

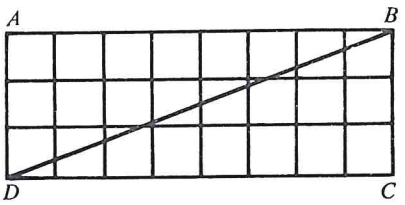



Name the letter for each point.

13. (2, 4) 15. (3, 3) 17. (0, 6)
14. (4, 2) 16. (5, 1) 18. (6, 0)

$ABCD$ is a rectangle.

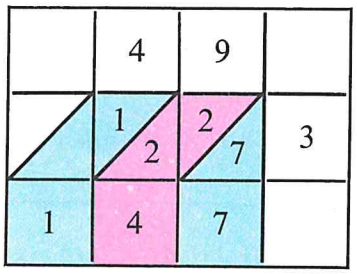
19. Name the perimeter of rectangle $ABCD$.
20. Name the perimeter of triangle $DCB$.
21. Name the area of rectangular region $ABCD$.
22. Name the area of triangular region $DCB$.



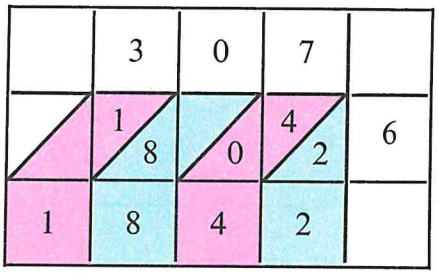

Name the sums or differences.

23. $46839 + 27406$
24. $73924 + 36561$
25. $50864 + 29657$
26. $624307 + 509806$
27. $183526 + 907477$
28. $347692 + 835038$
29. $3289 - 1706$
30. $7346 - 5827$
31. $5438 - 3439$
32. $42603 - 19846$
33. $83011 - 62108$
34. $64259 - 56179$

Look again at the method for addition which was shown on page 65. A similar method can be used for multiplication.




$$\begin{array}{r} 49 \\ \times\ 3 \\ \hline 27 \\ 120 \\ \hline 147 \end{array}$$




$$\begin{array}{r} 307 \\ \times\ 6 \\ \hline 42 \\ 0 \\ 1800 \\ \hline 1842 \end{array}$$

Use this method to name the products.

**1.** $\begin{array}{r} 63 \\ \times\ 6 \\ \hline \end{array}$ **2.** $\begin{array}{r} 13 \\ \times\ 7 \\ \hline \end{array}$ **3.** $\begin{array}{r} 38 \\ \times\ 5 \\ \hline \end{array}$ **4.** $\begin{array}{r} 283 \\ \times\ 2 \\ \hline \end{array}$ **5.** $\begin{array}{r} 357 \\ \times\ 3 \\ \hline \end{array}$ **6.** $\begin{array}{r} 807 \\ \times\ 8 \\ \hline \end{array}$

Study this example of the common factor property.

$5 \times 2 = 10$ and $3 \times 2 = 6$, so $2 \times 2 = 10 - 6 = 4$

Copy and complete the equations.

**7.** $6 \times 3 = a$ and $2 \times 3 = b$, so $4 \times 3 = c$

**8.** $9 \times 4 = d$ and $7 \times 4 = e$, so $2 \times 4 = f$

**9.** $8 \times 5 = g$ and $3 \times 5 = h$, so $5 \times 5 = z$

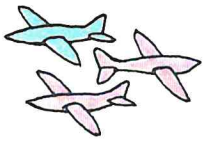
Copy and complete the equations.

**10.** $(8 \times 3) - (6 \times 3) = a - b = c$ $\quad (8 - 6) \times 3 = d \times 3 = e$

**11.** $(7 \times 4) - (4 \times 4) = f - g = h$ $\quad (7 - 4) \times 4 = i \times 4 = j$

**12.** $(5 \times 8) - (4 \times 8) = k - l = m$ $\quad (5 - 4) \times 8 = n \times 8 = p$

## Comparing subsets with sets



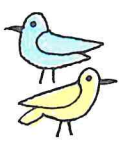
1 of 3 airplanes is blue.
$\frac{1}{3}$ of the airplanes is blue.

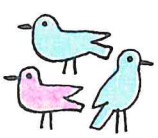
2 of 3 airplanes are gray.
$\frac{2}{3}$ of the airplanes are gray.

> $\frac{1}{3}$ and $\frac{2}{3}$ are **fractions.** They are numerals for **fractional numbers.** Fractional numbers may be used to compare the cardinal number of a subset with the cardinal number of the set.

### EXERCISES

For each set, name the fractional number which compares the number of blue objects with the total number of objects.

1. 

2. 

3.

4.

5.

6.

7.

8.

$\frac{1}{5}$ of the stars is red.

One fifth of the stars is red.

$\frac{1}{5}$ and *one fifth* are two names for the same fractional number.

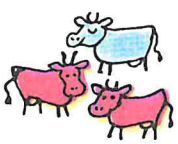
## EXERCISES

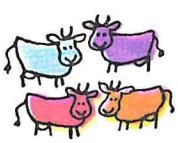
For each set, write a fraction to compare the number of red objects with the total number of objects.

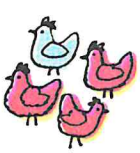
Write each fraction in two ways, as we wrote $\frac{1}{5}$ and *one fifth*.

1.

2.

3. 

4. 

5. 

6.

7.

8.

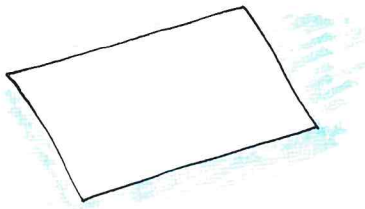
9.

10.

Complete the sentence.

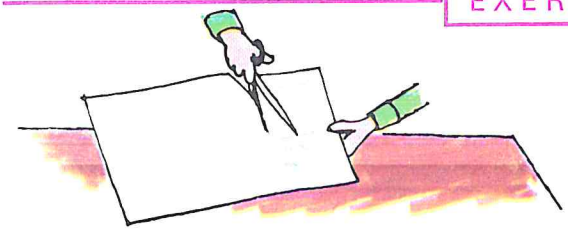
11. A ___?___ number may be used to compare a subset with the set.

## Comparing regions



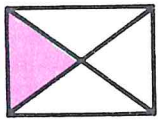
Peter had a piece of paper which he cut into two parts *of the same size*. He used *1 of the 2* pieces of paper. He used *one half* of the paper.

### EXERCISES



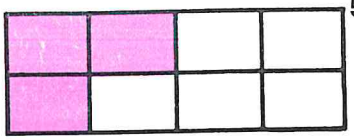
1. Can you say that one half of the paper was written on? Why?

For each figure, write a fraction to compare the shaded region with the total region.

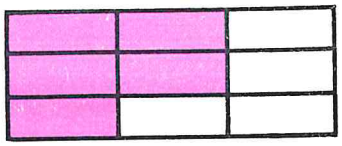
2. 

4.

6.

3. 

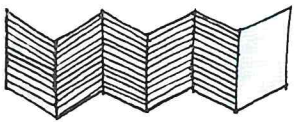
5.

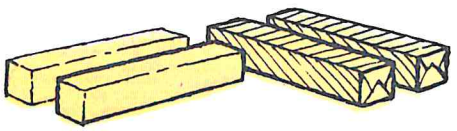
7. 

For each exercise, write a fraction to compare the shaded part with the whole object.

8.

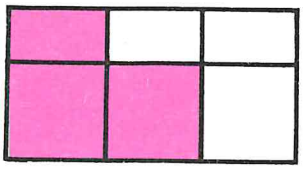
10.

9. 

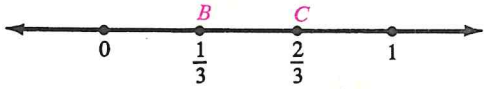
11. 

## Fractional number lines

Study the colored rods and the number line.



One third of the top rod is red.
Two thirds of the bottom rod are red.

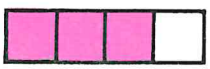


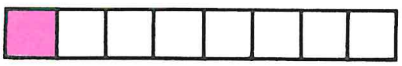

$B$ labels the point $\frac{1}{3}$.

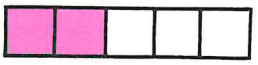
$C$ labels the point $\frac{2}{3}$.

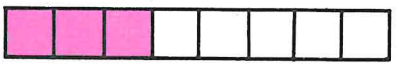
### EXERCISES

Write a fraction for the red part of each rod

1. 

2. 

3. 

4. 

Write a fraction for the red segment of each number line.

5.


6.


7.


8.


9.


10.


11.


12.

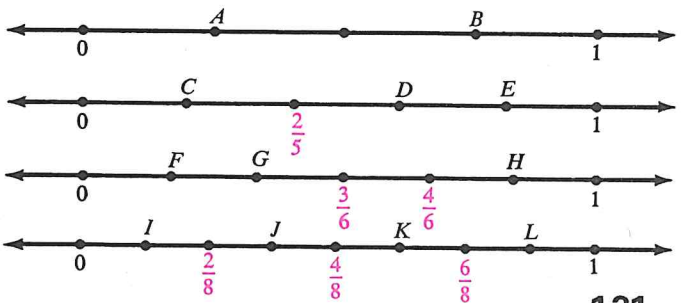

Write a fraction for each point labeled on the number lines.

13. $A$ 17. $E$ 21. $I$

14. $B$ 18. $F$ 22. $J$

15. $C$ 19. $G$ 23. $K$

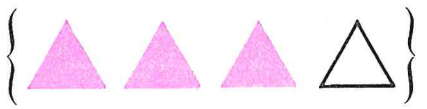
16. $D$ 20. $H$ 24. $L$



## The two numerals in a fraction

$\frac{3}{4}$ of the regions are red.

*Three fourths* of the regions are red.



The 4 names the number of objects in the set and names them *fourths*. It is called the **denominator.** The 3 names the number of objects being compared with the total set. It is called the **numerator.**

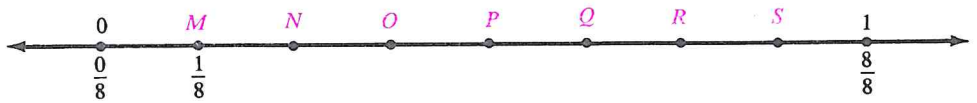
### EXERCISES

Write the numerator and the denominator of the fraction for each subset of red objects.

**1.** { C C C © } **2.** { ◆ ◇ ◇ ◇ } **3.** { ★ ★ ★ ★ ☆ }

| | | | |
|---|---|---|---|
| **numerator:** | $a$ | $c$ | $v$ |
| **denominator:** | $p$ | $e$ | $g$ |

Complete the chart by writing the numerator and the denominator for each fraction on the number line.




**4.**

| point | $M$ | $N$ | $O$ | $P$ | $Q$ | $R$ | $S$ |
|---|---|---|---|---|---|---|---|
| numerator | 1 | $a$ | $b$ | $d$ | $e$ | $g$ | $h$ |
| denominator | 8 | 8 | $c$ | 8 | $f$ | 8 | $i$ |

Write fractions with these numerators and denominators.

**5.**

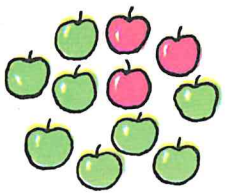
| numerator | 1 | 1 | 1 | 1 | 2 | 3 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| denominator | 2 | 3 | 4 | 5 | 3 | 4 | 5 | 5 | 5 |
| fraction | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ |

Name the numerator in

**6.** two fifths **7.** three fourths **8.** one fifth **9.** three fifths **10.** one half

**11–15.** Name the denominators in exercises 6 to 10.

Write a fraction to compare the number of red objects with the total number of objects.

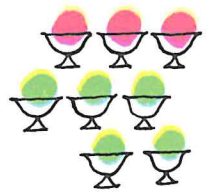
1. 

3.

5.

2.

4.

6. 

7. Name the numerator of each fraction in exercises 1 to 6.

8. Name the denominator of each fraction in exercises 1 to 6.

Write fractions to complete the sentences.

9. Two girls share a box of pennies evenly. Each girl takes ___?___ of the pennies.

10. Three boys share a bag of jelly beans evenly. Each boy takes ___?___ of the jelly beans.

11. If seven girls share the work evenly, each will do ___?___ of the work.

12. Of four boys, one is 10 years old and three are 11 years old. ___?___ of the boys are 11 years old.

13. In a family, four of the children are boys and one is a girl. ___?___ of the children are boys.

14. There are seven days in a week. Saturday and Sunday are ___?___ of the days in a week.

| JUNE | | | | | | |
|---|---|---|---|---|---|---|
| S | M | T | W | T | F | S |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |

## Joining subsets

$\frac{1}{3}$ of the regions is red.

$\frac{1}{3}$ of the regions is green.

$\frac{2}{3}$ of the regions are red or green.

$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

### EXERCISES

Study the sets. Then solve the equations.

1. $\frac{1}{4} + \frac{1}{4} = \frac{a}{4}$

2. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{b}{4}$

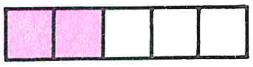
3. $\frac{1}{3} + \frac{1}{3} = \frac{c}{3}$

4. $\frac{1}{5} + \frac{1}{5} = \frac{d}{5}$

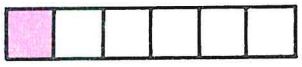
5. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{e}{5}$

6. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{f}{5}$

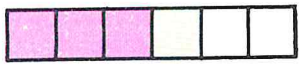
Study the regions. Then solve the equations.



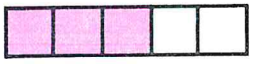
7. $\frac{2}{5} + \frac{1}{5} = \frac{a}{5}$



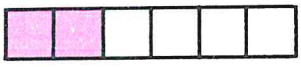
9. $\frac{1}{6} + \frac{1}{6} = \frac{c}{6}$



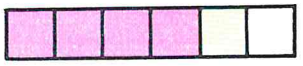
11. $\frac{3}{6} + \frac{1}{6} = \frac{e}{6}$



8. $\frac{3}{5} + \frac{1}{5} = \frac{b}{5}$



10. $\frac{2}{6} + \frac{1}{6} = \frac{d}{6}$



12. $\frac{4}{6} + \frac{1}{6} = \frac{f}{6}$

Study the sets. Then use fractions to complete the sentences.

13. ___?___ of the objects are colored.

14. ___?___ of the objects are colored.

15. ___?___ of the objects are colored.

16. ___?___ of the objects are colored.

Each piece is $\frac{1}{4}$, one fourth, of the pie. Three pieces form three fourths of the pie.

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4}$$

$$3 \times \frac{1}{4} = \frac{3}{4} \leftarrow \text{numerator}$$
$$\qquad\qquad\quad \leftarrow \text{denominator}$$

## EXERCISES

Rename each number in a simpler form.

**1.** $3 \times \frac{1}{5} = a$ **3.** $4 \times \frac{1}{5} = c$ **5.** $2 \times \frac{1}{3} = e$ **7.** $2 \times \frac{1}{5} = g$

**2.** $4 \times \frac{1}{6} = b$ **4.** $5 \times \frac{1}{6} = d$ **6.** $7 \times \frac{1}{10} = f$ **8.** $9 \times \frac{1}{10} = h$

In the number $3 \times \frac{1}{4}$, the 3 and $\frac{1}{4}$ are factors. Name each fractional number as the product of two factors.

**9.** $\frac{3}{5} = a$ **11.** $\frac{4}{5} = c$ **13.** $\frac{2}{3} = e$ **15.** $\frac{2}{5} = g$

**10.** $\frac{4}{6} = b$ **12.** $\frac{5}{6} = d$ **14.** $\frac{7}{10} = f$ **16.** $\frac{9}{10} = h$

Write another name for each fractional number.

**17.** $5 \times \frac{1}{10}$ **18.** $6 \times \frac{1}{7}$ **19.** $5 \times \frac{1}{6}$ **20.** $2 \times \frac{1}{6}$

Name each fractional number in two other ways.

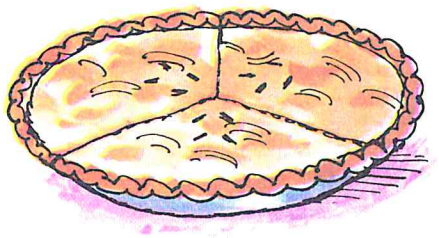
**21.** two thirds **23.** three fifths **25.** three fourths

**22.** four fifths **24.** five sixths **26.** seven eighths

Write two names for the fractional number to compare the number of red objects with the total number of objects in each set.

**27.** {● ● ○} **28.** {◆ ◇ ◇ ◇} **29.** {▲ ▲ ▲ △} **30.** {★ ★ ☆ ☆ ☆}

## The whole set



Ellen ate $\frac{2}{3}$ of the pie.

Bob ate $\frac{1}{3}$ of the pie.

Together they ate the whole pie.

$\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$ $\frac{3}{3}$ is another name for 1.

## EXERCISES

Study the sets. Then solve the equations.

1. { ● ● }

$\frac{1}{2} + \frac{1}{2} = \frac{a}{2} = b$

2. { ● ● ● ● }

$\frac{2}{4} + \frac{2}{4} = \frac{c}{4} = d$

3. { ● ● ● ● }

$\frac{3}{4} + \frac{1}{4} = \frac{e}{4} = f$

4. { ● ● ● ● ● }

$\frac{4}{5} + \frac{1}{5} = \frac{h}{5} = i$

5. { ● ● ● }

$\frac{1}{3} + \frac{2}{3} = \frac{k}{3} = l$

6. { ● ● ● ● ● }

$\frac{3}{5} + \frac{2}{5} = \frac{m}{5} = n$

Study the regions. Then solve the equations.

7.

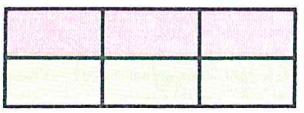
$\frac{1}{2} + \frac{1}{2} = \frac{a}{2} = b$

8.

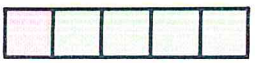
$\frac{3}{5} + \frac{2}{5} = \frac{c}{5} = d$

9.

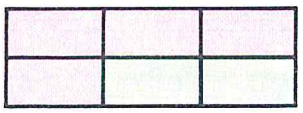
$\frac{1}{4} + \frac{3}{4} = \frac{e}{4} = f$

10. 

$\frac{3}{6} + \frac{3}{6} = \frac{h}{6} = i$

11. 

$\frac{1}{5} + \frac{4}{5} = \frac{k}{5} = l$

12. 

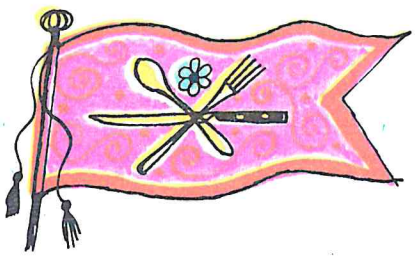
$\frac{4}{6} + \frac{2}{6} = \frac{m}{6} = n$

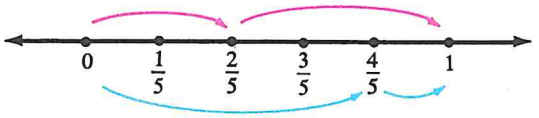
Solve the equations.

13. $1 = \frac{a}{2}$ 14. $1 = \frac{b}{3}$ 15. $1 = \frac{c}{4}$ 16. $1 = \frac{d}{5}$

Solve the problem.

17. Mary, Ellen and Sue each sewed one third of their club's flag. What part of the flag did they sew all together?



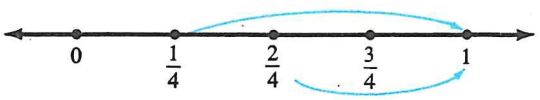



The red arrows show $\frac{2}{5} + \frac{3}{5} = \frac{5}{5} = 1$

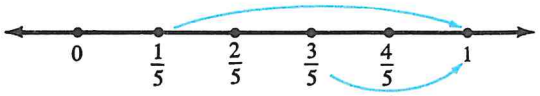
The blue arrows show $\frac{4}{5} + \frac{1}{5} = \frac{5}{5} = 1$

## EXERCISES

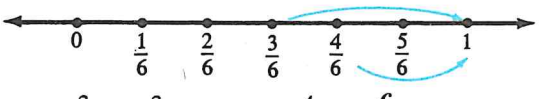
Use the number lines to solve the equations.




**1.** $\frac{1}{4} + \frac{3}{4} = a \qquad \frac{2}{4} + \frac{b}{4} = 1$

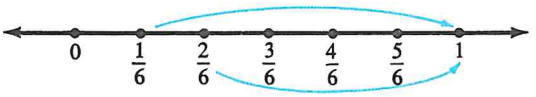



**2.** $\frac{1}{5} + \frac{4}{5} = c \qquad \frac{3}{5} + \frac{d}{5} = 1$

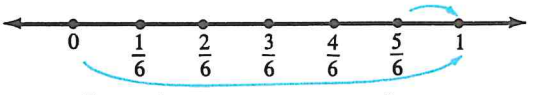



**3.** $\frac{3}{6} + \frac{3}{6} = e \qquad \frac{4}{6} + \frac{f}{6} = g$

**4.** $\frac{1}{3} + \frac{2}{3} = h \qquad \frac{2}{3} + \frac{i}{3} = 1$

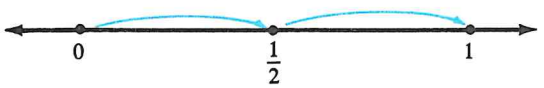



**5.** $\frac{1}{6} + \frac{5}{6} = j \qquad \frac{2}{6} + \frac{k}{6} = 1$

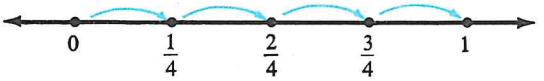



**6.** $\frac{5}{6} + \frac{1}{6} = l \qquad 0 + \frac{6}{6} = m$

Use the number lines to solve the equations.

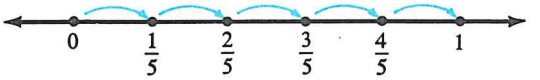



**7.** $2 \times \frac{1}{2} = a$




**8.** $b \times \frac{1}{4} = 1$

**9.** $c \times \frac{1}{3} = 1$




**10.** $5 \times \frac{1}{5} = d$

Complete this set of names for 1.

**11.** $\left\{\frac{2}{2}, \frac{a}{3}, \frac{b}{4}, \frac{5}{c}, \frac{d}{6}, \frac{7}{e}, \frac{f}{8}, \frac{9}{g}, \frac{h}{10}\right\}$

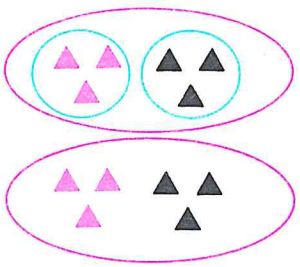
## Many names for one fractional number

*One* subset of red objects.
*Two* equivalent subsets in all.

The subset of red objects is $\frac{1}{2}$ of the set.

*Three* red objects.
*Six* objects in all.

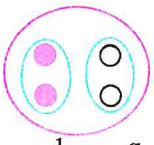
The red objects are $\frac{3}{6}$ of the set.



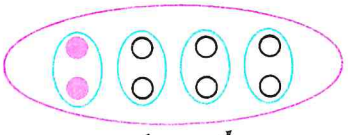
$\frac{1}{2} = \frac{3}{6}$

### EXERCISES

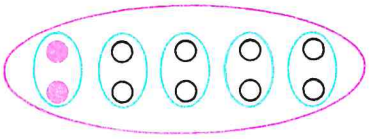
Study each set and solve the equation.



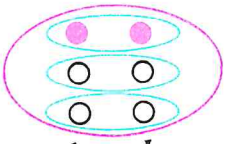
1. $\frac{1}{2} = \frac{a}{4}$



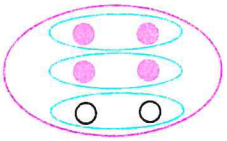
4. $\frac{1}{4} = \frac{d}{8}$



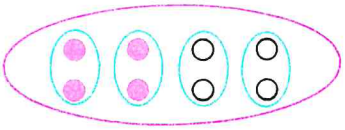
7. $\frac{1}{5} = \frac{i}{10}$



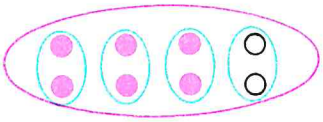
2. $\frac{1}{3} = \frac{b}{6}$



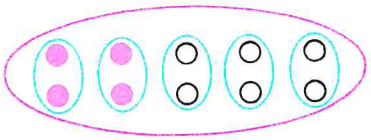
5. $\frac{2}{3} = \frac{e}{6}$



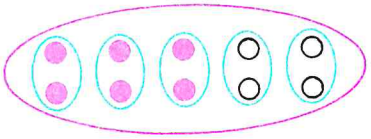
8. $\frac{1}{2} = \frac{k}{4} = \frac{l}{8}$



3. $\frac{3}{4} = \frac{c}{8}$



6. $\frac{2}{5} = \frac{f}{10}$

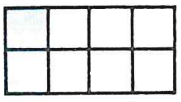


9. $\frac{3}{5} = \frac{m}{10}$

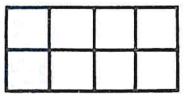
Study each region and solve the equation.

10. $\frac{1}{2} = \frac{2}{4}$

12. $\frac{4}{6} = \frac{b}{3}$

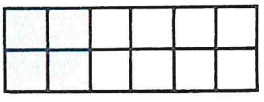


14. $\frac{1}{4} = \frac{d}{8}$



11. $\frac{2}{8} = \frac{a}{4}$

13. $\frac{3}{5} = \frac{c}{10}$



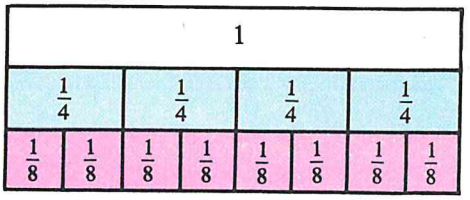
15. $\frac{4}{12} = \frac{e}{3}$

## Many names for one fractional number

The rods show

$\frac{1}{4} = \frac{2}{8}$ $\quad \frac{2}{4} = \frac{4}{8}$

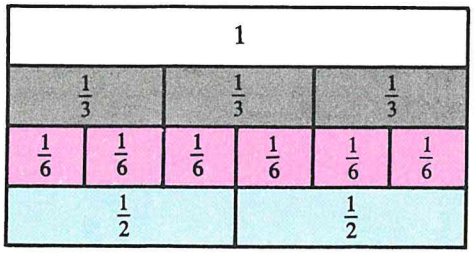
$\frac{3}{4} = \frac{6}{8}$ $\quad \frac{4}{4} = \frac{8}{8} = 1$




### EXERCISES

Solve the equations.

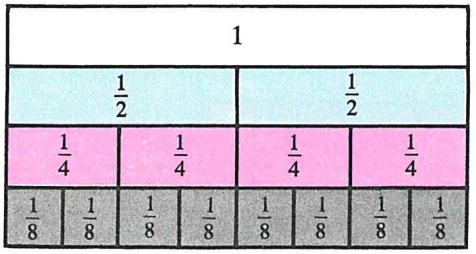
**1.**




$\frac{1}{3} = \frac{2}{6}$ $\quad \frac{2}{3} = \frac{a}{6}$

$\frac{1}{2} = \frac{b}{6}$ $\quad 1 = \frac{c}{6}$

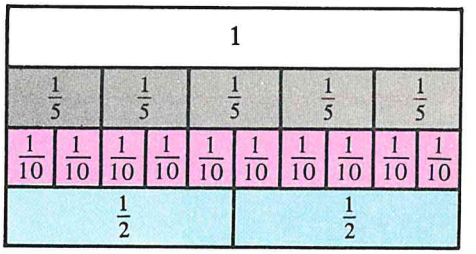
**2.**




$\frac{1}{2} = \frac{d}{4}$ $\quad \frac{1}{2} = \frac{e}{8}$

$\frac{1}{4} = \frac{f}{8}$ $\quad \frac{3}{4} = \frac{h}{8}$

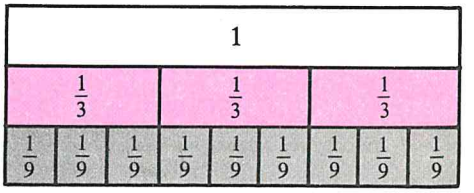
**3.**




$\frac{1}{5} = \frac{i}{10}$ $\quad \frac{2}{5} = \frac{k}{10}$

$\frac{1}{2} = \frac{l}{10}$ $\quad 1 = \frac{m}{10}$

**4.**




$\frac{1}{3} = \frac{n}{9}$ $\quad \frac{2}{3} = \frac{r}{9}$

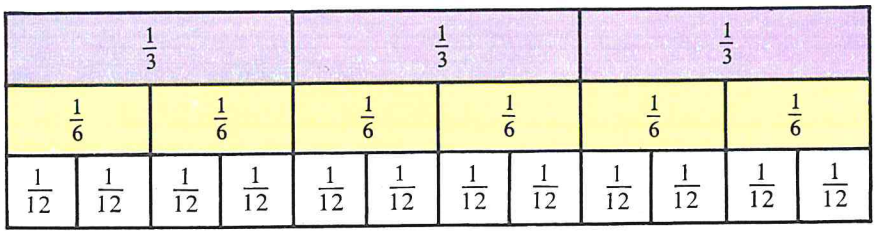
$\frac{3}{3} = \frac{s}{9}$ $\quad 1 = \frac{t}{9}$

Solve the problems.

**5.** Brenda said, "One fourth of the apples are green." Bill said, "Two eighths of the apples are green." Are Brenda and Bill talking about the same number of apples?

**6.** Sandra said, "Six eighths of the apples are red." Margo said, "Three fourths of the apples are red." Are Sandra and Margo talking about the same number of apples?

## Many names for one fractional number



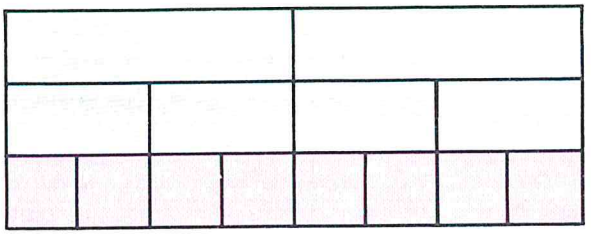

The colored rods show that $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{4}{12}$ are different names for the same fractional number. $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$

### EXERCISES

Use the rods above to solve the equations.

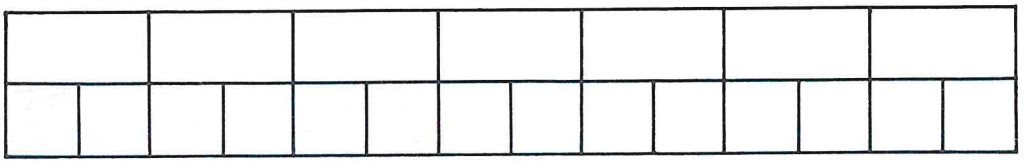
1. $\frac{2}{3} = \frac{a}{6}$ 2. $\frac{2}{3} = \frac{b}{12}$ 3. $\frac{4}{6} = \frac{c}{12}$

Study the rods and solve the equations.



4. $\frac{1}{4} = \frac{a}{8}$ 5. $\frac{3}{4} = \frac{b}{8}$ 6. $\frac{1}{2} = \frac{c}{4} = \frac{d}{8}$

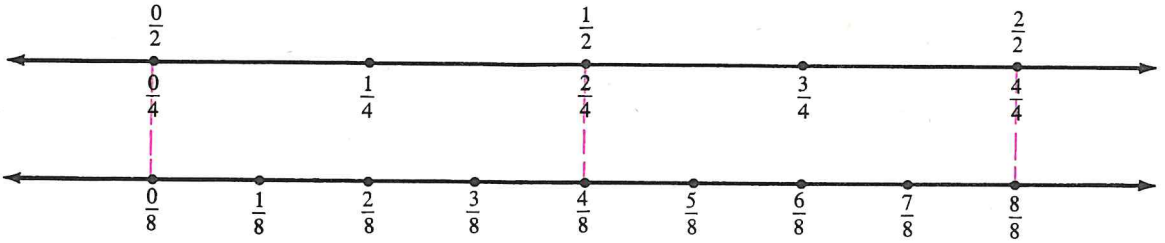
Study the rods and solve the equations.



7. $\frac{1}{7} = \frac{a}{14}$ 9. $\frac{3}{7} = \frac{c}{14}$ 11. $\frac{5}{7} = \frac{e}{14}$ 13. $\frac{8}{14} = \frac{h}{7}$

8. $\frac{2}{7} = \frac{b}{14}$ 10. $\frac{4}{7} = \frac{d}{14}$ 12. $\frac{6}{7} = \frac{f}{14}$ 14. $\frac{6}{14} = \frac{i}{7}$

## Many names for one fractional number



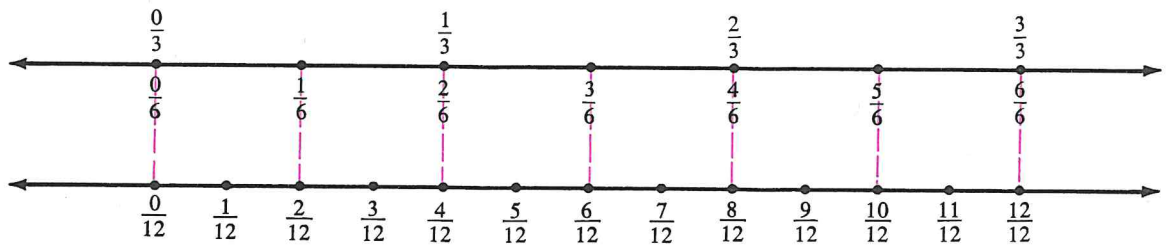

The number lines show that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ are different names for the same fractional number. $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

## EXERCISES

Use the number lines above to solve the equations.

**1.** $\frac{1}{4} = \frac{a}{8}$ **2.** $\frac{3}{4} = \frac{b}{8}$ **3.** $1 = \frac{c}{4}$ **4.** $1 = \frac{d}{8}$

Study the number lines and solve the equations.




**5.** $\frac{1}{3} = \frac{a}{6}$ **8.** $\frac{4}{6} = \frac{d}{3}$ **11.** $\frac{1}{3} = \frac{h}{12}$ **14.** $\frac{2}{3} = \frac{l}{12}$

**6.** $\frac{3}{6} = \frac{b}{12}$ **9.** $\frac{6}{12} = \frac{e}{4}$ **12.** $\frac{3}{6} = \frac{i}{4}$ **15.** $\frac{1}{2} = \frac{m}{4}$

**7.** $\frac{4}{12} = \frac{c}{3}$ **10.** $\frac{8}{12} = \frac{f}{3}$ **13.** $\frac{10}{12} = \frac{k}{6}$ **16.** $\frac{4}{12} = \frac{n}{6}$

Complete each set of names for the same fractional number.

**17.** $\left\{\frac{1}{2}, \frac{a}{4}, \frac{b}{8}, \frac{c}{6}, \frac{d}{12}\right\}$ **19.** $\left\{\frac{1}{3}, \frac{e}{6}, \frac{b}{12}\right\}$ **21.** $\left\{\frac{1}{4}, \frac{a}{8}, \frac{b}{12}\right\}$

**18.** $\left\{\frac{3}{4}, \frac{a}{8}, \frac{b}{12}\right\}$ **20.** $\left\{\frac{2}{3}, \frac{a}{6}, \frac{b}{12}\right\}$ **22.** $\left\{\frac{5}{6}, \frac{a}{12}, \frac{b}{18}\right\}$

## Equivalent fractions

$\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \cdots\right\}$

$\left\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \cdots\right\}$

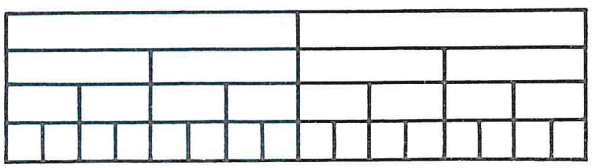
$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \cdots\right\}$

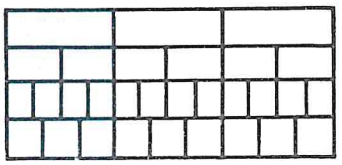
Fractions that name the same fractional number are **equivalent.** The sets on the left are sets of equivalent fractions.

In each set, the simplest numeral is in red. This is the fraction with the least denominator.

### EXERCISES

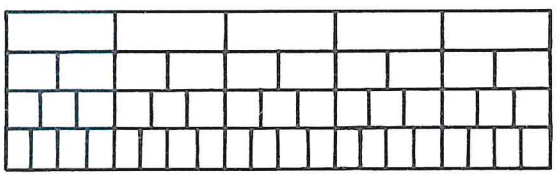
Study the rods. Then solve the equations.

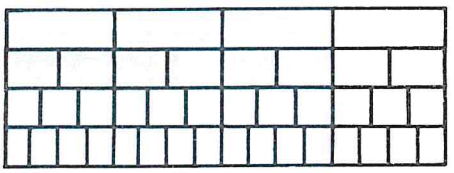




1. $\frac{1}{2} = \frac{a}{4} = \frac{b}{8} = \frac{c}{16}$

3. $\frac{1}{3} = \frac{i}{6} = \frac{k}{9} = \frac{l}{12}$

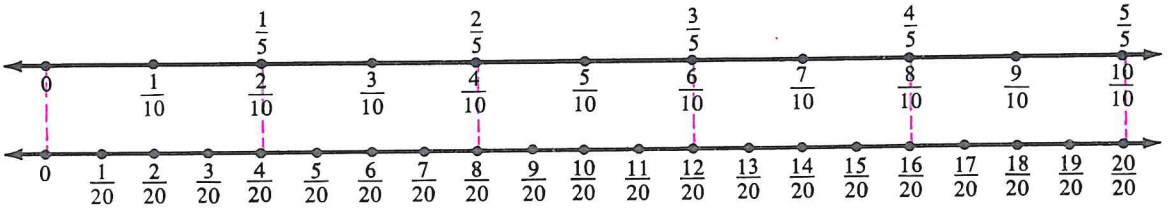




2. $\frac{1}{5} = \frac{d}{10} = \frac{e}{15} = \frac{h}{20}$

4. $\frac{3}{4} = \frac{m}{8} = \frac{n}{12} = \frac{r}{16}$

Study the number lines. Then solve the equations.




5. $\frac{1}{5} = \frac{a}{10} = \frac{b}{20}$

7. $\frac{3}{5} = \frac{r}{10} = \frac{s}{20}$

6. $\frac{2}{5} = \frac{c}{10} = \frac{d}{20}$

8. $\frac{4}{5} = \frac{h}{10} = \frac{i}{20}$

$\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \ldots\right\}$ is a set of equivalent fractions.

We see there is a pattern:

$\frac{2}{3} = \frac{1 \times 2}{1 \times 3}$ $\quad \frac{4}{6} = \frac{2 \times 2}{2 \times 3}$ $\quad \frac{6}{9} = \frac{3 \times 2}{3 \times 3}$ $\quad \frac{8}{12} = \frac{4 \times 2}{4 \times 3}$

It is easy to remember the pattern because all the equivalent fractions in the set $\left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \ldots\right\}$ are other names for 1.

## EXERCISES

Copy and complete these equations.

1. $\frac{4}{8} = \frac{a \times 1}{a \times 2}$
2. $\frac{5}{10} = \frac{v \times 1}{v \times 2}$
3. $\frac{6}{12} = \frac{c \times 1}{c \times 2}$
4. $\frac{4}{10} = \frac{r \times 2}{r \times 5}$
5. $\frac{6}{8} = \frac{2 \times c}{2 \times u}$
6. $\frac{4}{10} = \frac{2 \times e}{2 \times s}$
7. $\frac{6}{9} = \frac{3 \times e}{3 \times l}$
8. $\frac{8}{20} = \frac{v \times 2}{v \times 5}$
9. $\frac{6}{10} = \frac{e \times 3}{e \times 5}$
10. $\frac{9}{15} = \frac{r \times 3}{r \times 5}$
11. $\frac{9}{12} = \frac{i \times 3}{i \times 4}$
12. $\frac{9}{12} = \frac{3 \times k}{3 \times l}$
13. $\frac{4}{8} = \frac{4 \times m}{4 \times n}$
14. $\frac{8}{12} = \frac{4 \times d}{4 \times p}$
15. $\frac{15}{20} = \frac{s \times 3}{s \times 4}$
16. $\frac{30}{40} = \frac{z \times 3}{z \times 4}$
17. $\frac{10}{12} = \frac{m \times 5}{m \times 6}$
18. $\frac{50}{60} = \frac{n \times 5}{n \times 6}$
19. $\frac{5}{10} = \frac{5 \times s}{5 \times t}$
20. $\frac{10}{15} = \frac{5 \times u}{5 \times v}$
21. $\frac{15}{20} = \frac{5 \times w}{5 \times x}$

Pair the equivalent fractions in each set.

For example, for $\left\{\frac{3}{4}, \frac{6}{9}, \frac{3}{5}, \frac{6}{10}, \frac{9}{12}, \frac{4}{6}\right\}$, write the pairs $\left(\frac{3}{4}, \frac{9}{12}\right)$, $\left(\frac{6}{9}, \frac{4}{6}\right)$, and $\left(\frac{3}{5}, \frac{6}{10}\right)$.

22. $\left\{\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{9}{12}, \frac{3}{9}, \frac{5}{10}\right\}$
23. $\left\{\frac{3}{5}, \frac{4}{8}, \frac{6}{8}, \frac{6}{10}, \frac{2}{4}, \frac{9}{12}\right\}$
24. $\left\{\frac{1}{4}, \frac{3}{5}, \frac{3}{10}, \frac{5}{20}, \frac{12}{20}, \frac{6}{20}\right\}$
25. $\left\{\frac{2}{3}, \frac{4}{10}, \frac{3}{4}, \frac{8}{12}, \frac{15}{20}, \frac{2}{5}\right\}$

## Equivalent fractions

Study these steps for renaming $\frac{2}{3}$ as $\frac{4}{6}$.

$$\frac{2}{3} = \frac{2 \times 2}{2 \times 3}$$
$$= \frac{4}{6}$$

Multiply both the numerator and the denominator by 2. Rename the numerator and the denominator.

### EXERCISES

Copy and complete the equations.

1. $\frac{1}{3} = \frac{4 \times 1}{4 \times 3} = \frac{a}{c}$

2. $\frac{1}{4} = \frac{5 \times 1}{5 \times 4} = \frac{d}{e}$

3. $\frac{1}{5} = \frac{3 \times 1}{3 \times 5} = \frac{f}{g}$

4. $\frac{2}{5} = \frac{4 \times 2}{4 \times 5} = \frac{h}{i}$

5. $\frac{3}{4} = \frac{n \times 3}{n \times 4} = \frac{6}{8}$

6. $\frac{3}{4} = \frac{r \times 3}{r \times 4} = \frac{9}{12}$

7. $\frac{5}{8} = \frac{s \times 5}{s \times 8} = \frac{10}{16}$

8. $\frac{5}{8} = \frac{u \times 5}{u \times 8} = \frac{15}{24}$

Solve the equations.

9. $\frac{1}{3} = \frac{a}{12}$

10. $\frac{2}{3} = \frac{b}{12}$

11. $\frac{1}{3} = \frac{c}{15}$

12. $\frac{2}{3} = \frac{d}{15}$

13. $\frac{3}{5} = \frac{e}{10}$

14. $\frac{3}{5} = \frac{f}{15}$

15. $\frac{4}{5} = \frac{h}{10}$

16. $\frac{4}{5} = \frac{i}{15}$

17. $\frac{3}{8} = \frac{k}{32}$

18. $\frac{3}{8} = \frac{l}{40}$

19. $\frac{5}{8} = \frac{m}{16}$

20. $\frac{7}{8} = \frac{n}{24}$

Copy and complete each set of equivalent fractions.

21. $\left\{\frac{1}{2}, \frac{2}{4}, \frac{a}{12}, \frac{b}{16}, \frac{c}{18}, \frac{d}{20}\right\}$

22. $\left\{\frac{2}{3}, \frac{4}{6}, \frac{a}{9}, \frac{b}{12}, \frac{c}{18}, \frac{d}{21}\right\}$

23. $\left\{\frac{3}{4}, \frac{6}{8}, \frac{a}{12}, \frac{b}{16}, \frac{c}{20}, \frac{d}{24}\right\}$

24. $\left\{\frac{2}{5}, \frac{4}{10}, \frac{a}{15}, \frac{b}{20}, \frac{c}{25}, \frac{d}{30}\right\}$

Complete this sentence.

25. To rename a fractional number, we can ___?___ both the numerator and the denominator by the same factor.

In $\left\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \ldots\right\}$ we see

$$\frac{2}{3} = \frac{2 \div 1}{3 \div 1} = \frac{2}{3} \qquad \frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$$

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3} \qquad \frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

To rename fractional numbers like $\frac{4}{6}$, $\frac{6}{9}$, and $\frac{8}{12}$ we divide both the numerator and the denominator by the same number.

## EXERCISES

Copy and complete the equations.

1. $\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{a}{c}$
2. $\frac{6}{8} = \frac{6 \div 2}{8 \div e} = \frac{f}{g}$
3. $\frac{9}{12} = \frac{9 \div i}{12 \div 3} = \frac{3}{j}$
4. $\frac{4}{8} = \frac{4 \div k}{8 \div n} = \frac{1}{p}$
5. $\frac{3}{6} = \frac{r \div 3}{s \div 3} = \frac{1}{2}$
6. $\frac{4}{10} = \frac{u \div 2}{10 \div v} = \frac{2}{5}$

Use equations to rename the fractions.

7. $\frac{2}{4}$ as $\frac{x}{2}$
8. $\frac{2}{6}$ as $\frac{z}{3}$
9. $\frac{8}{10}$ as $\frac{a}{5}$
10. $\frac{3}{12}$ as $\frac{b}{4}$
11. $\frac{3}{6}$ as $\frac{c}{2}$
12. $\frac{4}{12}$ as $\frac{d}{3}$
13. $\frac{5}{10}$ as $\frac{e}{2}$
14. $\frac{10}{12}$ as $\frac{f}{6}$

Solve the equations.

15. $\frac{6}{12} = \frac{a}{2}$
16. $\frac{8}{16} = \frac{b}{2}$
17. $\frac{9}{15} = \frac{c}{5}$
18. $\frac{8}{12} = \frac{d}{6}$
19. $\frac{5}{20} = \frac{e}{4}$
20. $\frac{15}{20} = \frac{f}{4}$
21. $\frac{8}{20} = \frac{2}{g}$
22. $\frac{12}{20} = \frac{3}{s}$
23. $\frac{16}{20} = \frac{4}{i}$
24. $\frac{4}{20} = \frac{1}{j}$
25. $\frac{12}{16} = \frac{k}{4}$
26. $\frac{4}{16} = \frac{1}{u}$

Solve the equations.

27. $\frac{4}{8} = \frac{a}{4} = \frac{n}{2}$
28. $\frac{6}{12} = \frac{v}{4} = \frac{e}{2}$
29. $\frac{6}{12} = \frac{c}{6} = \frac{r}{2}$
30. $\frac{8}{16} = \frac{a}{8} = \frac{n}{4}$
31. $\frac{12}{16} = \frac{c}{8} = \frac{m}{4}$
32. $\frac{8}{20} = \frac{r}{10} = \frac{v}{5}$

## Renaming fractional numbers

Compare the methods.

| Renaming $\frac{2}{3}$ as $\frac{6}{9}$ | Renaming $\frac{6}{9}$ as $\frac{2}{3}$ |
|---|---|
| $\frac{2}{3} = \frac{3 \times 2}{3 \times 3}$ <br> $= \frac{6}{9}$ | $\frac{6}{9} = \frac{6 \div 3}{9 \div 3}$ <br> $= \frac{2}{3}$ |

### EXERCISES

Copy and complete the equations.

**1.** $\frac{3}{5} = \frac{4 \times 3}{4 \times 5} = a$

**2.** $\frac{2}{3} = \frac{6 \times 2}{6 \times 3} = b$

**3.** $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = c$

**4.** $\frac{4}{5} = \frac{3 \times 4}{3 \times 5} = d$

**5.** $\frac{3}{9} = \frac{3 \div 3}{9 \div 3} = e$

**6.** $\frac{1}{3} = \frac{10 \times 1}{10 \times 3} = f$

**7.** $\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = g$

**8.** $\frac{12}{30} = \frac{12 \div 6}{30 \div 6} = h$

**9.** $\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = i$

**10.** $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = j$

**11.** $\frac{3}{10} = \frac{4 \times 3}{4 \times 10} = k$

**12.** $\frac{3}{8} = \frac{4 \times 3}{4 \times 8} = l$

**13.** $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = m$

**14.** $\frac{8}{40} = \frac{8 \div 8}{40 \div 8} = n$

**15.** $\frac{16}{40} = \frac{16 \div 8}{40 \div 8} = p$

Write each fraction in the simplest form.

**16.** $\frac{2}{4}$ **17.** $\frac{4}{16}$ **18.** $\frac{3}{6}$ **19.** $\frac{5}{25}$ **20.** $\frac{8}{10}$ **21.** $\frac{15}{25}$ **22.** $\frac{10}{20}$ **23.** $\frac{12}{48}$ **24.** $\frac{30}{40}$ **25.** $\frac{16}{48}$ **26.** $\frac{40}{50}$ **27.** $\frac{15}{60}$

Solve the equations.

**28.** $\frac{1}{2} = \frac{a}{8}$ **29.** $\frac{1}{3} = \frac{c}{15}$ **30.** $\frac{2}{3} = \frac{b}{12}$ **31.** $\frac{3}{4} = \frac{s}{12}$ **32.** $\frac{2}{5} = \frac{n}{10}$ **33.** $\frac{3}{4} = \frac{u}{8}$ **34.** $\frac{3}{4} = \frac{r}{16}$ **35.** $\frac{3}{5} = \frac{z}{20}$

## Sharpshooting

Answer **A** as quickly as you can. Then see if you can answer **B** even more quickly.

### A

1. What is the cost of 6 pads of writing paper at 95¢ a pad?
2. $47 + 8 + 9 + 5 + 4 = n$
3. Name the perimeter of a rectangle 42 feet long and 26 feet wide.
4. What is the difference between 216 and 471?
5. What number is one million greater than 3,486,915?
6. If the cost per plant is 79¢, how much do 6 plants cost?
7. How many inches are there in 8 feet 7 inches?
8. Name the quotient and remainder when 83 is divided by 7.
9. $8 \times 27 = n$.
10. Name the multiple of ten nearest to 716.
11. Name the sums.
    a. $1 + 3 + 5 + 7 + 9$
    b. $2 + 4 + 6 + 8 + 10$
    c. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

### B

1. $4683 + 9214 = n$
2. What is the cost of 23 ice cream cones at 15¢ each?
3. Which costs more, 4 ties at 75¢ each or 5 ties at 55¢ each?
4. Name the product of 73 and 7.
5. What is the difference between the product of 6 and 7 and the sum of 6 and 7?
6. How many feet and inches are there in 39 inches?
7. List the members of $A \cup B$.
   $A = \{7, 3, 8, 6, 9\}$
   $B = \{5, 3, 1, 2, 4\}$
8. Write $4000 + 500 + 40 + 7$ as a compact numeral.
9. Which is greater, $9 \times 14$ or $10 \times 12$?
10. Name the sum of $(18 \times 3)$ and $(2 \times 3)$.
11. Name the sums.
    a. $11 + 13 + 15 + 17 + 19$
    b. $12 + 14 + 16 + 18 + 20$
    c. $11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20$

## Practice page

Complete the sentences. Write $>$, $<$, or $=$ to replace each ◯.

1. $4 + 19 \bigcirc 8 \times 3$
2. $5 \times 10 \bigcirc 6 \times 8$
3. $72 \div 9 \bigcirc 4 \times 2$
4. $6 + 17 \bigcirc 5 \times 4$
5. $3 + 9 \bigcirc 15 + 13$
6. $29 + 7 \bigcirc 8 \times 4$
7. $23 + 7 \bigcirc 6 \times 5$
8. $26 - 4 \bigcirc 6 \times 4$
9. $12 \times 5 \bigcirc 14 + 46$
10. $4 \times 9 \bigcirc 7 \times 15$
11. $12 \times 7 \bigcirc 184 - 100$
12. $63 \div 9 \bigcirc 64 \div 8$

Complete the equations. Write $+$, $-$, $\times$, or $\div$ to replace each △.

13. $18 \triangle 6 = 3$
14. $21 \triangle 7 = 3$
15. $14 \triangle 3 = 11$
16. $36 \triangle 9 = 4$
17. $6 \triangle 9 = 15$
18. $9 \triangle 6 = 3$
19. $3 \triangle 8 = 24$
20. $7 \triangle 6 = 42$
21. $48 \triangle 12 = 36$
22. $19 \triangle 11 = 30$
23. $19 \triangle 11 = 8$
24. $73 \triangle 32 = 41$

Name the sums or missing addends.

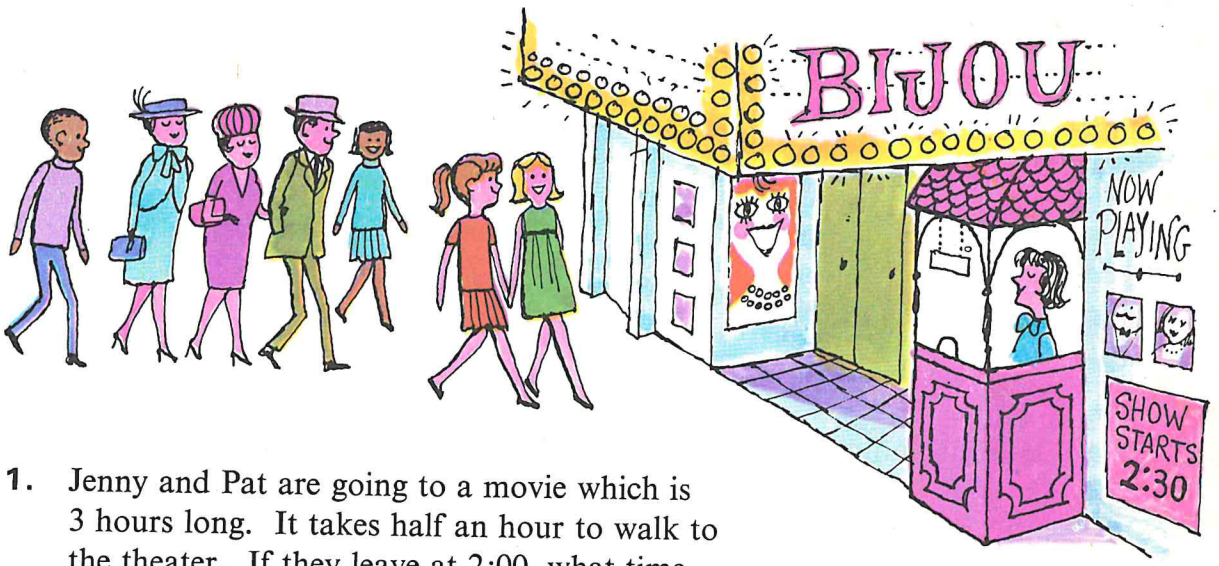
25. $34 + 16 = a$
26. $67 - 19 = i$
27. $36 + 16 = c$
28. $7 + 8 + 9 + 6 = e$
29. $\frac{1}{3} + \frac{1}{3} = m$
30. $29 + 47 = f$
31. $38 + 38 = g$
32. $49 - 49 = h$
33. $36 + d = 60$
34. $\frac{1}{4} + \frac{2}{4} = n$
35. $386 - 247 = j$
36. $17 + k = 83$
37. $376 - 48 = l$
38. $24 + b = 48$
39. $\frac{1}{5} + q = \frac{2}{5}$

Name the products or missing factors.

40. $3 \times 13 = a$
41. $3 \times 16 = b$
42. $4 \times 16 = c$
43. $6 \times 16 = d$
44. $3 \times 24 = e$
45. $4 \times 15 = f$
46. $6 \times 18 = g$
47. $7 \times 21 = h$
48. $3 \times i = 30$
49. $5 \times j = 50$
50. $7 \times k = 70$
51. $9 \times l = 90$
52. $\frac{1}{5} \times 3 = m$
53. $\frac{1}{4} \times 3 = n$
54. $3 \times q = \frac{3}{5}$
55. $\frac{1}{5} \times p = \frac{4}{5}$

Name the thousand just after and the thousand just before

56. 2914
57. 4286
58. 14,000
59. 5000
60. 384,000
61. 14,286
62. 297,684
63. 20,800




1. Jenny and Pat are going to a movie which is 3 hours long. It takes half an hour to walk to the theater. If they leave at 2:00, what time should they be home?

2. If your heart beats 70 times a minute, how many times will it beat in an hour?

3. It is 12 minutes before 2 o'clock. How many minutes from now until 25 minutes past 2 o'clock?

4. a. If you put eggs into boxes that hold 12 eggs each, how many boxes would you completely fill with 100 eggs?
   b. You will need another box to pack the remaining eggs. How many eggs will be in that box?
   c. How many boxes do you need to pack 100 eggs?

5. If you have 100 stamps to paste in your stamp book and have filled 8 pages with 8 stamps on each page, how many stamps do you still have to paste in your book?

6. How many 20 minute television programs can be shown in 4 hours?

7. If the area of a rectangle is 80 square inches and one side is 8 inches, how long is the other side?

## Separating sets

Five girls are selling tickets for their class concert. They have 20 tickets to sell. If each girl has the same number of tickets to sell, how many must one girl sell?

One fifth of 20 $= \frac{1}{5} \times 20 = 20 \div 5 = 4$

Each girl must sell 4 tickets.

### EXERCISES

Complete the sentences and solve the equations.

1. A set of 21 will form 3 sets of $a$.
   $\frac{1}{3} \times 21 = b$
2. A set of 18 will form 6 sets of $e$.
   $f \times 18 = 3$
3. A set of 24 will form 6 sets of $c$.
   $\frac{1}{6} \times 24 = d$
4. A set of 24 will form 8 sets of $g$.
   $h \times 24 = 3$

Copy and solve the equations.

5. $21 \div 3 = a$ $\quad \frac{1}{3} \times 21 = b$
6. $30 \div 6 = c$ $\quad \frac{1}{6} \times 30 = d$
7. $48 \div 6 = e$ $\quad \frac{1}{6} \times 48 = f$
8. $36 \div 4 = q$ $\quad \frac{1}{4} \times 36 = h$
9. $16 \div 4 = i$ $\quad \frac{1}{4} \times 16 = j$
10. $40 \div 4 = k$ $\quad \frac{1}{4} \times 40 = l$
11. $\frac{1}{5} \times 15 = 15 \div i = j$
12. $\frac{1}{4} \times 20 = 20 \div k = l$
13. $\frac{1}{6} \times 18 = 18 \div a = b$
14. $\frac{1}{4} \times 28 = 28 \div c = d$
15. $\frac{1}{2} \times 16 = 16 \div e = f$
16. $\frac{1}{2} \times 8 = 8 \div g = h$

Solve the equations.

17. one sixth of 36 $= a$
18. one third of 15 $= b$
19. one half of 20 $= c$
20. one fourth of 28 $= d$
21. one third of 36 $= e$
22. one fifth of 15 $= f$
23. one fourth of 40 $= g$
24. one seventh of 35 $= h$

## Showing division

Study the methods of showing division.

$3\overline{)18}$ $\quad$ $\frac{1}{3} \times 18 = 6$ $\quad$ $18 \div 3 = 6$ $\quad$ $\frac{18}{3} = 6$

$$18 \div 3 = \frac{18}{3} = 6$$

### EXERCISES

Name the quotients.

1. $3\overline{)12}$
2. $\frac{1}{3} \times 12$
3. $12 \div 3$
4. $6\overline{)24}$
5. $\frac{1}{6} \times 24$
6. $24 \div 6$
7. $5\overline{)15}$
8. $\frac{1}{5} \times 15$
9. $15 \div 5$
10. $8\overline{)16}$
11. $\frac{1}{8} \times 16$
12. $16 \div 8$

Name the quotients.

13. $\frac{12}{3}$
14. $\frac{14}{7}$
15. $\frac{24}{6}$
16. $\frac{25}{5}$
17. $\frac{15}{5}$
18. $\frac{32}{8}$
19. $\frac{16}{8}$
20. $\frac{14}{7}$
21. $\frac{30}{6}$
22. $\frac{20}{10}$
23. $\frac{72}{9}$
24. $\frac{40}{10}$
25. $\frac{48}{6}$
26. $\frac{60}{10}$

Solve the equations to complete the number patterns.

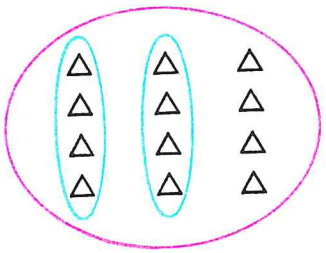
27. $\frac{16}{2} = a$
28. $\frac{8}{2} = b$
29. $\frac{4}{2} = c$
30. $\frac{2}{2} = d$
31. $1 \div 2 = e$
32. $\frac{48}{2} = f$
33. $\frac{24}{2} = g$
34. $\frac{12}{2} = h$
35. $\frac{6}{2} = i$
36. $3 \div 2 = j$
37. $\frac{96}{4} = k$
38. $\frac{48}{4} = l$
39. $\frac{24}{4} = m$
40. $\frac{12}{4} = n$
41. $3 \div 4 = u$
42. $\frac{81}{3} = p$
43. $\frac{27}{3} = q$
44. $\frac{9}{3} = r$
45. $\frac{3}{3} = s$
46. $1 \div 3 = t$

Solve the equations.

47. $15 \div 3 = \frac{a}{3} = b$
48. $30 \div 6 = \frac{m}{6} = \frac{n}{3} = r$
49. $16 \div 4 = \frac{c}{4} = d$
50. $48 \div 12 = \frac{e}{6} = \frac{h}{3} = k$

## Separating sets

Study the sets and the equations.



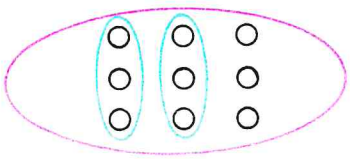
$$\frac{1}{3} \times 12 = 4$$

$$\begin{aligned}\frac{2}{3} \times 12 &= \left(2 \times \frac{1}{3}\right) \times 12 \\ &= 2 \times \left(\frac{1}{3} \times 12\right) \\ &= 2 \times 4 \\ &= 8\end{aligned}$$

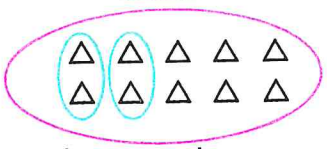
### EXERCISES

Study the sets. Then solve the equations.

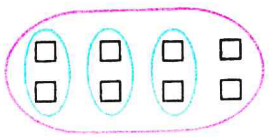
1. $\frac{2}{3} \times 9 = a$



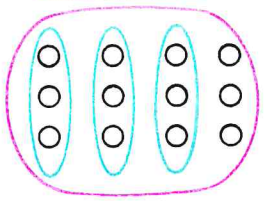
2. $\frac{2}{5} \times 10 = b$



3. $\frac{3}{4} \times 8 = c$



4. $\frac{3}{4} \times 12 = d$



Copy and complete the equations.

5. $\begin{aligned}\frac{2}{3} \times 15 &= 2 \times \left(\frac{1}{3} \times 15\right) \\ &= 2 \times a \\ &= b\end{aligned}$

6. $\begin{aligned}\frac{3}{4} \times 20 &= 3 \times \left(\frac{1}{4} \times 20\right) \\ &= 3 \times c \\ &= d\end{aligned}$

7. $\begin{aligned}\frac{3}{5} \times 10 &= 3 \times \left(\frac{1}{5} \times 10\right) \\ &= 3 \times e \\ &= f\end{aligned}$

8. $\begin{aligned}\frac{3}{5} \times 20 &= 3 \times \left(\frac{1}{5} \times 20\right) \\ &= 3 \times g \\ &= h\end{aligned}$

9. $\begin{aligned}\frac{3}{4} \times 16 &= 3 \times \left(\frac{1}{4} \times 16\right) \\ &= 3 \times i \\ &= j\end{aligned}$

10. $\begin{aligned}\frac{3}{8} \times 16 &= 3 \times \left(\frac{1}{8} \times 16\right) \\ &= 3 \times k \\ &= l\end{aligned}$

Solve the equations.

11. $\frac{2}{3} \times 18 = a$
12. $\frac{2}{5} \times 15 = b$
13. $\frac{2}{3} \times 24 = c$
14. $\frac{2}{5} \times 50 = d$
15. $\frac{2}{3} \times 30 = e$
16. $\frac{5}{6} \times 12 = f$
17. $\frac{3}{4} \times 40 = g$
18. $\frac{5}{6} \times 18 = h$

Study the equations.

$\frac{1}{6} \times 12 = 2$

$\frac{1}{3} \times 12 = 4$

$\frac{2}{6} \times 12 = 2 \times 2 = 4$

$\frac{1}{3} = \frac{2}{6}$

$\frac{2}{3} \times 12 = 2 \times 4 = 8$

$\frac{4}{6} \times 12 = 4 \times 2 = 8$

$\frac{2}{3} = \frac{4}{6}$

## EXERCISES

Solve the equations.

**1.** $\frac{1}{2} \times 12 = a$, $\frac{1}{4} \times 12 = b$

$\frac{2}{4} \times 12 = 2 \times c = d$

$\frac{1}{2} = \frac{e}{4}$

**2.** $\frac{1}{4} \times 8 = h$, $\frac{1}{8} \times 8 = i$

$\frac{2}{8} \times 8 = 2 \times j = k$

$\frac{1}{4} = \frac{l}{8}$

**3.** $\frac{1}{5} \times 10 = m$, $\frac{1}{10} \times 10 = p$

$\frac{2}{10} \times 10 = 2 \times n = r$

$\frac{1}{5} = \frac{b}{10}$

**4.** $\frac{6}{8} \times 8 = s$, $\frac{1}{4} \times 8 = u$

$\frac{3}{4} \times 8 = 3 \times v = w$

$\frac{6}{8} = \frac{x}{4}$

Solve the equations.

**5.** $\frac{1}{2} \times 8 = \frac{a}{4} \times 8$

**6.** $\frac{1}{3} \times 6 = \frac{b}{6} \times 6$

**7.** $\frac{1}{4} \times 16 = \frac{c}{8} \times 16$

**8.** $\frac{1}{4} \times 16 = \frac{d}{16} \times 16$

**9.** $\frac{1}{5} \times 10 = \frac{e}{10} \times 10$

**10.** $\frac{1}{5} \times 20 = \frac{f}{10} \times 20$

**11.** $\frac{2}{8} \times 8 = \frac{d}{4} \times 8$

**12.** $\frac{6}{8} \times 8 = \frac{h}{4} \times 8$

**13.** $\frac{2}{8} \times 24 = \frac{i}{4} \times 24$

**14.** $\frac{6}{8} \times 24 = \frac{n}{4} \times 24$

**15.** $\frac{9}{12} \times 24 = \frac{k}{4} \times 24$

**16.** $\frac{4}{8} \times 24 = \frac{8}{s} \times 24$

**17.** $\frac{6}{10} \times 10 = \frac{3}{m} \times 10$

**18.** $\frac{8}{10} \times 10 = \frac{n}{5} \times 10$

**19.** $\frac{10}{12} \times 12 = \frac{5}{m} \times 12$

**20.** $\frac{8}{12} \times 12 = \frac{4}{p} \times 12$

**21.** $\frac{6}{9} \times 18 = \frac{2}{q} \times 18$

**22.** $\frac{8}{12} \times 24 = \frac{2}{r} \times 24$

## Using sets and numbers

1. Ben had 6 apples. He gave half his apples to Bob. How many did Ben have then? How many did Bob have?
2. John travels 8 miles to school by bus.
   a. If he has gone one fourth of the way, how far has he gone?
   b. If he has gone three fourths of the way, how far has he gone?

**Remember**

You may use five steps to answer your problems.

1. Name the sets.
2. Ask what is happening to the sets.
3. Write an equation about the numbers in the sets.
4. Solve the equation.
5. Use the *number* from the *equation* to answer the *question* about the *sets*.

3. Mary travels to school by bus. After she has traveled 3 miles, she has traveled half the distance. How far does Mary travel to school?

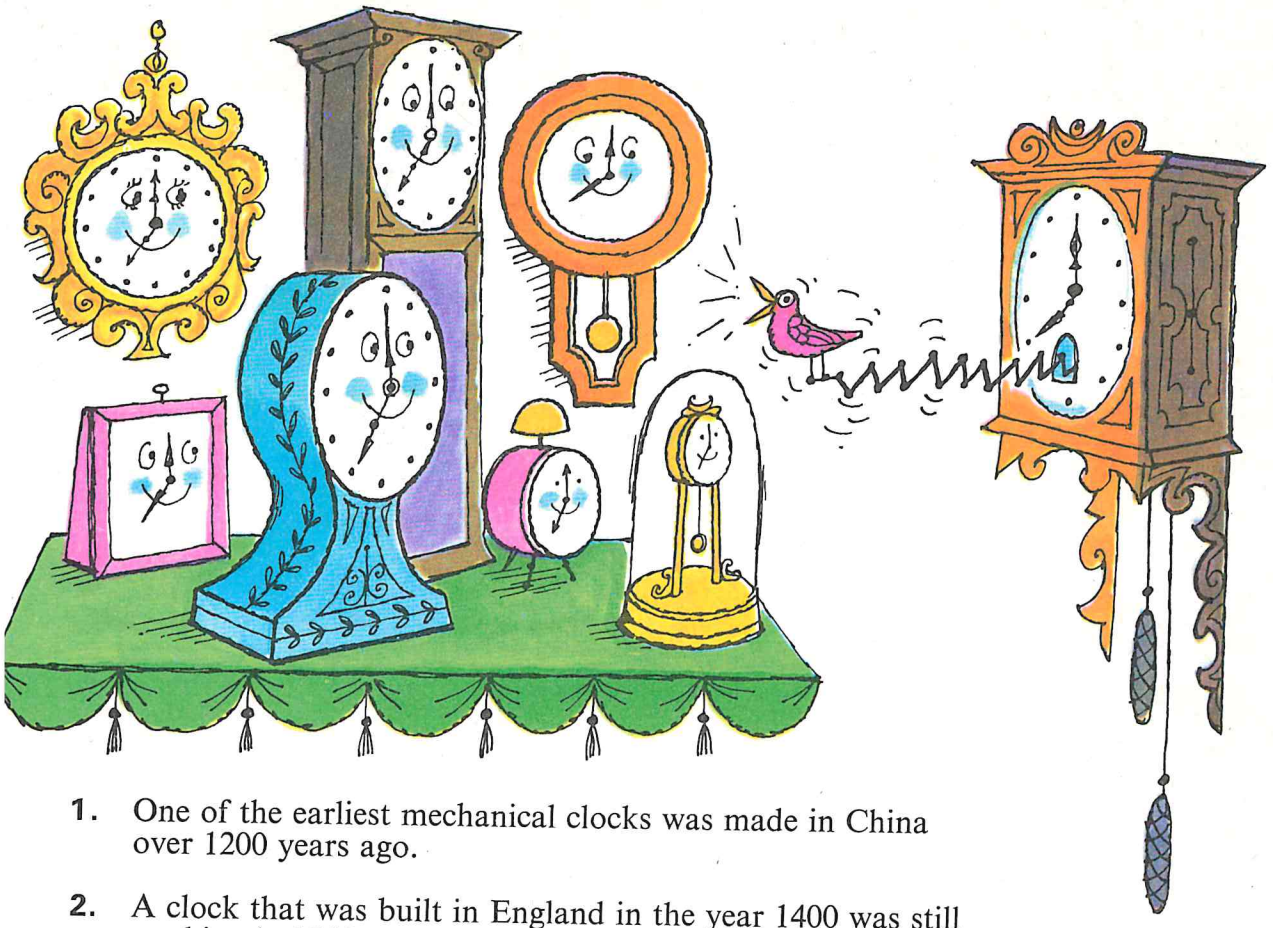
4. If you have 15¢ and give one third of your money to your brother, how much does your brother get?
5. If you have 18¢ and give one third away, how much do you keep?

| S | M | T | W | T | F | S |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| 29 | 30 | 31 | | | | |

6. There are 14 days in a fortnight. How many days are in one seventh of a fortnight?
7. There are 60 minutes in an hour. How many minutes are in one third of an hour? in two thirds of an hour?
8. There are 12 inches in a foot. How many inches are in one fourth of a foot? in three fourths of a foot?

9. If you have 21 days for your vacation and plan to spend one third of the time in the country and the rest of the time at the beach, how many days do you plan to spend at the beach?

10. If you have 20 boxes of blueberries and give three fifths of them away, how many boxes do you give away? How many boxes of blueberries do you have left?



1. One of the earliest mechanical clocks was made in China over 1200 years ago.

2. A clock that was built in England in the year 1400 was still working in 1960.

3. One of the world's largest clocks is in France. It was built about 100 years ago and is about 40 feet high, 20 feet wide, and 10 feet thick.

4. One of the most expensive wrist watches in the world cost about $1500. It was made in Switzerland.

5. One of the oldest watches in the world was made in Germany about 450 years ago. It can now be seen in the Memorial Hall in Philadelphia, Pennsylvania.

6. One of the smallest watches in the world is about $\frac{1}{2}$ inch long and $\frac{1}{4}$ inch wide. It weighs about $\frac{1}{4}$ ounce.

7. There is a clock in Milwaukee, Wisconsin that has a minute hand about 20 feet long. The diameter of the clock face is about 40 feet.

## Practice page

Name a compact numeral for

1. $6000 + 400 + 70 + 6$
2. forty four thousand, two hundred twenty seven
3. $26{,}000 + 380 + 7$
4. seven hundred sixty four thousand, two hundred two

Name the numbers three greater and three less than

5. 49,689
6. 30,098
7. 843,997
8. six thousand twenty seven

Name the sums or differences.

9. $368 + 469$
10. $862 - 289$
11. $765 + 429$
12. $314 - 289$
13. $1211 - 930$
14. $463 - 286$
15. $36857 + 26954 + 38612$
16. $27486 + 54293 + 34279$
17. $321070 + 640768 + 173248$
18. $628554 + 49206 + 486570$

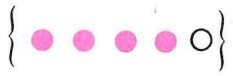
Name the products.

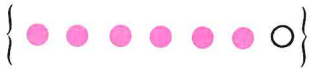
19. $463 \times 3$
20. $279 \times 8$
21. $574 \times 6$
22. $382 \times 9$
23. $609 \times 7$
24. $485 \times 5$

Name the amounts of money.

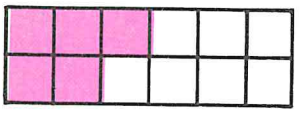
25. $\$3.86 + 2.54 + 3.60$
26. $\$14.76 - 3.84$
27. $\$63.54 - 9.37$
28. $\$5.35 \times 5$
29. $\$4.72 \times 3$
30. $\$14.86 + 2.93 + 17.54$
31. $\$19.63 - 12.84$
32. $\$52.92 - 13.95$
33. $\$9.54 \times 9$
34. $\$14.79 \times 6$
35. $\$4.83 + 9.06 + 13.89$
36. $\$73.48 - 29.49$
37. $\$67.52 - 29.63$
38. $\$18.64 \times 8$
39. $\$15.64 \times 4$
40. $\$4.07 + 16.82 + 14.84$
41. $\$13.76 - 12.84$
42. $\$94.66 - 51.73$
43. $\$16.43 \times 6$
44. $\$17.64 \times 7$
45. $\$19.83 + 12.50 + 14.60$
46. $\$16.73 - 4.89$
47. $\$78.57 - 25.68$
48. $\$17.52 \times 10$
49. $\$12.65 \times 8$

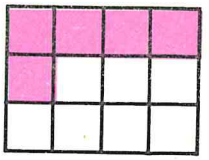
Write fractions to show what part of each set, region, and line segment is red.

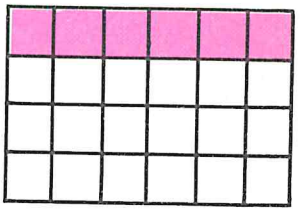
1. 

2. 

3.

4. 

5. 

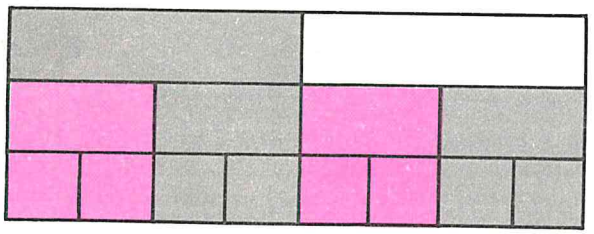
6. 

Write the fractions.

7.

| numerator | 1 | 4 | 3 | 5 | 6 | 5 | 7 |
|---|---|---|---|---|---|---|---|
| denominator | 2 | 7 | 8 | 9 | 10 | 8 | 11 |
| fraction | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ |

Use the rods to solve the equations.



8. $\frac{1}{2} = \frac{a}{4}$

9. $\frac{1}{4} = \frac{b}{8}$

10. $\frac{1}{2} = \frac{c}{8}$

11. $\frac{2}{4} = \frac{d}{8}$

12. $\frac{3}{4} = \frac{e}{8}$

13. $\frac{4}{4} = \frac{f}{8}$

Copy and complete the equations.

14. $\frac{4}{12} = \frac{4 \div a}{12 \div 4} = b$

15. $\frac{6}{10} = \frac{c \div 2}{e \div 2} = f$

16. $\frac{5}{15} = \frac{5 \div 5}{15 \div m} = n$

17. $\frac{7}{21} = \frac{7 \div r}{s \div 7} = t$

18. $\frac{2}{3} = \frac{4 \times 2}{4 \times 3} = \frac{x}{12}$

19. $\frac{2}{5} = \frac{3 \times z}{3 \times a} = \frac{b}{c}$

Complete each set of equivalent fractions.

20. $\left\{\frac{1}{2}, \frac{a}{4}, \frac{b}{6}, \frac{c}{8}, \frac{d}{10}\right\}$

21. $\left\{\frac{3}{5}, \frac{a}{10}, \frac{b}{15}, \frac{c}{20}\right\}$

22. $\left\{\frac{3}{4}, \frac{a}{8}, \frac{b}{12}, \frac{c}{16}\right\}$

23. $\left\{\frac{4}{5}, \frac{a}{10}, \frac{b}{15}, \frac{c}{20}\right\}$

24. $\left\{\frac{5}{6}, \frac{a}{12}, \frac{b}{18}, \frac{c}{24}\right\}$

*25. $\left\{\frac{1}{3}, \frac{a}{6}, \frac{b}{30}, \frac{c}{300}\right\}$

## LOOKING BACK

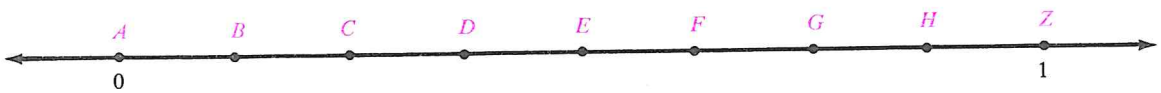
A **fractional number** compares the cardinal numbers of a subset and a set. A **fraction** is a numeral for a fractional number.

If we compare a subset of 4 with a set of 7, we can write the fraction $\frac{4}{7}$. 4 is the **numerator** of the fraction and 7 is the **denominator.**

Write fractions to compare the number of red objects with the total number of objects.

**1.** {● ● ○ ○ ○}  **2.** {▲ ▲ ▲ ▲ ▲ △}  **3.** {● ● ● ● ○ ○ ○}

Fractional numbers may be shown on a number line.




Study the number line. Then name the fractional numbers labeled

**4.** *B**5.** *C**6.** *D**7.** *E**8.** *G**9.** *H*

Fractions that name the same fractional number are called **equivalent fractions.**

**10–13.** Pair the equivalent fractions listed here.

$\frac{1}{2}$ $\frac{2}{5}$ $\frac{14}{16}$ $\frac{12}{20}$ $\frac{6}{12}$ $\frac{7}{8}$ $\frac{3}{5}$ $\frac{6}{15}$

To rename a fractional number we may multiply or divide both the numerator and the denominator by the same number.

Solve the equations.

**14.** $\frac{1}{2} = \frac{a}{8}$  **16.** $\frac{3}{5} = \frac{c}{50}$  **18.** $\frac{6}{8} = \frac{3}{e}$  **20.** $\frac{16}{20} = \frac{4}{g}$

**15.** $\frac{2}{5} = \frac{b}{25}$  **17.** $\frac{3}{10} = \frac{d}{60}$  **19.** $\frac{4}{10} = \frac{f}{5}$  **21.** $\frac{100}{200} = \frac{h}{2}$

Study this method to name the sum $\left(\frac{1}{2} \times 17\right) + \left(\frac{1}{2} \times 3\right)$.

$$\left(\frac{1}{2} \times 17\right) + \left(\frac{1}{2} \times 3\right) = \frac{1}{2} \times (17 + 3)$$ Use the distributive property.

$$= \frac{1}{2} \times 20$$ Name the sum $17 + 3$.

$$= 10$$ Name the product $\frac{1}{2} \times 20$.

Use the distributive property to name the sums.

1. $\left(\frac{1}{2} \times 15\right) + \left(\frac{1}{2} \times 5\right)$
2. $\left(\frac{1}{3} \times 19\right) + \left(\frac{1}{3} \times 11\right)$
3. $\left(\frac{1}{4} \times 9\right) + \left(\frac{1}{4} \times 7\right)$
4. $\left(\frac{1}{5} \times 36\right) + \left(\frac{1}{5} \times 14\right)$

Study this method to name the difference $\left(\frac{1}{2} \times 9\right) - \left(\frac{1}{2} \times 7\right)$.

$$\left(\frac{1}{2} \times 9\right) - \left(\frac{1}{2} \times 7\right) = \frac{1}{2} \times (9 - 7)$$ Use the distributive property.

$$= \frac{1}{2} \times 2$$ Name the difference $9 - 7$.

$$= 1$$ Name the product $\frac{1}{2} \times 2$.

Use the distributive property to name the differences.

5. $\left(\frac{1}{2} \times 17\right) - \left(\frac{1}{2} \times 7\right)$
6. $\left(\frac{1}{2} \times 64\right) - \left(\frac{1}{2} \times 44\right)$
7. $\left(\frac{1}{3} \times 21\right) - \left(\frac{1}{3} \times 18\right)$
8. $\left(\frac{1}{3} \times 160\right) - \left(\frac{1}{3} \times 130\right)$

*Just For Fun*

9. We often talk about a *quarter of a dollar*, a *quarter of an hour*, and an *eighth of an inch*. Make a list of other common uses of fractions.

10 Sizes for clothing are sometimes given in fractions.
   a. Make a list of fractions used to give sizes.
   b. Find out what is meant by a *size 13 shoe* and so forth.

## Basic multiplication–division facts

There are 36 **basic multiplication-division facts** to be memorized. On this page, they are shown in two different ways. You may use this page whenever you want more practice.

### EXERCISES

Name the products.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $2 \times 2$ | $3 \times 2$ | $4 \times 2$ | $3 \times 3$ | $5 \times 2$ | $6 \times 2$ | $4 \times 3$ | $7 \times 2$ | $5 \times 3$ |
| $4 \times 4$ | $8 \times 2$ | $9 \times 2$ | $6 \times 3$ | $5 \times 4$ | $7 \times 3$ | $8 \times 3$ | $6 \times 4$ | $5 \times 5$ |
| $9 \times 3$ | $7 \times 4$ | $6 \times 5$ | $8 \times 4$ | $7 \times 5$ | $6 \times 6$ | $9 \times 4$ | $8 \times 5$ | $7 \times 6$ |
| $9 \times 5$ | $8 \times 6$ | $7 \times 7$ | $9 \times 6$ | $8 \times 7$ | $9 \times 7$ | $8 \times 8$ | $9 \times 8$ | $9 \times 9$ |

Name the missing factors.

| | | | | | |
|---|---|---|---|---|---|
| $5\overline{)10}$ | $5\overline{)20}$ | $5\overline{)30}$ | $5\overline{)40}$ | $3\overline{)21}$ | $9\overline{)81}$ |
| $2\overline{)12}$ | $3\overline{)12}$ | $4\overline{)32}$ | $6\overline{)42}$ | $9\overline{)72}$ | $9\overline{)63}$ |
| $2\overline{)4}$ | $7\overline{)14}$ | $3\overline{)24}$ | $6\overline{)24}$ | $6\overline{)54}$ | $8\overline{)64}$ |
| $5\overline{)15}$ | $5\overline{)25}$ | $5\overline{)35}$ | $9\overline{)45}$ | $3\overline{)6}$ | $2\overline{)16}$ |
| $4\overline{)16}$ | $4\overline{)36}$ | $6\overline{)36}$ | $7\overline{)56}$ | $3\overline{)27}$ | $2\overline{)8}$ |
| $2\overline{)18}$ | $3\overline{)18}$ | $4\overline{)28}$ | $6\overline{)48}$ | $3\overline{)9}$ | $7\overline{)49}$ |

List the multiplication-division facts that you do not know and practice them until you are sure of them.

You should be able to name all these products and factors without using your paper.

## The Associative Property

Solve the equations.

1. $2 \times (2 \times 3) = 2 \times a = b$
   $(2 \times 2) \times 3 = c \times 3 = d$

2. $2 \times (3 \times 3) = 2 \times e = f$
   $(2 \times 3) \times 3 = g \times 3 = h$

3. $3 \times (2 \times 4) = 3 \times i = j$
   $(3 \times 2) \times 4 = k \times 4 = m$

4. $5 \times (2 \times 3) = 5 \times n = p$
   $(5 \times 2) \times 3 = r \times 3 = s$

Solve the equations.

5. $3 \times 30 = 3 \times (3 \times 10)$
   $= (3 \times 3) \times 10$
   $= a$

6. $4 \times 30 = 4 \times (3 \times 10)$
   $= (4 \times 3) \times 10$
   $= b$

## The Commutative Property

Solve the equations.

7. $3 \times 4 = a$ and $4 \times 3 = b$ so $3 \times 4 = 4 \times c$
8. $5 \times 7 = d$ and $7 \times 5 = e$ so $5 \times 7 = 7 \times f$
9. $6 \times 8 = g$ and $8 \times 6 = h$ so $6 \times 8 = i \times 6$

## The Common Factor Property (The Distributive Property)

Solve the equations.

10. $(2 \times 4) + (2 \times 3) = 8 + u = v$
    $(2 \times 4) + (2 \times 3) = 2 \times w = x$

11. $(3 \times 3) + (4 \times 3) = 9 + a = b$
    $(3 \times 3) + (4 \times 3) = c \times 3 = d$

12. $(4 \times 3) + (5 \times 3) = e \times 3 = f$
    $(3 \times 4) + (3 \times 5) = 3 \times g = h$

13. $(3 \times 5) + (3 \times 2) = 15 + i = j$
    $(3 \times 5) + (3 \times 2) = 3 \times k = l$

14. $(5 \times 6) + (3 \times 6) = m \times 6 = n$
    $(6 \times 5) + (6 \times 3) = 6 \times p = q$

15. $3 \times 6 = 18$
    $\frac{4 \times 6}{7 \times 6} = \frac{24}{r}$

16. $3 \times 7 = s$
    $\frac{5 \times 7}{8 \times 7} = \frac{u}{v}$

17. $10 \times 3 = w$
    $\frac{4 \times 3}{14 \times 3} = \frac{x}{y}$

18. $4 \times 3 = a$
    $\frac{5 \times 3}{9 \times 3} = \frac{i}{r}$

19. $6 \times 4 = b$
    $\frac{3 \times 4}{9 \times 4} = \frac{i}{t}$

20. $5 \times 8 = d$
    $\frac{3 \times 8}{8 \times 8} = \frac{e}{n}$

## Extending multiplication

Here are three ways of using the distributive property to name the product $6 \times 36$.

| Equations | Expanded numerals | Number patterns |
|---|---|---|
| $6 \times 36 = 6 \times (30 + 6)$<br>$= (6 \times 30) + (6 \times 6)$<br>$= 180 + 36$<br>$= 216$ | $36 = 30 + 6$<br>$\times 6 \quad \times 6$<br>$180 + 36 = 216$ | $6 \times 6 = 36$<br>$6 \times 30 = 180$<br>$6 \times 36 = 216$ |

### EXERCISES

Use equations to name the products.

**1.** $5 \times 28$ **2.** $7 \times 49$ **3.** $6 \times 84$ **4.** $9 \times 76$

Use expanded numerals to name the products.

**5.** $52 \times 4$ **6.** $33 \times 6$ **7.** $95 \times 2$ **8.** $37 \times 8$ **9.** $48 \times 7$ **10.** $94 \times 9$

Complete the number patterns.

**11.**
$3 \times 6 = a$
$10 \times 6 = c$
$13 \times 6 = e$

**12.**
$5 \times 9 = f$
$60 \times 9 = i$
$65 \times j = m$

**13.**
$7 \times 8 = n$
$30 \times 8 = r$
$37 \times 8 = s$

**14.**
$7 \times 4 = t$
$90 \times 4 = u$
$97 \times 4 = v$

**15.**
$9 \times 7 = w$
$40 \times 7 = x$
$y \times 7 = z$

**16.**
$3 \times 7 = a$
$80 \times 7 = c$
$83 \times e = g$

Name the products. Then complete the sentence.

**17.** $6 \times 0$ **18.** $29 \times 0$ **19.** $0 \times 64$ **20.** $0 \times 285$

**21.** When zero is a factor, the product is always ___?___.

Solve the equations.

**22.** $7 \times 2 = a$
**23.** $7 \times 20 = b$
**24.** $7 \times 200 = c$
**25.** $7 \times 2000 = d$

**26.** $6 \times 8 = e$
**27.** $6 \times 80 = f$
**28.** $6 \times 800 = g$
**29.** $6 \times 8000 = h$

**30.** $6 \times 5 = i$
**31.** $6 \times 50 = j$
**32.** $6 \times 500 = k$
**33.** $6 \times 5000 = m$

## Using the common factor property

Compare the two methods to name the product 647 × 6.

| Expanded numerals | Number patterns |
|---|---|
| $647 = 600 + 40 + 7$<br>$\times 6 \qquad\qquad \times 6$<br>$3600 + 240 + 42 = 3882$ | $7 \times 6 = 42$<br>$40 \times 6 = 240$<br>$600 \times 6 = 3600$<br>$647 \times 6 = 3882$ |

### EXERCISES

Solve the equations.

**1.**
$3 \times 7 = 21$
$10 \times 7 = 70$
$200 \times 7 = 1400$
$213 \times 7 = a$

**2.**
$4 \times 8 = b$
$30 \times 8 = c$
$400 \times 8 = e$
$434 \times 8 = g$

**3.**
$4 \times 6 = 24$
$20 \times 6 = 120$
$300 \times 6 = 1800$
$324 \times 6 = i$

**4.**
$5 \times 9 = k$
$30 \times 9 = m$
$600 \times 9 = n$
$635 \times 9 = p$

**5.**
$6 \times 9 = r$
$30 \times 9 = s$
$400 \times 9 = u$
$436 \times 9 = v$

**6.**
$4 \times 7 = w$
$40 \times 7 = x$
$400 \times 7 = z$
$444 \times 7 = a$

Use expanded numerals to name the products.

**7.** $146 \times 3$ **9.** $272 \times 2$ **11.** $528 \times 8$ **13.** $739 \times 6$ **15.** $645 \times 4$ **17.** $358 \times 7$

**8.** $586 \times 5$ **10.** $409 \times 9$ **12.** $760 \times 8$ **14.** $619 \times 6$ **16.** $998 \times 4$ **18.** $835 \times 7$

Solve the problems.

**19.** On Class Night the fourth grade class charged each visitor 5¢ to see their room. They wanted to raise money for a class trip. If 100 people came the first hour, 60 people came the second hour, and 9 people came the third hour, how many people visited in all? How much money was collected?

**20.** What is the cost of 200 candies at 7¢ each? of 60 candies at 7¢ each? of 8 candies at 7¢ each? of 268 candies at 7¢ each?

## Using the common factor property

Study the two methods to name the product $647 \times 6$.

| Four step | | Short method |
|---|---|---|
| $\begin{array}{r} 647 \\ \times\ 6 \\ \hline 42 \\ 240 \\ 3600 \\ \hline 3882 \end{array}$ | $\begin{aligned} 6 \times 7 &= 42 \\ 6 \times 40 &= 240 \\ 6 \times 600 &= 3600 \\ 6 \times 647 &= 3882 \end{aligned}$ | $\begin{array}{r} {\scriptstyle 24} \\ 647 \\ \times\ 6 \\ \hline 3882 \end{array}$ <br> $6 \times 7 = 42$ <br> Write 2 in the ones' place. <br> $6 \times 40 = 240, 240 + 40 = 280$ <br> Write 8 in the ten's place. <br> $6 \times 600 = 3600,$ <br> $3600 + 200 = 3800$ <br> Write 8 in the hundreds' place, and 3 in the thousands' place. |

### EXERCISES

Use the 4-step method to name the products.

| | | | | | |
|---|---|---|---|---|---|
| **1.** $\begin{array}{r} 457 \\ \times\ 2 \\ \hline \end{array}$ | **3.** $\begin{array}{r} 389 \\ \times\ 4 \\ \hline \end{array}$ | **5.** $\begin{array}{r} 271 \\ \times\ 8 \\ \hline \end{array}$ | **7.** $\begin{array}{r} 476 \\ \times\ 5 \\ \hline \end{array}$ | **9.** $\begin{array}{r} 674 \\ \times\ 7 \\ \hline \end{array}$ | **11.** $\begin{array}{r} 563 \\ \times\ 8 \\ \hline \end{array}$ |
| **2.** $\begin{array}{r} 247 \\ \times\ 3 \\ \hline \end{array}$ | **4.** $\begin{array}{r} 628 \\ \times\ 4 \\ \hline \end{array}$ | **6.** $\begin{array}{r} 435 \\ \times\ 7 \\ \hline \end{array}$ | **8.** $\begin{array}{r} 296 \\ \times\ 8 \\ \hline \end{array}$ | **10.** $\begin{array}{r} 557 \\ \times\ 6 \\ \hline \end{array}$ | **12.** $\begin{array}{r} 789 \\ \times\ 9 \\ \hline \end{array}$ |

Use the short method to name the products.

| | | | |
|---|---|---|---|
| **13.** $3 \times 247$ | **15.** $4 \times 628$ | **17.** $7 \times 435$ | **19.** $8 \times 296$ |
| **14.** $4 \times 367$ | **16.** $7 \times 286$ | **18.** $9 \times 546$ | **20.** $5 \times 557$ |

Name the products.

| | | | | | |
|---|---|---|---|---|---|
| **21.** $\begin{array}{r} 84 \\ \times\ 9 \\ \hline \end{array}$ | **23.** $\begin{array}{r} 325 \\ \times\ 6 \\ \hline \end{array}$ | **25.** $\begin{array}{r} 579 \\ \times\ 7 \\ \hline \end{array}$ | **27.** $\begin{array}{r} 308 \\ \times\ 5 \\ \hline \end{array}$ | **29.** $\begin{array}{r} 470 \\ \times\ 8 \\ \hline \end{array}$ | **31.** $\begin{array}{r} 909 \\ \times\ 9 \\ \hline \end{array}$ |
| **22.** $\begin{array}{r} 79 \\ \times\ 7 \\ \hline \end{array}$ | **24.** $\begin{array}{r} 790 \\ \times\ 7 \\ \hline \end{array}$ | **26.** $\begin{array}{r} 80 \\ \times\ 9 \\ \hline \end{array}$ | **28.** $\begin{array}{r} 804 \\ \times\ 9 \\ \hline \end{array}$ | **30.** $\begin{array}{r} 386 \\ \times\ 8 \\ \hline \end{array}$ | **32.** $\begin{array}{r} 444 \\ \times\ 6 \\ \hline \end{array}$ |

Solve the equations.

**33.** $7 \times 489 = (28 \times 100) + (56 \times 10) + 63 = p$

**34.** $3 \times 426 = (12 \times 100) + (6 \times 10) + 18 = e$

**35.** $4 \times 649 = (24 \times 100) + (16 \times 10) + 36 = a$

## Mr. Todd's Grocery Store

1. Mr. Todd has about 270 customers a day. How many customers does he have in 6 days?

2. Mr. Todd sells 425 pounds of lettuce in a week. If it costs Mr. Todd 9 cents a pound, how much does he pay for lettuce each week?

3. A crate holds 180 lemons. If Mr. Todd uses 9 crates of lemons a week, how many lemons does he sell in a week?

4. The grocery delivery truck travels about 376 miles a week. About how many miles does it travel in 4 weeks?

5. Mr. Todd collects $860 a day from selling groceries. If the store is open 6 days a week, how much money does Mr. Todd collect in a week?

6. If Larry earns $2.74 a day for helping Mr. Todd, how much does he earn in 3 days? in 5 days?

7. Sally also works for Mr. Todd after school. She earns $2.23 each day. How much money does Sally earn in 4 days? in 7 days?

8. Mr. Todd has a display of apples. If each table holds 285 apples, how many apples can be displayed on 3 tables?

## Naming missing digits

Study the examples.

| | |
|---|---|
| $\begin{array}{r} 2a \\ \times\ 6 \\ \hline 156 \end{array}$ | Since $6 \times 1 = 6$ and $6 \times 6 = 36$, $a$ might be 1 or 6.<br>If $a$ is 1: $\begin{array}{r} 21 \\ \times\ 6 \\ \hline 126 \end{array}$ If $a$ is 6: $\begin{array}{r} 26 \\ \times\ 6 \\ \hline 156 \end{array}$<br>The second multiplication gives the correct product, so $a$ must be 6. |
| $\begin{array}{r} b6 \\ \times\ 7 \\ \hline 392 \end{array}$ | Start to work the multiplication.<br>$7 \times 6 = 42$<br>$7 \times b = 39 - 4$<br>$= 35$<br>$\begin{array}{r} {\scriptstyle 4} \\ b6 \\ \times\ 7 \\ \hline 2 \end{array}$<br>$7 \times 5 = 35$, so $b$ must be 5<br>$\begin{array}{r} 56 \\ \times\ 7 \\ \hline 42 \\ 350 \\ \hline 392 \end{array}$ |

### EXERCISES

Complete the multiplications.

**1.** $\begin{array}{r} 8a \\ \times\ 6 \\ \hline 504 \end{array}$ **4.** $\begin{array}{r} 6d \\ \times\ 3 \\ \hline 192 \end{array}$ **7.** $\begin{array}{r} g9 \\ \times\ 4 \\ \hline 196 \end{array}$ **10.** $\begin{array}{r} j7 \\ \times\ 8 \\ \hline 536 \end{array}$ **13.** $\begin{array}{r} 12n \\ \times\ 4 \\ \hline 504 \end{array}$ **16.** $\begin{array}{r} 86t \\ \times\ 8 \\ \hline 6896 \end{array}$

**2.** $\begin{array}{r} 5b \\ \times\ 7 \\ \hline 392 \end{array}$ **5.** $\begin{array}{r} 6e \\ \times\ 7 \\ \hline 434 \end{array}$ **8.** $\begin{array}{r} h6 \\ \times\ 5 \\ \hline 430 \end{array}$ **11.** $\begin{array}{r} k8 \\ \times\ 6 \\ \hline 288 \end{array}$ **14.** $\begin{array}{r} 38r \\ \times\ 3 \\ \hline 1161 \end{array}$ **17.** $\begin{array}{r} 7u5 \\ \times\ 7 \\ \hline 5145 \end{array}$

**3.** $\begin{array}{r} 7c \\ \times\ 8 \\ \hline 576 \end{array}$ **6.** $\begin{array}{r} 7f \\ \times\ 4 \\ \hline 296 \end{array}$ **9.** $\begin{array}{r} i4 \\ \times\ 8 \\ \hline 592 \end{array}$ **12.** $\begin{array}{r} l7 \\ \times\ 9 \\ \hline 603 \end{array}$ **15.** $\begin{array}{r} 23s \\ \times\ 6 \\ \hline 1422 \end{array}$ **18.** $\begin{array}{r} v92 \\ \times\ 5 \\ \hline 4460 \end{array}$

Solve the problems.

**19.** There are 635 seats in the movie theater. A movie was shown for 5 nights. If the movie theater was full each night, how many people saw the movie?

**20.** If 426 cars cross the bridge every hour, how many cars cross the bridge in 7 hours?

## Camping

The six members of the Albert family went camping on their vacation.

1. Mr. Albert bought 6 sleeping bags for $19 each. How much did he pay in all?

2. The family took 4 days to reach the mountains. If they traveled 345 miles a day, how many miles from home are the mountains?

3. The highest mountain in the area is 14,256 feet high. If the Alberts' camp site was 6,824 feet high, how far were they from the top of the highest mountain?

4. Mr. Albert said that each of the 6 fishing rods should have 132 feet of line on it. How many feet of fishing line should they buy?

5. It cost the Alberts $2.65 a day to camp in the State Park. If they stayed 8 days, what was their total bill?

6. Because of the fish they caught, Mrs. Albert spent only $7.40 each day for food for the 8 camping days. How much did she spend in all?

7. The Alberts saw a bear and 3 bear cubs. Mr. Albert thought that the large bear weighed about 550 pounds and that the cubs weighed about 175 pounds each. Would the 3 cubs together weigh more or less than the large bear?

## Multiples of ten

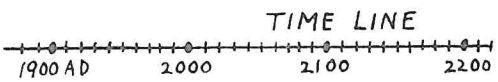
Name the products.

1. $7 \times 10 = a$
2. $8 \times 10 = b$
3. $9 \times 10 = c$
4. $10 \times 10 = d$
5. $12 \times 10 = e$
6. $7 \times 100 = f$
7. $8 \times 100 = g$
8. $9 \times 100 = h$
9. $10 \times 100 = i$
10. $12 \times 100 = j$
11. $20 \times 10 = k$
12. $50 \times 10 = l$
13. $60 \times 10 = m$
14. $20 \times 20 = n$
15. $20 \times 40 = z$
16. $30 \times 30 = p$
17. $30 \times 40 = q$
18. $40 \times 80 = r$
19. $50 \times 70 = s$
20. $60 \times 80 = t$

Name the missing factors.

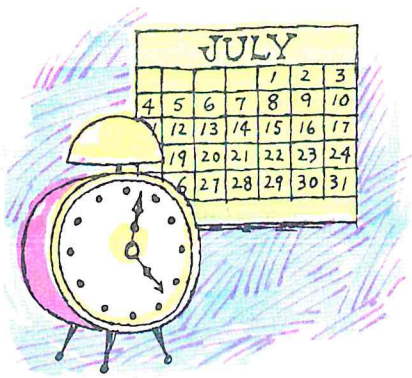
21. $100 \div 10 = a$
22. $200 \div 10 = b$
23. $700 \div 10 = c$
24. $800 \div 20 = d$
25. $1000 \div 20 = e$
26. $1200 \div 20 = f$
27. $600 \div 30 = g$
28. $800 \div 40 = h$
29. $270 \div 30 = i$
30. $500 \div 50 = j$
31. $1000 \div 20 = k$
32. $40 \div 10 = l$

Solve the problems.




33. **Century** is a name for 100 years. How many years are in 19 centuries?

34. **Decade** is a name for 10 years. How many decades are in a century? How many decades are in 800 years? How many centuries are in 800 years?




35. A very famous mathematician named Euclid lived about 2200 years ago. How many centuries ago was this?

36. There are 60 seconds in a minute. How many seconds are in 4 minutes? in 7 minutes? in 10 minutes?

37. There are 60 minutes in an hour. How many minutes are in 6 hours? in 9 hours? in 12 hours?

38. How many seconds are in one hour?

39. How many minutes are in one day?

40. **Score** is an old name for 20. How many years are in 2 score years? in 3 score years? in 3 score and 10 years?

41. If a man lived 100 years, how many score years did he live?

42. September has 30 days. How many hours are in September?

Study the methods to name the product $20 \times 48$.

| Equations | Expanded numerals |
|---|---|
| $20 \times 48 = (10 \times 2) \times 48$<br>$= 10 \times (2 \times 48)$<br>$= 10 \times 96$<br>$= 960$ | $48 = 40 + 8$<br>$\times 20 \quad \times 20$<br>$800 + 160 = 960$ |

## EXERCISES

Copy and complete the equations.

**1.** $30 \times 43 = (10 \times 3) \times 43$
$= 10 \times (3 \times a)$
$= 10 \times b$
$= c$

**2.** $50 \times 86 = (10 \times 5) \times 86$
$= 10 \times (d \times 86)$
$= 10 \times e$
$= f$

**3.** $30 \times 29 = 30 \times (20 + 9)$
$= (30 \times 20) + (30 \times a)$
$= 600 + b$
$= c$

**4.** $70 \times 92 = 70 \times (d + 2)$
$= (70 \times e) + (70 \times 2)$
$= f + g$
$= h$

Use expanded numerals to name the products.

**5.** $56 \times 30$ **6.** $83 \times 20$ **7.** $36 \times 60$ **8.** $59 \times 70$ **9.** $75 \times 40$ **10.** $97 \times 80$ **11.** $67 \times 90$

Complete the number patterns.

**12.**

| × | 7 | 70 | 700 |
|---|---|---|---|
| 37 | *a* | *b* | *c* |

**13.**

| × | 8 | 80 | 800 |
|---|---|---|---|
| 84 | *d* | *e* | *f* |

**14.**

| × | 5 | 50 | 500 |
|---|---|---|---|
| 67 | *g* | *h* | *i* |

**15.**

| × | 9 | 90 | 900 |
|---|---|---|---|
| 75 | *j* | *k* | *l* |

Name the products.

**16.** $36 \times 50$ **17.** $53 \times 40$ **18.** $69 \times 70$ **19.** $97 \times 30$ **20.** $85 \times 80$ **21.** $94 \times 90$ **22.** $82 \times 60$

## One factor between 10 and 20

Study the methods to name the product $36 \times 14$.

| Equations | Expanded numerals |
|---|---|
| $14 \times 36 = (10 + 4) \times 36$<br>$= (10 \times 36) + (4 \times 36)$<br>$= 360 + 144$<br>$= 504$ | $\begin{array}{r} 36 \\ \times\ 14 \\ \hline \end{array} = \begin{array}{r} 36 \\ \times\ 10 \\ \hline \end{array} + \begin{array}{r} 36 \\ \times\ 4 \\ \hline \end{array}$<br>$360 + 144 = 504$ |

### EXERCISES

Copy and complete the examples.

1. $\begin{array}{r} 24 \\ \times\ 16 \\ \hline \end{array} = \begin{array}{r} 24 \\ \times\ 10 \\ \hline \end{array} + \begin{array}{r} 24 \\ \times\ 6 \\ \hline \end{array}$
$a + c = d$

2. $\begin{array}{r} 48 \\ \times\ 16 \\ \hline \end{array} = \begin{array}{r} 48 \\ \times\ 10 \\ \hline \end{array} + \begin{array}{r} 48 \\ \times\ 6 \\ \hline \end{array}$
$e + g = h$

Solve the equations.

3. $10 \times 24 = 240$ and $3 \times 24 = 72$ so $13 \times 24 = 312$
4. $10 \times 32 = a$ and $2 \times 32 = b$ so $12 \times 32 = c$
5. $10 \times 21 = d$ and $3 \times 21 = e$ so $13 \times 21 = f$
6. $10 \times 42 = g$ and $4 \times 42 = h$ so $14 \times 42 = i$
7. $10 \times 32 = j$ and $4 \times 32 = k$ so $14 \times 32 = l$

8. $4 \times 26 = a$
$\underline{10 \times 26 = c}$
$14 \times 26 = e$

9. $9 \times 32 = f$
$\underline{10 \times 32 = i}$
$19 \times 32 = j$

10. $3 \times 57 = k$
$\underline{10 \times 57 = m}$
$13 \times 57 = n$

11. $5 \times 67 = v$
$\underline{10 \times 67 = s}$
$15 \times 67 = u$

12. $6 \times 35 = v$
$\underline{10 \times 35 = w}$
$16 \times 35 = x$

13. $3 \times 48 = z$
$\underline{10 \times 48 = a}$
$13 \times 48 = c$

14. $8 \times 52 = d$
$\underline{10 \times 52 = e}$
$18 \times 52 = g$

15. $2 \times 59 = h$
$\underline{10 \times 59 = i}$
$12 \times 59 = j$

Complete the number patterns.

16.

| × | 24 |
|---|---|
| 3 | $a$ |
| 10 | $c$ |
| 13 | $e$ |

17.

| × | 36 |
|---|---|
| 5 | $i$ |
| 10 | $m$ |
| 15 | $n$ |

18.

| × | 45 |
|---|---|
| 6 | $r$ |
| 10 | $s$ |
| 16 | $u$ |

19.

| × | 38 |
|---|---|
| 4 | $w$ |
| 10 | $x$ |
| 14 | $z$ |

20.

| × | 42 |
|---|---|
| 7 | $a$ |
| 10 | $c$ |
| 17 | $e$ |

21.

| × | 57 |
|---|---|
| 6 | $i$ |
| 10 | $m$ |
| 16 | $n$ |

Study the methods to name the product $14 \times 36$.

| Equations | Three step |
|---|---|
| $14 \times 36 = (10 + 4) \times 36$<br>$= (10 \times 36) + (4 \times 36)$<br>$= 360 + 144$<br>$= 504$ | $\begin{array}{r} 36 \\ \times\ 14 \\ \hline 144 \\ 360 \\ \hline 504 \end{array}$ $\begin{array}{l} \\ = 4 \times 36 \\ = 10 \times 36 \\ = 14 \times 36 \end{array}$ |

## EXERCISES

Use the 3-step method to name the products.

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** $\begin{array}{r} 27 \\ \times\ 12 \end{array}$ | **4.** $\begin{array}{r} 24 \\ \times\ 13 \end{array}$ | **7.** $\begin{array}{r} 39 \\ \times\ 14 \end{array}$ | **10.** $\begin{array}{r} 56 \\ \times\ 16 \end{array}$ | **13.** $\begin{array}{r} 29 \\ \times\ 12 \end{array}$ | **16.** $\begin{array}{r} 25 \\ \times\ 14 \end{array}$ | **19.** $\begin{array}{r} 38 \\ \times\ 12 \end{array}$ |
| **2.** $\begin{array}{r} 37 \\ \times\ 12 \end{array}$ | **5.** $\begin{array}{r} 26 \\ \times\ 13 \end{array}$ | **8.** $\begin{array}{r} 49 \\ \times\ 14 \end{array}$ | **11.** $\begin{array}{r} 54 \\ \times\ 16 \end{array}$ | **14.** $\begin{array}{r} 19 \\ \times\ 12 \end{array}$ | **17.** $\begin{array}{r} 35 \\ \times\ 12 \end{array}$ | **20.** $\begin{array}{r} 68 \\ \times\ 12 \end{array}$ |
| **3.** $\begin{array}{r} 37 \\ \times\ 17 \end{array}$ | **6.** $\begin{array}{r} 48 \\ \times\ 16 \end{array}$ | **9.** $\begin{array}{r} 96 \\ \times\ 14 \end{array}$ | **12.** $\begin{array}{r} 57 \\ \times\ 12 \end{array}$ | **15.** $\begin{array}{r} 62 \\ \times\ 11 \end{array}$ | **18.** $\begin{array}{r} 73 \\ \times\ 15 \end{array}$ | **21.** $\begin{array}{r} 85 \\ \times\ 18 \end{array}$ |

Solve the equations.

**22.** $16 \times 48 = (10 + 6) \times 48$
$= (10 \times 48) + (a \times 48)$
$= 480 + b$
$= c$

**23.** $12 \times 63 = (10 + 2) \times 63$
$= (e \times 63) + (2 \times 63)$
$= f + 126$
$= g$

Solve the problems.

**24.** There are 12 inches in one foot. How many inches are in 27 feet?

**25.** There are 16 ounces in one pound. How many ounces are in 38 pounds?

**26.** There are 36 inches in one yard. How many inches are in 17 yards?

**27.** There are 12 objects in one dozen. How many objects are in 25 dozen?

## Both factors greater than 10

Study the methods to name the product $32 \times 48$.

| Equations | Number patterns |
|---|---|
| $32 \times 48 = (30 + 2) \times 48$<br>$= (30 \times 48) + (2 \times 48)$<br>$= 1440 + 96$<br>$= 1536$ | $2 \times 48 = 96$<br>$30 \times 48 = 1440$<br>$32 \times 48 = 1536$ |

### EXERCISES

Solve the equations.

1. $20 \times 24 = 480$ and $3 \times 24 = 72$ so $23 \times 24 = 552$
2. $40 \times 32 = a$ and $3 \times 32 = b$ so $43 \times 32 = c$
3. $40 \times 21 = d$ and $3 \times 21 = e$ so $43 \times 21 = f$
4. $30 \times 42 = g$ and $6 \times 42 = h$ so $36 \times 42 = i$
5. $50 \times 32 = j$ and $4 \times 32 = k$ so $54 \times 32 = l$
6. $60 \times 27 = m$ and $5 \times 27 = n$ so $65 \times 27 = z$
7. $20 \times 41 = p$ and $4 \times 41 = q$ so $24 \times 41 = r$

Solve the equations.

8. $4 \times 26 = a$
   $30 \times 26 = c$
   $34 \times 26 = e$

9. $9 \times 32 = f$
   $20 \times 32 = i$
   $29 \times 32 = j$

10. $3 \times 57 = k$
    $20 \times 57 = m$
    $23 \times 57 = n$

11. $5 \times 67 = r$
    $40 \times 67 = s$
    $45 \times 67 = u$

12. $4 \times 35 = v$
    $20 \times 35 = w$
    $24 \times 35 = x$

13. $3 \times 48 = z$
    $40 \times 48 = a$
    $43 \times 48 = c$

14. $4 \times 52 = e$
    $50 \times 52 = i$
    $54 \times 52 = j$

15. $2 \times 59 = k$
    $60 \times 59 = m$
    $62 \times 59 = n$

Complete the number patterns.

16.

| × | 24 |
|---|---|
| 1 | $a$ |
| 20 | $c$ |
| 21 | $e$ |

17.

| × | 23 |
|---|---|
| 2 | $i$ |
| 30 | $m$ |
| 32 | $n$ |

18.

| × | 36 |
|---|---|
| 3 | $r$ |
| 20 | $s$ |
| 23 | $u$ |

19.

| × | 27 |
|---|---|
| 2 | $v$ |
| 40 | $w$ |
| 42 | $x$ |

20.

| × | 44 |
|---|---|
| 6 | $z$ |
| 50 | $a$ |
| 56 | $c$ |

21.

| × | 28 |
|---|---|
| 3 | $e$ |
| 60 | $i$ |
| 63 | $m$ |

Study the methods to name the product $24 \times 38$.

**Expanded numerals**

$$\begin{array}{r} 38 \\ \times\ 24 \\ \hline \end{array} = \begin{array}{r} 38 \\ \times\ 20 \\ \hline 760 \end{array} + \begin{array}{r} 38 \\ \times\ 4 \\ \hline 152 \end{array} = 912$$

**Three step**

$$\begin{array}{r} 38 \\ \times\ 24 \\ \hline 152 \\ 760 \\ \hline 912 \end{array} \begin{array}{l} \\ \\ = 4 \times 38 \\ = 20 \times 38 \\ = 24 \times 38 \end{array}$$

## EXERCISES

Use expanded numerals to name the products.

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** $40 \times 36$ | **3.** $60 \times 27$ | **5.** $50 \times 65$ | **7.** $20 \times 49$ | **9.** $22 \times 49$ | **11.** $57 \times 24$ | **13.** $63 \times 31$ |
| **2.** $76 \times 52$ | **4.** $83 \times 47$ | **6.** $95 \times 38$ | **8.** $69 \times 69$ | **10.** $77 \times 79$ | **12.** $86 \times 78$ | **14.** $94 \times 88$ |

Use the 3-step method to name the products.

| | | | | | | |
|---|---|---|---|---|---|---|
| **15.** $46 \times 32$ | **17.** $68 \times 27$ | **19.** $38 \times 34$ | **21.** $72 \times 64$ | **23.** $36 \times 45$ | **25.** $29 \times 14$ | **27.** $38 \times 47$ |
| **16.** $36 \times 48$ | **18.** $43 \times 67$ | **20.** $29 \times 43$ | **22.** $87 \times 26$ | **24.** $75 \times 32$ | **26.** $68 \times 41$ | **28.** $94 \times 35$ |

Complete each sentence. Write $<$, $>$, or $=$ to replace the ◯.

**29.** $12 \times 14$ ◯ $14 \times 12$

**30.** $47 \times 10$ ◯ $40 \times 17$

**31.** $3 \times (40 - 1)$ ◯ $3 \times 39$

**32.** $18 \times (20 + 4)$ ◯ $(18 \times 20) + (18 \times 4)$

Name the products.

| | | | |
|---|---|---|---|
| **33.** $7 \times 7$ | **35.** $47 \times 47$ | **37.** $8 \times 8$ | **39.** $38 \times 38$ |
| **34.** $17 \times 17$ | **36.** $77 \times 77$ | **38.** $18 \times 18$ | **40.** $88 \times 88$ |

## Practice with multiplication

Name the products.

**1.**
$$\begin{array}{r} 40 \\ \times\ 7 \\ \hline a \end{array} + \begin{array}{r} 7 \\ \times\ 7 \\ \hline c \end{array} = \begin{array}{r} 47 \\ \times\ 7 \\ \hline e \end{array}$$

**2.**
$$\begin{array}{r} 50 \\ \times\ 9 \\ \hline g \end{array} + \begin{array}{r} 4 \\ \times\ 9 \\ \hline i \end{array} = \begin{array}{r} 54 \\ \times\ 9 \\ \hline j \end{array}$$

**3.**
$$\begin{array}{r} 38 \\ \times\ 20 \\ \hline m \end{array} + \begin{array}{r} 38 \\ \times\ 4 \\ \hline n \end{array} = \begin{array}{r} 38 \\ \times\ 24 \\ \hline p \end{array}$$

**4.**
$$\begin{array}{r} 30 \\ \times\ 50 \\ \hline q \end{array} + \begin{array}{r} 30 \\ \times\ 6 \\ \hline r \end{array} = \begin{array}{r} 30 \\ \times\ 56 \\ \hline s \end{array}$$

**5.**
$$\begin{array}{r} 70 \\ \times\ 70 \\ \hline u \end{array} + \begin{array}{r} 70 \\ \times\ 8 \\ \hline v \end{array} = \begin{array}{r} 70 \\ \times\ 78 \\ \hline w \end{array}$$

**6.**
$$\begin{array}{r} 90 \\ \times\ 80 \\ \hline x \end{array} + \begin{array}{r} 90 \\ \times\ 5 \\ \hline y \end{array} = \begin{array}{r} 90 \\ \times\ 85 \\ \hline z \end{array}$$

Name the products.

**7.** $\begin{array}{r} 560 \\ \times\ 8 \\ \hline \end{array}$ **8.** $\begin{array}{r} 392 \\ \times\ 4 \\ \hline \end{array}$ **9.** $\begin{array}{r} 579 \\ \times\ 7 \\ \hline \end{array}$

**10.** $\begin{array}{r} 608 \\ \times\ 8 \\ \hline \end{array}$ **11.** $\begin{array}{r} 171 \\ \times\ 9 \\ \hline \end{array}$ **12.** $\begin{array}{r} 969 \\ \times\ 8 \\ \hline \end{array}$

**13.** $\begin{array}{r} 27 \\ \times\ 32 \\ \hline \end{array}$ **14.** $\begin{array}{r} 45 \\ \times\ 57 \\ \hline \end{array}$ **15.** $\begin{array}{r} 38 \\ \times\ 74 \\ \hline \end{array}$

**16.** $\begin{array}{r} 89 \\ \times\ 16 \\ \hline \end{array}$ **17.** $\begin{array}{r} 93 \\ \times\ 36 \\ \hline \end{array}$ **18.** $\begin{array}{r} 57 \\ \times\ 78 \\ \hline \end{array}$

**19.** $94 \times 47$

**20.** $34 \times 69$

**21.** $41 \times 18$

**22.** $73 \times 46$

**23.** $82 \times 63$

In each exercise, some digits are missing.
Name the missing digits.

**24.**
$$\begin{array}{r} 48 \\ \times\ 32 \\ \hline 96 \\ 1a40 \\ \hline 15c6 \end{array}$$

**25.**
$$\begin{array}{r} 74 \\ \times\ 36 \\ \hline 4e4 \\ 2\,l20 \\ \hline 2n64 \end{array}$$

**26.**
$$\begin{array}{r} 63 \\ \times\ 43 \\ \hline 1r9 \\ 2520 \\ \hline 2us9 \end{array}$$

**27.**
$$\begin{array}{r} 64 \\ \times\ 77 \\ \hline 448 \\ 4v80 \\ \hline 4zx8 \end{array}$$

**28.**
$$\begin{array}{r} 96 \\ \times\ 44 \\ \hline 3a4 \\ 3840 \\ \hline 4ec4 \end{array}$$

**29.**
$$\begin{array}{r} 78 \\ \times\ 36 \\ \hline 468 \\ 2\,i40 \\ \hline 2pn8 \end{array}$$

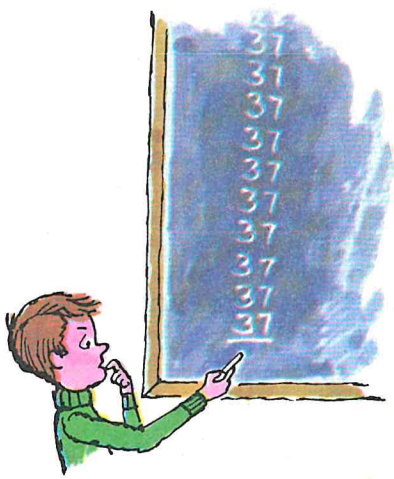
**30.**
$$\begin{array}{r} 84 \\ \times\ 34 \\ \hline 336 \\ 25r0 \\ \hline 28us \end{array}$$

**31.**
$$\begin{array}{r} 67 \\ \times\ 45 \\ \hline 3v5 \\ 2680 \\ \hline z0x5 \end{array}$$

1. There are 31 days in March and 24 hours in each day. How many hours are there in March?

2. If you spend 24 cents a day for traveling to school and travel 42 days in October and November, how much money do you spend?

3. If you travel to and from school 14 miles each day on 42 days in October and November, how far do you travel?




4. To name the sum of 24 addends which are all 37, you may write 37 twenty four times and then add. Name the sum by a quicker method.

5. Each can of cherries contains 35 cherries. How many cherries are in 60 cans?

6. If your watch ticks 300 times a minute, how many times does it tick in an hour?

7. If you deliver 48 newspapers per day, how many do you deliver in 30 days?

## Practice page

Name the sums.

**1.**
$$\begin{array}{r} 36 \\ 24 \\ 38 \\ 43 \\ 29 \\ +\ 17 \\ \hline \end{array}$$

**2.**
$$\begin{array}{r} 343 \\ 246 \\ 117 \\ 90 \\ 578 \\ +\ 279 \\ \hline \end{array}$$

**3.**
$$\begin{array}{r} 340 \\ 269 \\ 82 \\ 920 \\ 74 \\ +\ 703 \\ \hline \end{array}$$

**4.**
$$\begin{array}{r} 361 \\ 284 \\ 493 \\ 685 \\ 274 \\ +\ 389 \\ \hline \end{array}$$

**5.**
$$\begin{array}{r} 34561 \\ 29685 \\ +\ 34867 \\ \hline \end{array}$$

**6.**
$$\begin{array}{r} 34629 \\ 23504 \\ +\ 73896 \\ \hline \end{array}$$

Name the differences.

**7.** $\begin{array}{r} 826 \\ -\ 554 \\ \hline \end{array}$ **8.** $\begin{array}{r} 362 \\ -\ 259 \\ \hline \end{array}$ **9.** $\begin{array}{r} 279 \\ -\ 170 \\ \hline \end{array}$ **10.** $\begin{array}{r} 934 \\ -\ 806 \\ \hline \end{array}$ **11.** $\begin{array}{r} 728 \\ -\ 189 \\ \hline \end{array}$ **12.** $\begin{array}{r} 64281 \\ -\ 34729 \\ \hline \end{array}$

Name the products.

**13.** $\begin{array}{r} 64 \\ \times\ 64 \\ \hline \end{array}$ **15.** $\begin{array}{r} 89 \\ \times\ 94 \\ \hline \end{array}$ **17.** $\begin{array}{r} 68 \\ \times\ 39 \\ \hline \end{array}$ **19.** $\begin{array}{r} 386 \\ \times\ 9 \\ \hline \end{array}$ **21.** $\begin{array}{r} 381 \\ \times\ 9 \\ \hline \end{array}$ **23.** $\begin{array}{r} 436 \\ \times\ 6 \\ \hline \end{array}$

**14.** $\begin{array}{r} 57 \\ \times\ 57 \\ \hline \end{array}$ **16.** $\begin{array}{r} 37 \\ \times\ 62 \\ \hline \end{array}$ **18.** $\begin{array}{r} 47 \\ \times\ 81 \\ \hline \end{array}$ **20.** $\begin{array}{r} 478 \\ \times\ 6 \\ \hline \end{array}$ **22.** $\begin{array}{r} 627 \\ \times\ 6 \\ \hline \end{array}$ **24.** $\begin{array}{r} 578 \\ \times\ 9 \\ \hline \end{array}$

Copy and complete the examples.

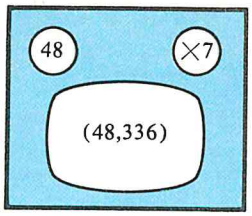
**25.** $\begin{array}{r} 74 \\ +\ 2a \\ \hline 102 \end{array}$ **27.** $\begin{array}{r} 382 \\ +\ 4c4 \\ \hline 876 \end{array}$ **29.** $\begin{array}{r} 246 \\ +\ c6e \\ \hline 711 \end{array}$ **31.** $\begin{array}{r} 84 \\ -\ 2e \\ \hline 62 \end{array}$ **33.** $\begin{array}{r} 46a \\ -\ 2u8 \\ \hline 223 \end{array}$ **35.** $\begin{array}{r} c38 \\ -\ 2r5 \\ \hline 463 \end{array}$

**26.** $\begin{array}{r} 38 \\ +\ 49 \\ \hline r7 \end{array}$ **28.** $\begin{array}{r} 385 \\ +\ ae4 \\ \hline 829 \end{array}$ **30.** $\begin{array}{r} 246 \\ +\ n2s \\ \hline 675 \end{array}$ **32.** $\begin{array}{r} 854 \\ -\ n62 \\ \hline 392 \end{array}$ **34.** $\begin{array}{r} 4n8 \\ -\ k56 \\ \hline 162 \end{array}$ **36.** $\begin{array}{r} sr5 \\ -\ 369 \\ \hline 476 \end{array}$

Complete each sentence. Write $<$, $>$, or $=$ to replace the ◯.

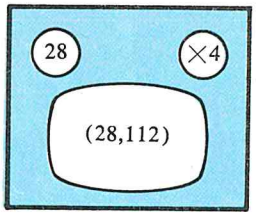
**37.** $328 + 465 \bigcirc 269 + 641$
**38.** $743 + 268 \bigcirc 831 + 180$
**39.** $683 + 509 \bigcirc 817 + 750$
**40.** $134 + 681 \bigcirc 296 + 542$
**41.** $836 - 497 \bigcirc 947 - 610$
**42.** $7 \times 67 \bigcirc 6 \times 76$
**43.** $8 \times 43 \bigcirc 7 \times 49$
**44.** $2 \times 58 \bigcirc 3 \times 46$
**45.** $45 \times 68 \bigcirc 89 \times 29$
**46.** $77 \times 49 \bigcirc 35 \times 54$

The TV shows the tuned in number to be 48. The rule is *multiply by 7*. The number pair is (48, 336).

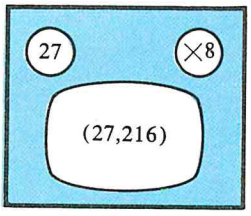



## EXERCISES

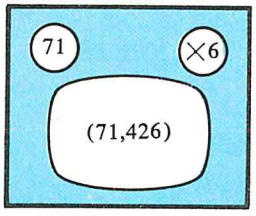
Use the TV to complete the number pairs in each set.



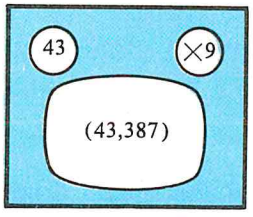

1. {(28, 112), (31, *a*), (49, *b*), (54, *c*), (217, *d*) (231, *e*), (424, *f*)}




3. {(27, 216), (35, *m*), (46, *n*), (54, *z*), (28, *p*), (53, *q*),(77, *r*)}




2. {(71, 426), (36, *g*), (49, *h*), (156, *i*), (248, *j*) (353, *k*),(465, *l*)}




4. {(43, 387), (18, *s*), (47, *t*), (23, *u*), (39, *v*), (57, *w*), (68, *x*)}

Use the rule to complete the number pairs in each set.

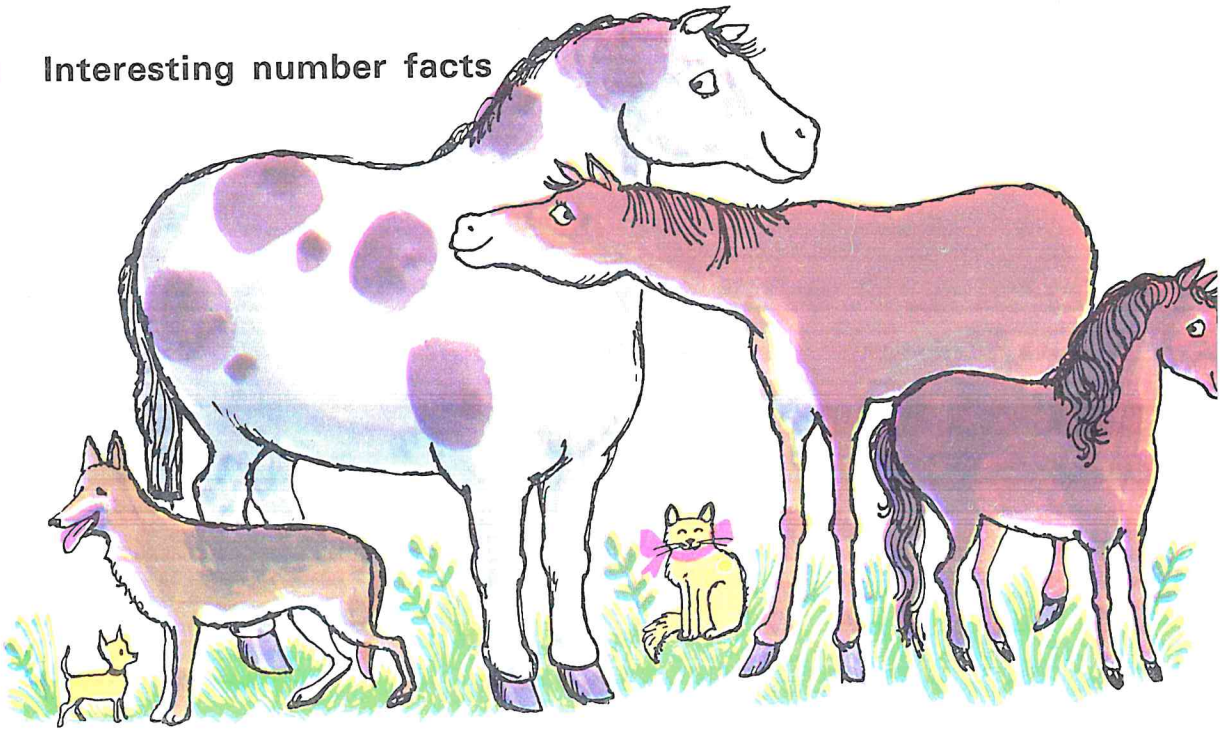
5. *Multiply by 10:*
{(24, 240), (31, *a*), (57, *b*), (83, *c*), (95, *d*), (120, *e*)}

6. *Multiply by 7:*
{(123, 861), (284, *f*), (396, *g*), (487, *h*), (705, *i*), (964, *j*)}

7. *Multiply by 15:*
{(24, 360), (34, *k*), (75, *l*), (61, *m*), (89, *n*), (94, *z*)}

8. *Multiply by 34:*
{(42, 1428), (21, *p*), (39, *q*), (56, *r*), (18, *s*), (0, *t*)}

## Interesting number facts



1. One of the largest horses ever known was 7 feet tall. It weighed about 3000 pounds.

2. The longest mane of a horse was about 12 feet. The longest tail of a horse was about 10 feet.

3. One of the oldest dogs died at the age of 27.

4. The Chihuahua from Mexico is said to be the world's smallest dog. Some Chihuahuas weigh as little as one pound.

5. A police dog can jump a wall that is over 9 feet tall.

6. A cat has been known to live as long as 35 years.

7. The heaviest rabbit ever known weighed about 20 pounds.

Can you find other interesting number facts about animals?

$4 \times 7 = a$
$7 \times 4 = a$
$a \div 4 = 7$
$a \div 7 = 4$

In these equations, the product is missing. Multiplication is naming two factors as the product.

$4 \times a = 28$
$a \times 4 = 28$
$28 \div 4 = a$
$28 \div a = 4$

In these equations, a factor is missing. Division is naming the product and a factor as the missing factor.

Another word for **missing factor** is **quotient.**

## EXERCISES

Name the quotients.

**1.** $3 \times a = 12$ **2.** $b \times 7 = 49$ **3.** $12 \div 6 = d$ **4.** $20 \div 5 = e$ **5.** $63 \div f = 7$ **6.** $36 \div g = 9$ **7.** $72 \div h = 8$ **8.** $i \div 8 = 3$

Name the quotients.

**9.** $3\overline{)15}$ **10.** $6\overline{)36}$ **11.** $6\overline{)42}$ **12.** $9\overline{)54}$ **13.** $4\overline{)32}$ **14.** $8\overline{)64}$ **15.** $3\overline{)21}$ **16.** $9\overline{)81}$ **17.** $8\overline{)56}$ **18.** $5\overline{)45}$

Complete the tables.

**19.**

| × | 6 |
|---|---|
| 4 | 24 |
| $a$ | 18 |
| $b$ | 42 |
| $c$ | 30 |
| $d$ | 12 |

**20.**

| × | 4 |
|---|---|
| $e$ | 16 |
| $f$ | 36 |
| $g$ | 20 |
| $h$ | 28 |
| $i$ | 32 |

**21.**

| × | 8 |
|---|---|
| $j$ | 32 |
| $k$ | 64 |
| $l$ | 24 |
| $m$ | 48 |
| $n$ | 56 |

**22.**

| × | 7 |
|---|---|
| $z$ | 49 |
| $p$ | 28 |
| $q$ | 42 |
| $r$ | 63 |
| $s$ | 35 |

**23.**

| × | 9 |
|---|---|
| $t$ | 45 |
| $u$ | 72 |
| $v$ | 27 |
| $w$ | 54 |
| $x$ | 81 |

**24.**

| **factor** | 2 | 3 | 4 | $d$ | $e$ | 7 | 8 | $h$ | $i$ | 7 | $k$ | $l$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **factor** | $a$ | $b$ | $c$ | 5 | 6 | $f$ | $g$ | 9 | 8 | $j$ | 6 | 5 |
| **product** | 16 | 24 | 20 | 45 | 42 | 56 | 48 | 63 | 32 | 56 | 36 | 25 |

## Separating sets

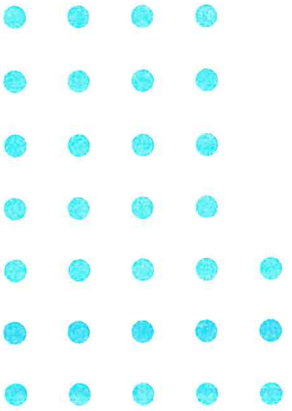
To find how many sets of 7 may be formed from a set of 31, you may think like this:

$4 \times 7 = 28 \qquad 5 \times 7 = 35$

31 is between 28 and 35.
There are more than 4 sets of 7 in 31, but not enough for 5 sets of 7.

$31 = (4 \times 7) + 3$

A set of 31 forms 4 sets of 7 with a remainder of 3.



### EXERCISES

Solve the equations.

1. $57 = (9 \times a) + 3$
2. $49 = (6 \times b) + 1$
3. $44 = (7 \times c) + 2$
4. $45 = (6 \times 7) + d$
5. $22 = (5 \times 4) + e$
6. $19 = (h \times 3) + f$
7. $38 = (7 \times g) + 3$
8. $47 = (h \times 8) + 7$
9. $75 = (i \times 9) + 3$

Complete the sentences.

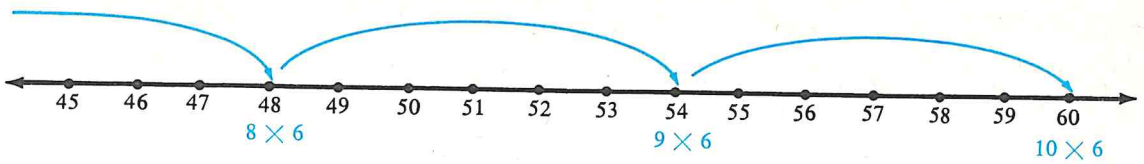
10. A set of 14 forms 4 sets of 3 with a remainder of ___?___.
11. A set of 25 forms 6 sets of 4 with a remainder of ___?___.
12. A set of 23 forms 3 sets of 7 with a remainder of ___?___.
13. A set of 38 forms 4 sets of 8 with a remainder of ___?___.
14. A set of 47 forms 9 sets of 5 with a remainder of ___?___.
15. A set of 59 forms 8 sets of 7 with a remainder of ___?___.
16. A set of 53 forms 7 sets of 7 with a remainder of ___?___.
17. A set of 61 forms 6 sets of 9 with a remainder of ___?___.

Complete this chart.

18.

| size of set | 34 | 26 | 25 | 20 | 28 | 30 | 32 | 29 | 35 |
|---|---|---|---|---|---|---|---|---|---|
| number of subsets | 6 | 4 | 3 | $c$ | $e$ | $q$ | 6 | $k$ | $m$ |
| size of each subset | 5 | 6 | 8 | 6 | 5 | 4 | $i$ | 3 | 4 |
| number of objects remaining | 4 | $a$ | $b$ | $d$ | $f$ | $h$ | $j$ | $l$ | $n$ |

## Separating sets on the number line




To find how many sets of 6 may be formed from a set of 55, we may count by sixes on the number line as above. 55 is between $9 \times 6$ and $10 \times 6$. There are 9 sets of 6 in 55 and 1 remaining. 9 is the **quotient** and 1 is the **remainder.**

### EXERCISES

Use the number line above to complete the number sentences.

1. 55 is between $(a \times 8)$ and $(b \times 8)$.
2. 49 is between $(c \times 8)$ and $(d \times 8)$.
3. 50 is between $(e \times 6)$ and $(f \times 6)$.
4. 54 is between $(g \times 7)$ and $(h \times 7)$.
5. 48 is between $(i \times 9)$ and $(j \times 9)$.
6. 51 is between $(k \times 7)$ and $(l \times 7)$.

Use the number line to complete the number sentences.

7. 28 is between $(a \times 6)$ and $(b \times 6)$.
8. 22 is between $(c \times 3)$ and $(d \times 3)$.
9. 29 is between $(e \times 8)$ and $(f \times 8)$.
10. 23 is between $(g \times 4)$ and $(h \times 4)$.

Solve the equations.

11. $23 = (4 \times 5) + a$
12. $31 = (5 \times 6) + b$
13. $25 = (6 \times 4) + c$
14. $34 = (4 \times 8) + d$
15. $26 = (6 \times 4) + e$
16. $25 = (8 \times 3) + f$

Name the quotients and remainders.

17. 33 divided **a.** by 4 **b.** by 6 **c.** by 5 **d.** by 8
18. 59 divided **a.** by 8 **b.** by 7 **c.** by 9 **d.** by 6
19. 47 divided **a.** by 5 **b.** by 6 **c.** by 7 **d.** by 8
20. 46 divided **a.** by 6 **b.** by 7 **c.** by 8 **d.** by 9
21. 33 divided **a.** by 6 **b.** by 7 **c.** by 8 **d.** by 9
22. 29 divided **a.** by 4 **b.** by 6 **c.** by 3 **d.** by 8
23. 37 divided **a.** by 5 **b.** by 6 **c.** by 8 **d.** by 9
24. 28 divided **a.** by 6 **b.** by 3 **c.** by 9 **d.** by 7

## Quotients and remainders

$3\overline{)20}$ is the working form for asking "How many sets of 3 can be formed from a set of 20?"

$3\overline{)20}$ $\quad 6 \times 3 = 18. \quad 7 \times 3 = 21$
20 is between 18 and 21.

$$\begin{array}{r} 6 \\ 3\overline{)20} \\ \underline{18} \\ 2 \end{array}$$

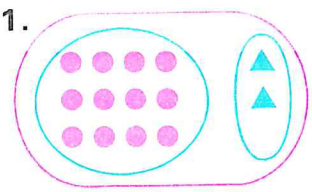
$6 \times 3 = 18$
$20 - 18 = 2$

6 is the **quotient.** 2 is the **remainder.**

### EXERCISES

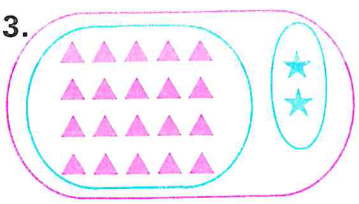
Use the sets to solve the equations.

1.



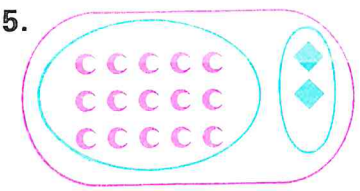
$(4 \times 3) + 2 = a$

3.



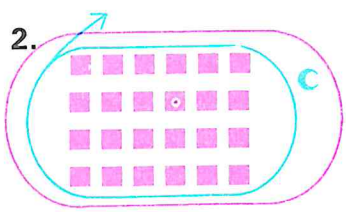
$(5 \times 4) + 2 = c$

5.



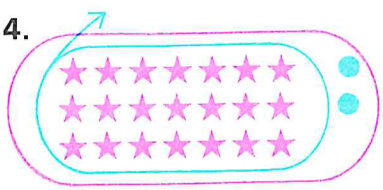
$(5 \times 3) + 2 = e$

2.



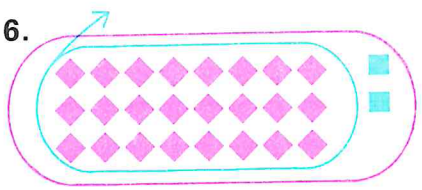
$25 - (6 \times 4) = b$

4.



$23 - (7 \times 3) = d$

6.



$26 - (8 \times 3) = f$

Complete the examples.

**7.**
$$\begin{array}{r} 3 \\ 6\overline{)20} \\ \underline{18} \\ a \end{array}$$

**8.**
$$\begin{array}{r} b \\ 5\overline{)17} \\ \underline{15} \\ 2 \end{array}$$

**9.**
$$\begin{array}{r} 5 \\ 8\overline{)43} \\ \underline{40} \\ c \end{array}$$

**10.**
$$\begin{array}{r} d \\ 6\overline{)45} \\ \underline{42} \\ e \end{array}$$

**11.**
$$\begin{array}{r} f \\ 7\overline{)38} \\ \underline{35} \\ g \end{array}$$

**12.**
$$\begin{array}{r} h \\ 4\overline{)38} \\ \underline{i} \\ j \end{array}$$

Name the quotients and remainders.

**13.** $4\overline{)17}$ **14.** $5\overline{)23}$ **15.** $7\overline{)45}$ **16.** $6\overline{)58}$ **17.** $8\overline{)49}$ **18.** $9\overline{)68}$

## The working form for division

Study the equations and working forms.

| Addition | Subtraction | Multiplication | Division |
|---|---|---|---|
| $14 + 16 = 30$ | $42 - 28 = 14$ | $3 \times 27 = 81$ | $34 = (5 \times 6) + 4$ |
| $\begin{array}{r} 14 \\ +\ 16 \\ \hline 30 \end{array}$ | $\begin{array}{r} 42 \\ -\ 28 \\ \hline 14 \end{array}$ | $\begin{array}{r} 27 \\ \times\ 3 \\ \hline 81 \end{array}$ | $\begin{array}{r} 5 \\ 6\overline{)34} \\ 30 \\ \hline 4 \end{array}$ |

### EXERCISES

Complete the working forms for the equations.

**1.** $20 = (6 \times 3) + 2$

$$\begin{array}{r} a \\ 3\overline{)20} \\ 18 \\ \hline b \end{array}$$

**3.** $22 = (5 \times 4) + 2$

$$\begin{array}{r} e \\ 4\overline{)22} \\ 20 \\ \hline f \end{array}$$

**5.** $45 = (7 \times 6) + 3$

$$\begin{array}{r} i \\ 6\overline{)45} \\ 42 \\ \hline j \end{array}$$

**2.** $52 = (5 \times 9) + 7$

$$\begin{array}{r} c \\ 9\overline{)52} \\ 45 \\ \hline d \end{array}$$

**4.** $53 = (8 \times 6) + 5$

$$\begin{array}{r} g \\ 6\overline{)53} \\ 48 \\ \hline h \end{array}$$

**6.** $69 = (8 \times 8) + 5$

$$\begin{array}{r} k \\ 8\overline{)69} \\ 64 \\ \hline l \end{array}$$

Write equations for the working forms.

**7.** $\begin{array}{r} 9 \\ 3\overline{)28} \\ 27 \\ \hline 1 \end{array}$ **9.** $\begin{array}{r} 7 \\ 4\overline{)31} \\ 28 \\ \hline 3 \end{array}$ **11.** $\begin{array}{r} 5 \\ 6\overline{)34} \\ 30 \\ \hline 4 \end{array}$ **13.** $\begin{array}{r} 5 \\ 8\overline{)45} \\ 40 \\ \hline 5 \end{array}$ **15.** $\begin{array}{r} 5 \\ 5\overline{)27} \\ 25 \\ \hline 2 \end{array}$ **17.** $\begin{array}{r} 8 \\ 7\overline{)58} \\ 56 \\ \hline 2 \end{array}$

**8.** $\begin{array}{r} 9 \\ 4\overline{)39} \\ 36 \\ \hline 3 \end{array}$ **10.** $\begin{array}{r} 5 \\ 7\overline{)41} \\ 35 \\ \hline 6 \end{array}$ **12.** $\begin{array}{r} 8 \\ 6\overline{)52} \\ 48 \\ \hline 4 \end{array}$ **14.** $\begin{array}{r} 6 \\ 9\overline{)62} \\ 54 \\ \hline 8 \end{array}$ **16.** $\begin{array}{r} 7 \\ 8\overline{)61} \\ 56 \\ \hline 5 \end{array}$ **18.** $\begin{array}{r} 9 \\ 4\overline{)37} \\ 36 \\ \hline 1 \end{array}$

Name the quotients and remainders.

**19.** $4\overline{)23}$ **20.** $3\overline{)28}$ **21.** $5\overline{)37}$ **22.** $7\overline{)46}$ **23.** $8\overline{)42}$ **24.** $6\overline{)53}$

## Two step division

$6\overline{)102}$

We want to know how many sixes are in 102.

$$\begin{array}{r|l} 6\overline{)102} & \\ 54 & 9 \\ \hline 48 & \end{array}$$

Our basic multiplication facts for 6 only go up to $9 \times 6 = 54$. So we can subtract at least 9 sixes from 102. $102 - (9 \times 6) = 48$.

$$\begin{array}{r|l} 6\overline{)102} & \\ 54 & 9 \\ \hline 48 & \\ 48 & 8 \\ \hline 0 & 17 \end{array}$$

Now we can subtract 8 more sixes.
$48 - (8 \times 6) = 0$.

There are $9 + 8 = 17$ sixes in 102.
The quotient is 17 and the remainder is 0.

We can check our work like this:

$$\begin{array}{rcl} 9 \times 6 & = & 54 \\ \underline{8 \times 6} & = & \underline{48} \\ 17 \times 6 & = & 102 \end{array}$$

### EXERCISES

Copy and complete the divisions. Check your work by solving the equations.

**1.**

$$\begin{array}{r|l} 7\overline{)112} & \\ 63 & a \\ \hline 49 & \\ 49 & b \\ \hline 0 & c \end{array}$$

$$\begin{array}{rcl} a \times 7 & = & 63 \\ \underline{b \times 7} & = & \underline{49} \\ (a + b) \times 7 & = & 112 \end{array}$$

**2.**

$$\begin{array}{r|l} 9\overline{)135} & \\ 54 & d \\ \hline 81 & \\ 81 & e \\ \hline 0 & g \end{array}$$

$$\begin{array}{rcl} d \times 9 & = & 54 \\ \underline{e \times 9} & = & \underline{81} \\ g \times 9 & = & 135 \end{array}$$

**3.**

$$\begin{array}{r|l} 8\overline{)120} & \\ 56 & h \\ \hline 64 & \\ 64 & i \\ \hline 0 & j \end{array}$$

$$\begin{array}{rcl} h \times 8 & = & 56 \\ \underline{i \times 8} & = & \underline{64} \\ j \times 8 & = & 120 \end{array}$$

**4.**

$$\begin{array}{r|l} 6\overline{)96} & \\ 60 & k \\ \hline 36 & \\ 36 & n \\ \hline 0 & p \end{array}$$

$$\begin{array}{rcl} k \times 6 & = & 60 \\ \underline{n \times 6} & = & \underline{36} \\ p \times 6 & = & 96 \end{array}$$

Solve the problem.

**5.** There are 95 children going on a school trip. They will go in cars. If 5 children can go in each car, how many cars will be needed?

## Two step division

Arthur and Sylvia used different steps to name $144 \div 9$.

**Arthur wrote**

$$\begin{array}{r|l} 9\overline{)144} & \\ 63 & 7 \\ \hline 81 & \\ 81 & 9 \\ \hline 0 & 16 \end{array}$$

$7 \times 9 = 63$

$9 \times 9 = 81$

$16 \times 9 = 144$

**Sylvia wrote**

$$\begin{array}{r|l} 9\overline{)144} & \\ 90 & 10 \\ \hline 54 & \\ 54 & 6 \\ \hline 0 & 16 \end{array}$$

$10 \times 9 = 90$

$6 \times 9 = 54$

$16 \times 9 = 144$

Which method do you think is better, Arthur's or Sylvia's?

### EXERCISES

Copy and complete the examples.

**1.**
$$\begin{array}{r|l} 5\overline{)65} & \\ 50 & 10 \\ \hline 15 & \\ 15 & 3 \\ \hline 0 & a \end{array}$$

$10 \times 5 = b$

$3 \times 5 = c$

$13 \times 5 = e$

**2.**
$$\begin{array}{r|l} 6\overline{)108} & \\ 60 & 10 \\ \hline 48 & \\ 48 & 8 \\ \hline 0 & j \end{array}$$

$10 \times 6 = h$

$8 \times 6 = i$

$18 \times 6 = j$

**3.**
$$\begin{array}{r|l} 7\overline{)84} & \\ 70 & 10 \\ \hline 14 & \\ 14 & 2 \\ \hline 0 & m \end{array}$$

$10 \times 7 = n$

$2 \times 7 = r$

$12 \times 7 = s$

**4.**
$$\begin{array}{r|l} 8\overline{)144} & \\ 80 & 10 \\ \hline 64 & \\ 64 & 8 \\ \hline 0 & u \end{array}$$

$10 \times 8 = v$

$8 \times 8 = w$

$18 \times 8 = x$

Name the quotients.

**5.** $3\overline{)57}$

**6.** $6\overline{)78}$

**7.** $5\overline{)75}$

**8.** $4\overline{)76}$

**9.** $3\overline{)51}$

**10.** $6\overline{)90}$

**11.** $4\overline{)52}$

**12.** $7\overline{)98}$

**13.** $6\overline{)84}$

**14.** $5\overline{)90}$

**15.** $4\overline{)68}$

**16.** $7\overline{)91}$

**17.** $8\overline{)104}$

**18.** $7\overline{)126}$

**19.** $9\overline{)117}$

**20.** $8\overline{)136}$

**21.** $7\overline{)119}$

**22.** $8\overline{)152}$

**23.** $9\overline{)126}$

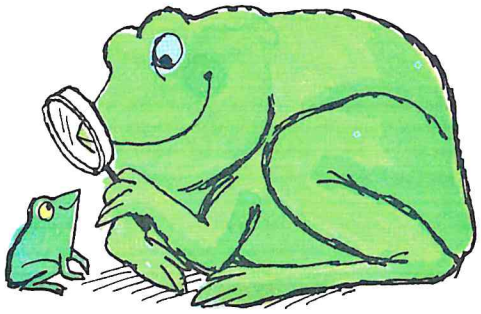
**24.** $6\overline{)114}$

**25.** $8\overline{)128}$

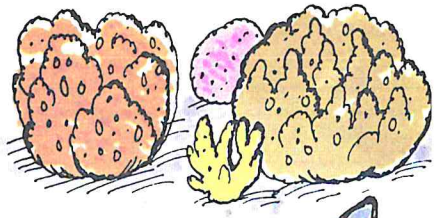
**26.** $7\overline{)133}$

**27.** $9\overline{)162}$

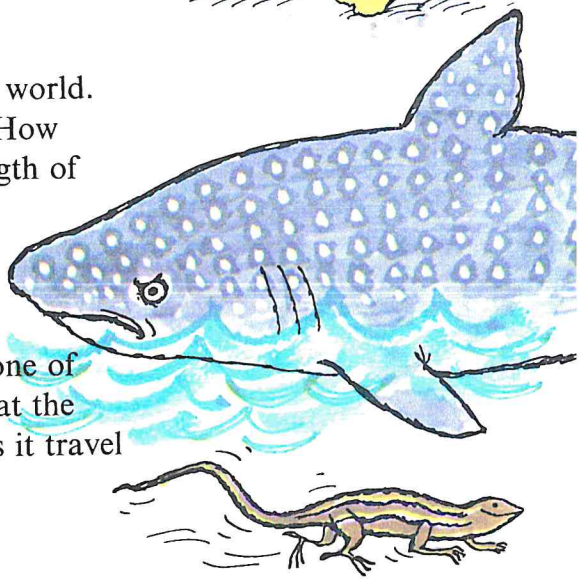
**28.** $9\overline{)171}$



1. The giant frog found in Japan grows to over 5 feet in length and weighs up to 90 pounds. How many 2 pound frogs match the weight of one 90 pound frog?

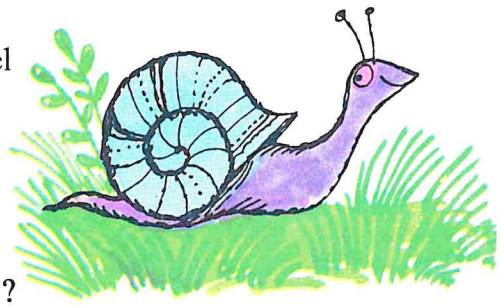


2. The tallest sponge is 4 feet in height. How many sponges that size match the height of a building that is 36 feet high?



3. The whale shark is the largest fish in the world. One whale shark was 45 feet in length. How many 5 foot sharks would match the length of that whale shark?

4. The race runner lizard from Georgia is one of the fastest reptiles on land. If it travels at the speed of 15 miles per hour, how far does it travel in 7 hours?



5. A snail's slowest pace is 23 inches every hour. At this pace, how many inches would he travel in 8 hours?

*6. A fast snail travels 220 yards in 4 hours.
   a. How many yards does he travel each hour?
   b. How many inches does he travel each hour?

## Sharpshooting

Answer the questions in **A** as quickly as you can. Then see if you can answer the questions in **B** even more quickly.

**A**

1. $46 + 9 + 8 + 7 + 4 = n$
2. $3 \times 487 = n$
3. Name the sum: $(9 \times 3) + (9 \times 10)$
4. Name the quotient and the remainder when 53 is divided by 8.
5. What is the cost of 8 balloons at 17¢ each?
6. A dozen eggs costs 72¢. What is the cost per egg?
7. What is the total value of the 7 in 7863?
8. One half of a number is 12. What is the number?
9. Is 103 closer to $(9 \times 11)$ or to $(10 \times 11)$?
10. Name the perimeter of a square with each side 14 inches long.
11. How many sides does a quadrilateral have?
12. $\frac{3}{4} = \frac{n}{12}$

**B**

1. $43 - 27 = n$
2. $153 \div 9 = n$
3. Name the cardinal number of {the first three days in January}.
4. $9 \times 8 = 72$, so $18 \times 8 = n$
5. $74 + 3 + 6 + 9 + 8 = n$
6. What is one fifth of sixty?
7. Name the product of 39 and 21.
8. Which is greater, $(9 \times 12)$ or $(10 \times 10)$?
9. What is the cost of a dozen candy bars at 9¢ each?
10. Which is greater, 397,462 or three hundred ninety five thousand, four hundred sixty-two.
11. Name the number of miles traveled in 8 hours at the rate of 47 miles per hour.
12. Rename $\frac{2}{3}$ as a fraction with a denominator of 15.

### Be Careful!

1. If I can see 4 cars using 1 eye, how many cars can I see using 2 eyes?
2. If a boy of 9 eats 3 meals in a day, how many meals does a man of 36 eat in a day?

## CHECKING UP

Name the products.

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** $38 \times 7$ | **3.** $36 \times 9$ | **5.** $293 \times 5$ | **7.** $681 \times 9$ | **9.** $35 \times 40$ | **11.** $39 \times 23$ | **13.** $57 \times 68$ |
| **2.** $46 \times 2$ | **4.** $48 \times 6$ | **6.** $473 \times 8$ | **8.** $39 \times 20$ | **10.** $57 \times 60$ | **12.** $35 \times 46$ | **14.** $67 \times 39$ |

Name the quotients.

**15.** $7\overline{)63}$ **16.** $6\overline{)48}$ **17.** $8\overline{)64}$ **18.** $5\overline{)45}$ **19.** $3\overline{)27}$ **20.** $6\overline{)54}$

Solve the equations.

| | | |
|---|---|---|
| **21.** $(9 \times 3) + 2 = a$ | **25.** $(3 \times 8) + 7 = f$ | **29.** $37 = (5 \times 7) + c$ |
| **22.** $(4 \times 6) + 3 = b$ | **26.** $(6 \times 9) + 3 = g$ | **30.** $30 = (7 \times 4) + e$ |
| **23.** $(7 \times 5) + 4 = c$ | **27.** $55 = (9 \times 6) + a$ | **31.** $40 = (6 \times 6) + f$ |
| **24.** $(6 \times 6) + 4 = e$ | **28.** $43 = (7 \times 6) + b$ | **32.** $75 = (8 \times 9) + g$ |

Copy and complete the divisions.

**33.**
$$\begin{array}{r} 4 \\ 7\overline{)30} \\ \underline{28} \\ a \end{array}$$

**34.**
$$\begin{array}{r} 7 \\ 5\overline{)36} \\ \underline{35} \\ b \end{array}$$

**35.**
$$\begin{array}{r} 6 \\ 6\overline{)40} \\ \underline{36} \\ c \end{array}$$

**36.**
$$\begin{array}{r} 6 \\ 7\overline{)45} \\ \underline{42} \\ d \end{array}$$

**37.**
$$\begin{array}{r} 6 \\ 8\overline{)50} \\ \underline{48} \\ e \end{array}$$

**38.**
$$\begin{array}{r} 9 \\ 5\overline{)49} \\ \underline{45} \\ f \end{array}$$

**39.**
$$\begin{array}{r} a \\ 6\overline{)38} \\ \underline{36} \\ 2 \end{array}$$

**40.**
$$\begin{array}{r} b \\ 8\overline{)75} \\ \underline{72} \\ 3 \end{array}$$

**41.**
$$\begin{array}{r} c \\ 9\overline{)59} \\ \underline{54} \\ 5 \end{array}$$

**42.**
$$\begin{array}{r} d \\ 8\overline{)37} \\ \underline{32} \\ 5 \end{array}$$

**43.**
$$\begin{array}{r} e \\ 5\overline{)34} \\ \underline{30} \\ 4 \end{array}$$

**44.**
$$\begin{array}{r} f \\ 4\overline{)31} \\ \underline{28} \\ 3 \end{array}$$

**45.**
$$\begin{array}{r|l} 7\overline{)84} & 10 \\ \underline{70} & \\ 14 & \\ \underline{14} & 2 \\ 0 & a \end{array}$$

**46.**
$$\begin{array}{r|l} 3\overline{)54} & b \\ \underline{30} & \\ 24 & \\ \underline{24} & c \\ 0 & d \end{array}$$

**47.**
$$\begin{array}{r|l} 4\overline{)68} & e \\ \underline{40} & \\ 28 & \\ \underline{28} & f \\ 0 & g \end{array}$$

**48.**
$$\begin{array}{r|l} 8\overline{)104} & \\ \underline{80} & h \\ 24 & \\ \underline{24} & i \\ 0 & j \end{array}$$

**49.**
$$\begin{array}{r|l} 7\overline{)119} & \\ \underline{70} & k \\ 49 & \\ \underline{49} & l \\ 0 & m \end{array}$$

**50.**
$$\begin{array}{r|l} 5\overline{)95} & \\ \underline{50} & n \\ 45 & \\ \underline{45} & r \\ 0 & p \end{array}$$

The distributive property is used in multiplications with factors greater than 10.

Name the products.

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** $23 \times 7$ | **3.** $97 \times 3$ | **5.** $457 \times 9$ | **7.** $487 \times 4$ | **9.** $43 \times 16$ | **11.** $57 \times 38$ | **13.** $69 \times 37$ |
| **2.** $46 \times 8$ | **4.** $48 \times 6$ | **6.** $237 \times 5$ | **8.** $57 \times 18$ | **10.** $69 \times 17$ | **12.** $43 \times 26$ | **14.** $26 \times 54$ |

Division is the opposite of multiplication.

Name the quotients.

**15.** $4\overline{)28}$ **16.** $6\overline{)30}$ **17.** $7\overline{)42}$ **18.** $9\overline{)72}$ **19.** $8\overline{)64}$ **20.** $5\overline{)35}$

Some divisions have remainders.

Name the quotients and the remainders.

| | | | | |
|---|---|---|---|---|
| **21.** $7\overline{)51}$ | **23.** $6\overline{)45}$ | **25.** $4\overline{)31}$ | **27.** $4\overline{)39}$ | **29.** $9\overline{)89}$ |
| **22.** $8\overline{)59}$ | **24.** $3\overline{)22}$ | **26.** $5\overline{)29}$ | **28.** $8\overline{)70}$ | **30.** $6\overline{)39}$ |

The distributive property is used in divisions with factors greater than 10.

Copy and complete the examples. Name the quotients.

**31.**
$5\overline{)70}$
$50 \mid 10$
$20$
$20 \mid 4$
$0 \mid a$

**32.**
$3\overline{)45}$
$30 \mid b$
$15$
$15 \mid c$
$0 \mid d$

**33.**
$4\overline{)56}$
$40 \mid e$
$16$
$16 \mid f$
$0 \mid g$

**34.**
$6\overline{)108}$
$60 \mid h$
$48$
$48 \mid i$
$0 \mid j$

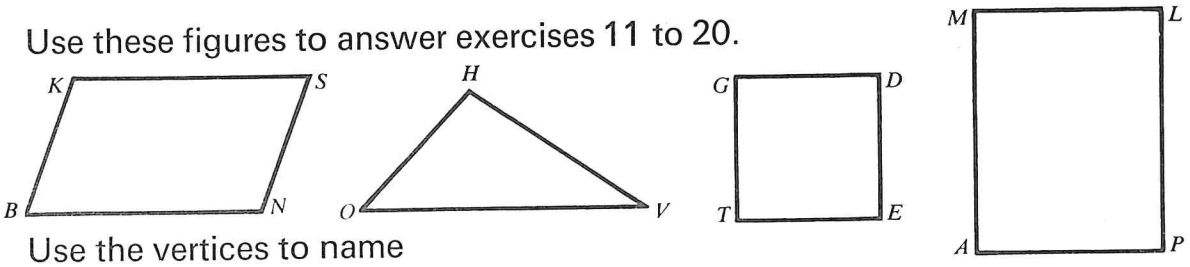
**35.**
$7\overline{)133}$
$70 \mid k$
$63$
$63 \mid l$
$0 \mid m$

## LOOKING WAY BACK

Name the sums and differences.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | 56 + 32 | **3.** | 124 + 51 | **5.** | 569 + 453 | **7.** | 45382 + 36459 | **9.** | 85067 + 38278 |
| **2.** | 72 − 47 | **4.** | 534 − 219 | **6.** | 321 − 123 | **8.** | 59146 − 32958 | **10.** | 94361 − 28495 |

Use these figures to answer exercises 11 to 20.




Use the vertices to name

**11.** the triangle **12.** the square **13.** the parallelogram **14.** the rectangle

Measure each line segment in inches.

**15.** m($\overline{OV}$) **16.** m($\overline{ML}$) **17.** m($\overline{KS}$) **18.** m($\overline{ED}$)

Name

**19.** a line segment congruent to $\overline{ML}$ **20.** a line segment congruent to $\overline{SN}$

Name the products.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **21.** | 48 × 7 | **22.** | 54 × 2 | **23.** | 76 × 9 | **24.** | 396 × 5 | **25.** | 578 × 7 | **26.** | 72 × 36 | **27.** | 54 × 98 |

Name the quotients and remainders.

**28.** $8\overline{)56}$ **29.** $3\overline{)26}$ **30.** $9\overline{)65}$ **31.** $6\overline{)84}$ **32.** $7\overline{)72}$ **33.** $9\overline{)121}$

Complete the sentences.

**34.** $26 + 78 = a$

**35.** $53 + b = 128$

**36.** $(5 \times 4) + 2 = c$

**37.** $3 \times 124 = d$

**38.** $156 - e = 78$

**39.** $5 \times f = 85$

**40.** $(9 \times 7) + 4 = g$

**41.** $56 \div h = 8$

**42.** $25 + 57 \bigcirc 27 + 55$

**43.** $1234 \bigcirc 9876 - 8642$

**44.** $3 \times 157 \bigcirc 4 \times 117$

**45.** $1 + 2 + 3 \bigcirc 4 \times 15 \times 0$

**46.** $\frac{1}{2} = \frac{a}{4}$

**47.** $\frac{2}{3} = \frac{b}{12}$

**48.** $\frac{5}{6} = \frac{20}{c}$

**49.** $\frac{8}{18} = \frac{d}{9}$

To name the product $18 \times 89$ we may use the fact that $18 = 3 \times 6$.

$$\begin{aligned} 18 \times 89 &= (3 \times 6) \times 89 \\ &= 3 \times (6 \times 89) \\ &= 3 \times 534 \\ &= 1602 \end{aligned}$$

Use this method to name the products.

**1.** $12 \times 69 = a$ **3.** $14 \times 58 = c$ **5.** $24 \times 87 = e$ **7.** $25 \times 43 = g$

**2.** $16 \times 57 = b$ **4.** $24 \times 63 = d$ **6.** $21 \times 69 = f$ **8.** $25 \times 98 = h$

To name the product $18 \times 89$ we may use the fact that $89 = 90 - 1$.

$$\begin{aligned} 18 \times 89 &= 18 \times (90 - 1) \\ &= (18 \times 90) - (18 \times 1) \\ &= 1620 - 18 \\ &= 1602 \end{aligned}$$

Use this method to name the products.

**9.** $16 \times 59 = a$ **11.** $12 \times 49 = c$ **13.** $12 \times 79 = e$ **15.** $15 \times 49 = g$

**10.** $16 \times 69 = b$ **12.** $14 \times 49 = d$ **14.** $14 \times 39 = f$ **16.** $15 \times 79 = h$

*Just For Fun*

**17.** Name the numbers from 1 to 40 by adding and subtracting only the numbers 1, 3, 9, and 27. For each number, do not use 1, 3, 9, or 27 more than once.
For example:

$1 = 1$
$2 = 3 - 1$
$3 = 3$
$7 = 9 - 3 + 1$
$14 = 27 - 9 - 3 - 1$
$25 = 27 - 3 + 1$

**18.** Name the numbers from 1 to 15 by adding 1, 2, 4, and 8. You may use all of these addends, but you must not use any addend more than once. For example:

$7 = 1 + 2 + 4$ $12 = 8 + 4$

## Intersection of sets

$A$ = {children in Sarah's class with red hair}
= {Nelly, Jim, Sarah, Eddie}
$B$ = {children in Sarah's class who come to school by bus}
= {Tom, Joan, Eddie, Sarah, Steve, Herman, Bob}

Eddie and Sarah are *common* to the two sets.

> The **intersection** of sets $A$ and $B$ is {Eddie, Sarah}.
> We may write this: $A \cap B$ = {Eddie, Sarah}.

### EXERCISES

List the intersection of each pair of sets.

1. {Jim, Jerry, Sally, Hal, Sam} and {Ida, Jerry, Kelly, Sally}
2. {lamp, fork, cookie, stamp, knife} and {lamp, stamp, fork, spoon}
3. {0, 2, 4, 6, 8} and {0, 1, 2, 3, 4, 5}
4. {2, 4, 8, 12, 16, 20, 24, 28, 32} and {8, 16, 24, 32}

Study the sets. Then list the members of the intersections.

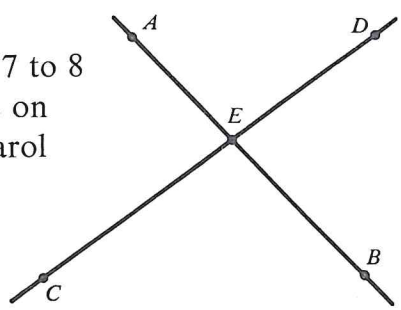
$A$ = {apple, apricot, peach, plum}
$B$ = {pear, apple, raisin, peach, plum, yam}
$C$ = {car, bus, train, subway, ship}
$D$ = {rickshaw, bicycle, car, wagon, bus, streetcar}
$E$ = {2, 3, 5, 7, 11, 13, 17, 19}
$F$ = {2, 4, 6, 8, 10}
$G$ = {0, 2, 4, 6, 8, 10}
$H$ = {1, 3, 5, 7, 9}

5. $A \cap B$ 6. $C \cap D$ 7. $E \cap F$ 8. $G \cap H$

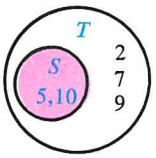
Answer the question and complete the sentence.

9. Mary said, "My favorite TV program is on from 7 to 8 on Friday." Carol said, "My favorite program is on from 7:30 to 8:30 on Friday." Can Mary and Carol see both these programs on the same set?

10. In the diagram, the point $E$ is common to $\overline{AB}$ and $\overline{CD}$. It is called the ___?___ of $\overline{AB}$ and $\overline{CD}$.

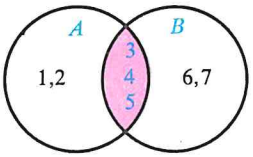


## Set diagrams




The diagram shows that $S$ is a **subset** of $T$.

$S = \{5, 10\}$
$T = \{2, 5, 7, 9, 10\}$




The **intersection** of $A$ and $B$ is a **subset** of both $A$ and $B$.

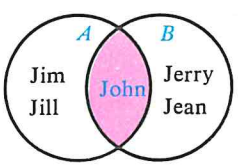
$A = \{1, 2, 3, 4, 5\}$
$B = \{3, 4, 5, 6, 7\}$
$A \cap B = \{3, 4, 5\}$
$\{3, 4, 5\}$ is a subset of both $A$ and $B$

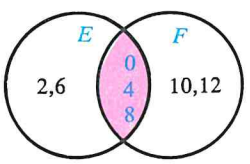
## EXERCISES

Draw diagrams to show

1. {Fred, Frank, Fran} is a subset of {Fred, Tom, Fran, Don, Frank}
2. {car} is a subset of {car, train, airplane}
3. {0, 3, 6} is a subset of {0, 1, 2, 3, 4, 5, 6}
4. {0, 2, 4, 6, 8} is a subset of {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Study the diagrams. Then list the members of the sets.



5. $A$ 6. $B$ 7. $A \cap B$
8. $E$ 9. $F$ 10. $E \cap F$
11. $S$ 12. $T$ 13. $S \cap T$

Draw a diagram to show each pair of sets.

14. $M$ = {Ed, Esther, Lois, Lenny} and $N$ = {Louis, Ned, Ed, Carol}
15. $C$ = {triangle, square, circle} and $L$ = {rectangle, trapezoid, circle}
16. $P = \{1, 4, 7\}$ and $Q = \{2, 4, 6, 8, 10, 12\}$
17. $H = \{10, 30, 70, 50, 90\}$ and $J = \{5, 10, 15, 30, 25, 20\}$
18. The boys in the Rock Club = {Stan, Dick, Joe, Sam, Al} and the boys in the Bug Club = {Sam, Don, Len, Dick, Bob, Tom}
From your diagram list the boys who are in both clubs.

## Multiples

| factor | × | factor | = | product |
|---|---|---|---|---|
| 3 | × | 0 | = | 0 |
| 3 | × | 1 | = | 3 |
| 3 | × | 2 | = | 6 |
| 3 | × | 3 | = | 9 |

and so on.

0, 3, 6, and 9 are **multiples** of 3 because they all have 3 as a factor. The set of multiples of 3 is $\{0, 3, 6, 9, 12, \ldots\}$. It is an infinite set.

### EXERCISES

Name the multiples in each set.

1. $\{0, 2, 4, 6, 8, 10, a, b, c, 18\}$
2. $\{0, 3, 6, 9, 12, 15, d, e, f, 27\}$
3. $\{0, 4, 8, 12, 16, g, h, i, j, 36\}$
4. $\{0, 5, 10, 15, 20, k, l, m, n, 45\}$
5. $\{0, 6, 12, 18, 24, 30, r, p, q, 54\}$
6. $\{0, 7, 14, 21, 28, 35, r, s, t, 63\}$
7. $\{0, 8, 16, 24, 32, u, v, w, x, 72\}$
8. $\{0, 9, 18, 27, 36, x, y, z, a, 81\}$

Use addition to name the multiples in each set.

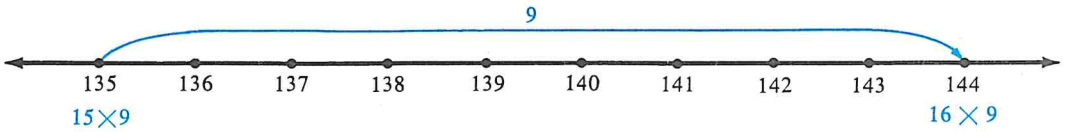
9. To name the multiples of 5 in $\{45, 50, 55, a, b, c, d, e, f, 90\}$, start with $45 = 9 \times 5$ and add 5 until you reach 90. Each time you add, name the sum. Each sum is a multiple of 5.
10. Start with $9 \times 6$, add 6 until you reach 108. $\{54, 60, a, b, c, d, e, f, g, 108\}$
11. Start with $9 \times 7$, add 7 until you reach 126. $\{63, 70, a, b, c, d, e, f, g, 126\}$
12. Start with $9 \times 8$. Add 8 until you reach 144. $\{72, 80, a, b, c, d, e, f, g, 144\}$
13. Start with $9 \times 9$. Add 9 until you reach 162. $\{81, 90, a, b, c, d, e, f, g, 162\}$

Copy and complete the number pattern.

In exercises 9 to 13, you named multiples of 5, 6, 7, 8, and 9. These multiples are set up as a number pattern.

14.

| × | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 50 | 55 | 60 | *a* | 70 | *b* | *c* | *d* | 90 |
| 6 | 60 | 66 | *e* | *f* | *g* | 90 | *h* | *i* | 108 |
| 7 | 70 | 77 | *j* | *k* | *l* | *m* | *n* | 119 | 126 |
| 8 | 80 | 88 | *p* | *q* | *r* | 120 | *s* | *t* | 144 |
| 9 | 90 | 99 | *u* | *v* | *w* | *x* | *y* | *z* | 162 |



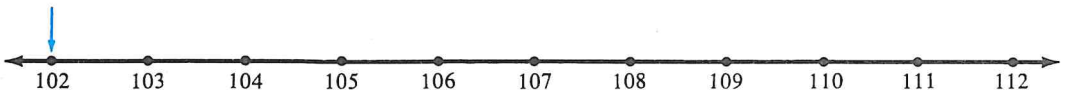

$135 = 15 \times 9$ 135 is a **multiple** of 9.

$144 = 135 + 9$ 144 is also a **multiple** of 9.

## EXERCISES

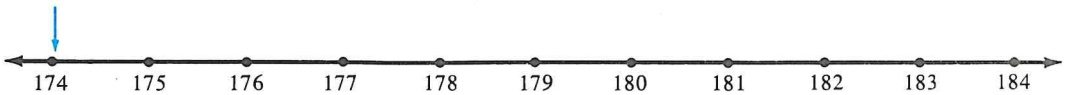
A multiple of 6 is marked on each number line. Name the next multiple of 6.

**1.**



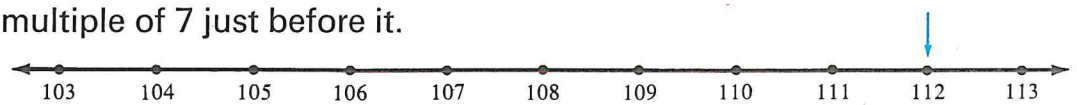

**2.**

**3.**



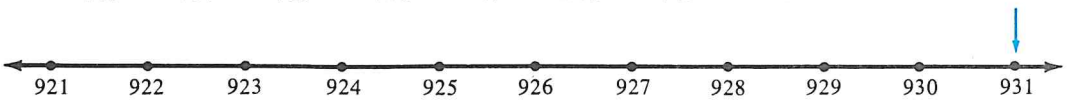

A multiple of 7 is marked on each number line. Name the multiple of 7 just before it.

**4.**




**5.**

**6.**




Use the first equation to solve the other equations in the row.

| | | | | |
|---|---|---|---|---|
| **7.** | $15 \times 8 = 120$ | $16 \times 8 = a$ | $17 \times 8 = b$ | $18 \times 8 = c$ |
| **8.** | $20 \times 8 = 160$ | $21 \times 8 = d$ | $22 \times 8 = e$ | $23 \times 8 = f$ |
| **9.** | $20 \times 7 = 140$ | $21 \times 7 = g$ | $22 \times 7 = h$ | $23 \times 7 = i$ |
| **10.** | $30 \times 9 = 270$ | $31 \times 9 = j$ | $32 \times 9 = k$ | $33 \times 9 = l$ |
| **11.** | $16 \times 8 = 128$ | $15 \times 8 = m$ | $14 \times 8 = n$ | $13 \times 8 = x$ |
| **12.** | $25 \times 9 = 225$ | $24 \times 9 = p$ | $23 \times 9 = q$ | $22 \times 9 = r$ |
| **13.** | $30 \times 7 = 210$ | $29 \times 7 = s$ | $28 \times 7 = t$ | $27 \times 7 = u$ |

## Common multiples

$A$ = {multiples of 3, less than 30} = {0, 3, 6, 9, 12, 15, 18, 21, 24, 27}
$B$ = {multiples of 4, less than 30} = {0, 4, 8, 12, 16, 20, 24, 28}

These sets have three **common** members. $A \cap B$ = {0, 12, 24}
The **common multiples** of 3 and 4, less than 30 are 0, 12, and 24.

### EXERCISES

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |

Use the chart above to help you.

1. Name the numbers less than 21 that are common multiples of
   **a.** 2 and 3 **b.** 2 and 5 **c.** 2 and 6
2. Name the numbers less than 31 that are common multiples of
   **a.** 3 and 4 **b.** 3 and 5 **c.** 3 and 6
3. Name the numbers less than 41 that are common multiples of
   **a.** 4 and 5 **b.** 4 and 6 **c.** 4 and 8

| × | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 36 | 42 | 48 | 54 | 60 | 66 | 72 | 78 | 84 | 90 | 96 | 102 | 108 | 114 | 120 |
| 7 | 42 | 49 | 56 | 63 | 70 | 77 | 84 | 91 | 98 | 105 | 112 | 119 | 126 | 133 | 140 |
| 8 | 48 | 56 | 64 | 72 | 80 | 88 | 96 | 104 | 112 | 120 | 128 | 136 | 144 | 152 | 160 |
| 9 | 54 | 63 | 72 | 81 | 90 | 99 | 108 | 117 | 126 | 135 | 144 | 153 | 162 | 171 | 180 |

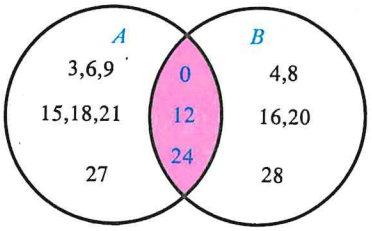
From the chart above, name the numbers between 50 and 120 that are common multiples of

4. 6 and 7
5. 6 and 8
6. 6 and 9
7. 7 and 8
8. 7 and 9
9. 8 and 9

## Diagrams to show common multiples

$A = \{0, 3, 6, 9, 12, 15, 18, 21, 24, 27\}$
$B = \{0, 4, 8, 12, 16, 20, 24, 28\}$

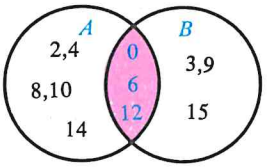
The shaded region shows the common multiples of 3 and 4, less than 30.



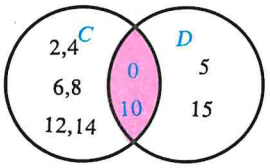

## EXERCISES

Study the diagrams and list the members of each set.

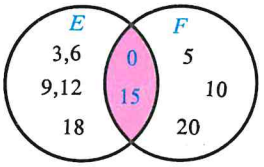
1. $A$
2. $B$
3. $A \cap B$



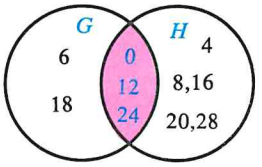

4. $C$
5. $D$
6. $C \cap D$




7. $E$
8. $F$
9. $E \cap F$




10. $G$
11. $H$
12. $G \cap H$




Use the diagrams above to list the sets.

13. the common multiples of 2 and 3, less than 16
14. the common multiples of 2 and 5, less than 16
15. the common multiples of 3 and 5, less than 21
16. the common multiples of 4 and 6, less than 30

Draw diagrams to show the intersections.

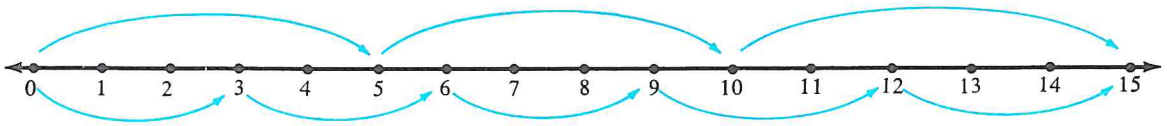
17. $M = \{0, 4, 8, 12, 16, 20, 24, 28\}$
    $N = \{0, 5, 10, 15, 20, 25, 30\}$
    $M \cap N$

18. $R = \{0, 6, 12, 18, 24, 30\}$
    $S = \{0, 8, 16, 24, 32\}$
    $R \cap S$

Answer the questions.

19. One common multiple of 4 and 6 is 24. What is a common multiple of 4 and 6 that is less than 24?
20. What is the **least** common multiple of 4 and 6 (other than 0)?
*21. 60 is a common multiple of 3, 5, and 6. Is 60 the **least** common multiple of 3, 5, and 6 (other than 0)?

## Factors and multiples

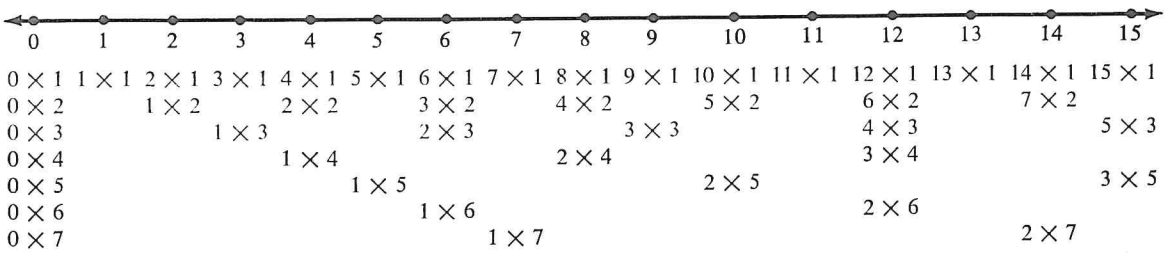



15 is a **multiple** of 5 and of 3.
5 and 3 are **factors** of 15.
1 and 15 are also factors of 15.

| factors | multiple | factors |
|---|---|---|
| $5 \times 3 = 15$ | $15 \div 3 = 5$ | |
| $3 \times 5 = 15$ | $15 \div 5 = 3$ | |
| $1 \times 15 = 15$ | $15 \div 15 = 1$ | |
| $15 \times 1 = 15$ | $15 \div 1 = 15$ | |

### EXERCISES

Study the number line.




Name the multiples less than 16 of

**1.** 2 **2.** 3 **3.** 4 **4.** 5 **5.** 6 **6.** 7

Use the number line to name the factors of

**7.** 4 **8.** 6 **9.** 8 **10.** 9 **11.** 12 **12.** 13

Solve the equations.

**13.** $12 = (2 \times 2) \times 3 = a \times 3$ $\quad 12 = 2 \times (2 \times 3) = 2 \times b$

**14.** $18 = (2 \times 3) \times 3 = c \times 3$ $\quad 18 = 2 \times (3 \times 3) = 2 \times d$

**15.** $20 = (2 \times 2) \times 5 = e \times 5$ $\quad 20 = 2 \times (2 \times 5) = 2 \times f$

**16.** $28 = (2 \times 2) \times 7 = g \times 7$ $\quad 28 = 2 \times (2 \times 7) = 2 \times h$

**17.** $50 = (2 \times 5) \times 5 = i \times 5$ $\quad 50 = 2 \times (5 \times 5) = 2 \times j$

**18.** $45 = (3 \times 3) \times 5 = k \times 5$ $\quad 45 = 3 \times (3 \times 5) = 3 \times m$

Use exercises 13 to 18 to name the factors of

**19.** 12 **20.** 18 **21.** 20 **22.** 28 **23.** 50 **24.** 45

$1 \times 15 = 15$
$3 \times 5 = 15$
$5 \times 3 = 15$
$15 \times 1 = 15$
factor $\times$ factor = product

15 is a **multiple** of 1, 3, 5, and 15.

$15 \div 15 = 1$
$15 \div 5 = 3$
$15 \div 3 = 5$
$15 \div 1 = 15$
product $\div$ factor = factor

The **factors** of 15 are 1, 3, 5, and 15.

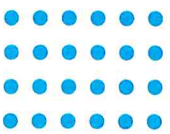
## EXERCISES

Solve the equations. Then complete the sentences.

1. $1 \times 6 = a$, $2 \times 3 = b$. The factors of 6 are $c, d, e, f$.
2. $1 \times 12 = g$, $2 \times 6 = h$, $3 \times 4 = i$. The factors of 12 are $j, k, l, m, n, a$.
3. $1 \times 18 = p$, $2 \times 9 = q$, $3 \times 6 = r$. The factors of 18 are $s, t, u, v, w, x$.
4. $1 \times 20 = a$, $2 \times 10 = b$, $4 \times 5 = c$. The factors of 20 are ___?___.
5. $1 \times 28 = e$, $2 \times 14 = f$, $4 \times 7 = g$. The factors of 28 are ___?___.
6. $1 \times 16 = i$, $2 \times 8 = j$, $4 \times 4 = k$. The factors of 16 are ___?___.
7. $1 \times 32 = m$, $2 \times 16 = n$, $4 \times 8 = a$. The factors of 32 are ___?___.
8. $1 \times a = 24$, $2 \times b = 24$, $3 \times c = 24$, $4 \times d = 24$.
   The factors of 24 are ___?___.
9. $1 \times f = 30$, $2 \times g = 30$, $3 \times h = 30$, $5 \times i = 30$.
   The factors of 30 are ___?___.
10. $1 \times k = 42$, $2 \times l = 42$, $3 \times m = 42$, $6 \times n = 42$.
    The factors of 42 are ___?___.

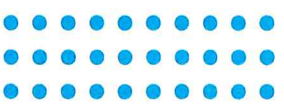
Show all the ways each set of dots can be arranged with an equal number of dots in every row.

11.



12.

13.



## Common factors

$A$ = {factors of 12} = {1, 2, 3, 4, 6, 12}
$B$ = {factors of 18} = {1, 2, 3, 6, 9, 18}

$A \cap B$ = {**common factors** of 12 and 18} = {1, 2, 3, 6}

## EXERCISES

Use the sets to help you complete the sentences.

**1.** {factors of 24} = {1, 2, 3, 4, 6, 8, 12, 24}
{factors of 30} = {1, 2, 3, 5, 6, 10, 15, 30}
The common factors of 24 and 30 are ___?___.

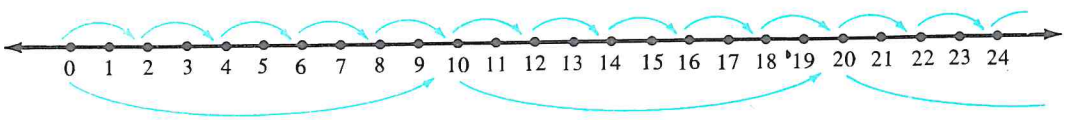
**2.** {factors of 24} = {1, 2, 3, 4, 6, 8, 12, 24}
{factors of 36} = {1, 2, 3, 4, 6, 9, 12, 18, 36}
The common factors of 24 and 36 are ___?___.

**3.** {factors of 32} = {1, 4, 8, 16, 32}
{factors of 48} = {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}
The common factors of 32 and 48 are ___?___.

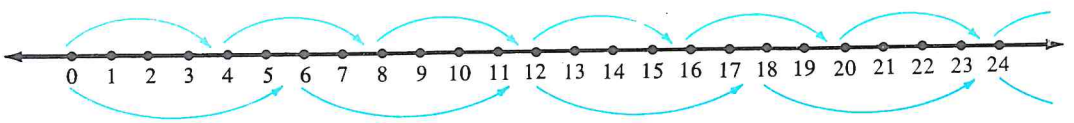
**4.** {factors of 40} = {1, 2, 4, 5, 8, 10, 20, 40}
{factors of 50} = {1, 2, 5, 10, 25, 50}
The common factors of 40 and 50 are ___?___.

Use the number lines to name the numbers less than 25

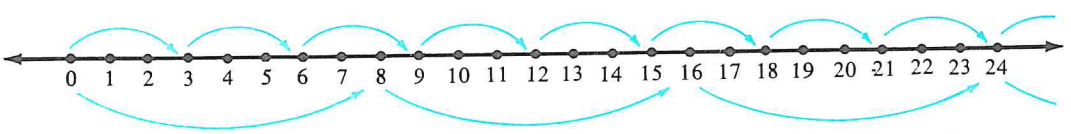
**5.** that have 2 and 10 as common factors.




**6.** that have 4 and 6 as common factors.




**7.** that have 3 and 8 as common factors.




Use the number lines in exercises 5 to 7 to name the common factors of

**8.** 8 and 24

**9.** 12 and 24

**10.** 16 and 24

**11.** 18 and 24

**12.** 20 and 24

**13.** 18 and 20

When addends have a common factor we may use the **distributive property** to help us name the sum.

24 and 32 have 8 as a common factor. We multiply the common factor and the sum of the other factors.

$$\begin{aligned} 24 + 32 &= (3 \times 8) + (4 \times 8) \\ &= (3 + 4) \times 8 \\ &= 7 \times 8 \\ &= 56 \end{aligned}$$

## EXERCISES

Solve the equations.

1. $\begin{aligned} 15 + 20 &= (3 \times 5) + (4 \times 5) \\ &= (3 + 4) \times a \\ &= 7 \times b = c \end{aligned}$

2. $\begin{aligned} 12 + 16 &= (3 \times d) + (4 \times 4) \\ &= (3 + 4) \times e \\ &= 7 \times f = g \end{aligned}$

3. $\begin{aligned} 21 + 24 &= (7 \times 3) + (8 \times h) \\ &= (7 + 8) \times i \\ &= 15 \times j = k \end{aligned}$

4. $\begin{aligned} 28 + 42 &= (4 \times 7) + (6 \times l) \\ &= (4 + 6) \times m \\ &= 10 \times n = z \end{aligned}$

5. $\begin{aligned} 48 + 12 &= (12 \times p) + (3 \times q) \\ &= (12 + 3) \times r \\ &= s \times 4 = t \end{aligned}$

6. $\begin{aligned} 56 + 32 &= (u \times 8) + (v \times 8) \\ &= (w + x) \times 8 \\ &= y \times 8 = z \end{aligned}$

Solve the equations.

7. $\begin{aligned} 27 \times 3 &= (20 + a) \times 3 \\ &= (20 \times 3) + (b \times 3) \\ &= \quad c \quad + \quad d \\ &= \quad e \end{aligned}$

8. $\begin{aligned} 480 \times 5 &= (400 + f) \times 5 \\ &= (400 \times 5) + (g \times 5) \\ &= \quad h \quad + \quad i \\ &= \quad j \end{aligned}$

9. $\begin{aligned} 690 \times 6 &= (k + 90) \times 6 \\ &= (l \times 6) + (m \times 6) \\ &= \quad n \quad + \quad a \\ &= \quad p \end{aligned}$

10. $\begin{aligned} 530 \times 7 &= (q + 30) \times 7 \\ &= (r \times 7) + (s \times 7) \\ &= \quad t \quad + \quad u \\ &= \quad y \end{aligned}$

11. $\begin{aligned} 50 + 30 &= (5 \times 10) + (3 \times 10) \\ &= (5 + 3) \times a \\ &= \quad b \end{aligned}$

12. $\begin{aligned} 60 + 70 &= (6 \times 10) + (7 \times 10) \\ &= (6 + 7) \times c \\ &= \quad d \end{aligned}$

13. $\begin{aligned} 700 + 200 &= (7 \times 100) + (2 \times 100) \\ &= (7 + 2) \times e \\ &= \quad f \end{aligned}$

14. $\begin{aligned} 800 + 900 &= (8 \times 100) + (9 \times 100) \\ &= (8 + 9) \times g \\ &= \quad h \end{aligned}$

## Numbers with only two factors

$2 = 2 \times 1$ $3 = 3 \times 1$ $5 = 5 \times 1$ $7 = 7 \times 1$ $11 = 11 \times 1$

The equations show the *only* factors for these numbers.

> Numbers that have *only* two different factors are called **prime numbers.** 2, 3, 5, 7 and 11 are prime numbers.

### EXERCISES

Name the factors of

**1.** 12 **3.** 14 **5.** 16 **7.** 18 **9.** 20 **11.** 22

**2.** 13 **4.** 15 **6.** 17 **8.** 19 **10.** 21 **12.** 23

**13.** Use your answers to exercises 1 to 12 to list all prime numbers between 11 and 24.

A famous Greek mathematician named Eratosthenes invented a method for finding primes.

*Step 1:* List all of the numbers as far as you want to go. (We listed the numbers from 1 to 40.)

*Step 2:* Cross off 1 since it is not a prime number.

*Step 3:* Circle 2 since it is a prime number. Then cross off all the other multiples of 2 since they are not prime numbers. (We show this in red.)

*Step 4:* Circle 3. Then cross off all the other multiples of 3 not already crossed off. (We show this in blue.)

*Step 5:* Circle 5. Then cross off all the other multiples of 5 not already crossed off. (We show this in green.)

For the numbers to 40, the result looks like this:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ~~1~~ | (2) | (3) | ~~4~~ | (5) | ~~6~~ | 7 | ~~8~~ | ~~9~~ | ~~10~~ |
| 11 | ~~12~~ | 13 | ~~14~~ | ~~15~~ | ~~16~~ | 17 | ~~18~~ | 19 | ~~20~~ |
| ~~21~~ | ~~22~~ | 23 | ~~24~~ | ~~25~~ | ~~26~~ | ~~27~~ | ~~28~~ | 29 | ~~30~~ |
| 31 | ~~32~~ | ~~33~~ | ~~34~~ | ~~35~~ | ~~36~~ | 37 | ~~38~~ | ~~39~~ | ~~40~~ |

**14.** Name the primes less than 40.

**15.** Why are multiples of 2 (other than 2) not primes?

**16.** Use Eratosthenes' method to name the primes less than 60.

Study this number chart. The prime numbers have been ringed.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | (2) | (3) | 4 | (5) | 6 | (7) | 8 | 9 | 10 |
| (11) | 12 | (13) | 14 | 15 | 16 | (17) | 18 | (19) | 20 |
| 21 | 22 | (23) | 24 | 25 | 26 | 27 | 28 | (29) | 30 |
| (31) | 32 | 33 | 34 | 35 | 36 | (37) | 38 | 39 | 40 |
| (41) | 42 | (43) | 44 | 45 | 46 | (47) | 48 | 49 | 50 |
| 51 | 52 | (53) | 54 | 55 | 56 | 57 | 58 | (59) | 60 |
| (61) | 62 | 63 | 64 | 65 | 66 | (67) | 68 | 69 | 70 |

## EXERCISES

Use the chart to answer the questions.

**1.** List the primes less than 70 that have a compact numeral ending in

**a.** 1 **b.** 3 **c.** 7 **d.** 9

**2.** Can the compact numeral for any prime (greater than 2) end in 2, 4, 6, 8, or 0? Why not?

**3.** Can the compact numeral for any prime (greater than 5) end in 5? Why not?

***4.** 1 is not a prime number. Why not?

Name the sums and complete the sentence.

**5.** $3 + 5 = a$ **8.** $5 + 7 = d$ **11.** $7 + 13 = g$ **14.** $3 + 19 = j$

**6.** $3 + 7 = b$ **9.** $5 + 11 = e$ **12.** $5 + 17 = h$ **15.** $3 + 29 = k$

**7.** $3 + 11 = c$ **10.** $7 + 11 = f$ **13.** $5 + 19 = i$ **16.** $7 + 19 = l$

**17.** The sum of two primes greater than 2 is an ___?___ number.

Name each even number as the sum of two primes.

**18.** 8 **20.** 12 **22.** 16 **24.** 20 **26.** 24 **28.** 28 **30.** 32

**19.** 10 **21.** 14 **23.** 18 **25.** 22 **27.** 26 **29.** 30 **31.** 34

*Name each odd number as the sum of three primes.

**32.** 9 **34.** 13 **36.** 17 **38.** 21 **40.** 25 **42.** 29 **44.** 33

**33.** 11 **35.** 15 **37.** 19 **39.** 23 **41.** 27 **43.** 31 **45.** 35

## Primes as factors

Study the equations.

$$\begin{aligned} 24 &= 2 \times 12 \\ &= 2 \times (2 \times 6) \\ &= 2 \times 2 \times (2 \times 3) \end{aligned}$$

We have renamed 24 as the **product of prime factors.**

### EXERCISES

Solve the equations and complete the sentence.

**1.** $14 = 2 \times a$

**2.** $15 = 3 \times b$

**3.** $21 = 3 \times c$

**4.** $35 = 7 \times d$

**5.** $22 = 2 \times e$

**6.** $33 = 3 \times f$

**7.** $55 = 5 \times g$

**8.** $12 = 3 \times 4$
$= 3 \times (2 \times a)$

**9.** $18 = 3 \times 6$
$= 3 \times (2 \times b)$

**10.** $30 = 3 \times 10$
$= 3 \times (2 \times c)$

**11.** $42 = 3 \times 14$
$= 3 \times (2 \times d)$

**12.** $20 = 2 \times 10$
$= 2 \times (2 \times e)$

**13.** $28 = 2 \times 14$
$= 2 \times (2 \times f)$

**14.** $44 = 2 \times 22$
$= 2 \times (2 \times g)$

**15.** $45 = 3 \times 15$
$= 3 \times (3 \times h)$

**16.** You renamed each number as the product of ___?___ factors.

Name the products and complete the sentence.

**17.** $2 \times 3 = a$

**18.** $2 \times 5 = b$

**19.** $2 \times 7 = c$

**20.** $2 \times 11 = d$

**21.** $3 \times 3 = f$

**22.** $3 \times 5 = g$

**23.** $3 \times 7 = h$

**24.** $3 \times 11 = i$

**25.** $5 \times 3 = k$

**26.** $5 \times 5 = l$

**27.** $5 \times 7 = m$

**28.** $5 \times 11 = n$

**29.** $7 \times 3 = p$

**30.** $7 \times 5 = q$

**31.** $7 \times 7 = r$

**32.** $7 \times 11 = s$

**33.** The products you named in exercises 17 to 32 all have two ___?___ factors.

Name the products and complete the sentence.

**34.** $2 \times 2 \times 2 = a$

**35.** $2 \times 2 \times 3 = b$

**36.** $2 \times 2 \times 5 = c$

**37.** $2 \times 3 \times 3 = d$

**38.** $2 \times 3 \times 5 = e$

**39.** $2 \times 3 \times 7 = f$

**40.** $3 \times 3 \times 3 = g$

**41.** $3 \times 3 \times 5 = h$

**42.** $3 \times 3 \times 7 = i$

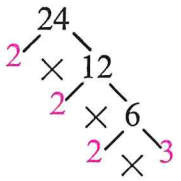
**43.** $2 \times 5 \times 5 = j$

**44.** $2 \times 5 \times 7 = k$

**45.** $2 \times 5 \times 11 = l$

**46.** The products you named in exercises 34 to 45 all have ___?___ prime factors.

**Factor tree**




**Equations**

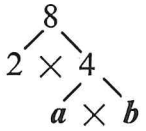
$$\begin{aligned} 24 &= 2 \times 12 \\ &= 2 \times (2 \times 6) \\ &= 2 \times 2 \times (2 \times 3) \end{aligned}$$

We may use a factor tree or equations to name the prime factors of 24.

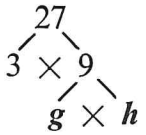
## EXERCISES

Copy and complete each factor tree.

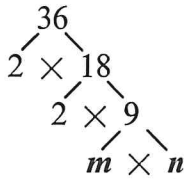
**1.**



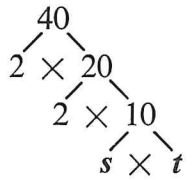

**4.**



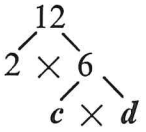

**7.**



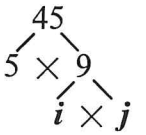

**10.**



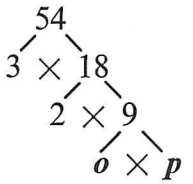

**2.**



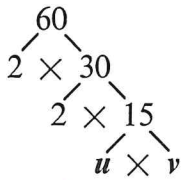

**5.**



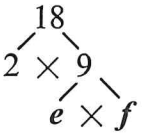

**8.**



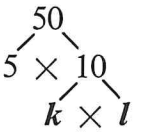

**11.**



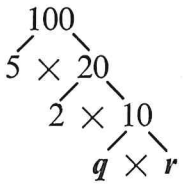

**3.**



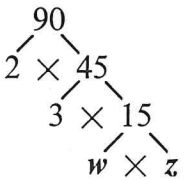

**6.**




**9.**




**12.**




Use the factor trees in exercises 1 to 6 to name each number as the product of three prime factors.
For example, $30 = 2 \times 3 \times 5$.

**13.** 8 **14.** 18 **15.** 27 **16.** 12 **17.** 45 **18.** 50

Use the factor trees in exercises 7 to 12 to name each number as the product of four prime factors.

**19.** 36 **20.** 54 **21.** 100 **22.** 40 **23.** 60 **24.** 90

## Two stage problems

What is the total cost of 7 quarts of milk at 29 cents a quart and 8 quarts of orange juice at 39 cents a quart?

Here is how we use the five-step method to solve this problem:

*Step 1:* There are 7 sets of 29 and 8 sets of 39.

*Step 2:* The sets are being combined. So we will multiply to find the cost of the milk and the cost of the orange juice. Then we will add to find the total cost.

*Step 3:* Write the equation for the problem:
$(7 \times 29) + (8 \times 39) = n$

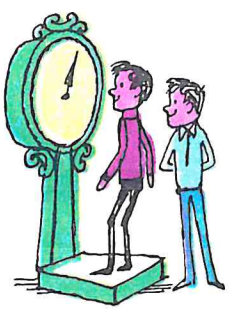
*Step 4:* Solve the equation: $7 \times 29 = 203$ and $8 \times 39 = 312$, so $n = 203 + 312 = 515$.

*Step 5:* The total cost is 515 cents, or \$5.15.

The two parts in *Step 2* show why problems of this kind are called **two stage problems.**

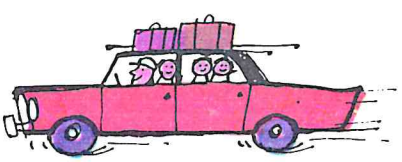
### EXERCISES

1. If there are 4 boxes each containing 5 balls and 6 boxes each containing 6 balls, how many balls are there in all?



2. Bob weighed 43 pounds at the beginning of the year and 56 pounds at the end of the year. Rich weighed 45 pounds at the beginning of the year and 57 pounds at the end of the year. How much weight did they gain in all?

3. Yesterday we traveled for 3 hours at the rate of 50 miles an hour. Today we traveled for 4 hours at the rate of 45 miles an hour. How far did we travel during the two days?



Write four equations for each set of two addends and the sum.

1. $\{2, 3, 5\}$
2. $\{3, 5, 8\}$
3. $\{7, 2, 9\}$
4. $\{3, 6, 9\}$
5. $\{3, 8, 11\}$
6. $\{4, 9, 13\}$
7. $\{4, 7, 11\}$
8. $\{6, 4, 10\}$
9. $\{6, 1, 7\}$
10. $\{9, 8, 17\}$
11. $\{8, 7, 15\}$
12. $\{6, 9, 15\}$

Write four equations for each set of two factors and the product.

13. $\{2, 3, 6\}$
14. $\{3, 5, 15\}$
15. $\{7, 2, 14\}$
16. $\{3, 6, 18\}$
17. $\{3, 8, 24\}$
18. $\{4, 9, 36\}$
19. $\{4, 7, 28\}$
20. $\{6, 4, 24\}$
21. $\{6, 1, 6\}$
22. $\{9, 3, 27\}$
23. $\{7, 3, 21\}$
24. $\{5, 9, 45\}$

Complete the equations. Write $+$, $-$, $\times$, or $\div$ to replace the $\triangle$.

25. $2 \triangle 7 = 14$
26. $2 \triangle 7 = 9$
27. $7 \triangle 2 = 5$
28. $6 \triangle 3 = 18$
29. $6 \triangle 3 = 9$
30. $24 \triangle 4 = 6$
31. $12 \triangle 8 = 20$
32. $36 \triangle 6 = 6$
33. $18 \triangle 8 = 10$
34. $24 \triangle 8 = 3$
35. $9 \triangle 4 = 36$
36. $9 \triangle 7 = 2$
37. $3 \triangle 10 = 30$
38. $7 \triangle 4 = 11$
39. $3 \triangle 10 = 13$

Complete the number sentences. Write $=$, $<$, or $>$ to replace the $\bigcirc$.

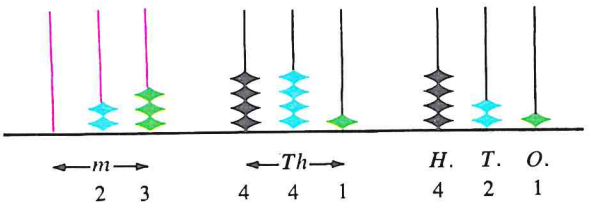
40. $5 + 9 \bigcirc 7 + 7$
41. $3 + 4 \bigcirc 8 + 2$
42. $4 \times 7 \bigcirc 5 \times 5$
43. $9 \times 7 \bigcirc 8 \times 8$
44. $1 \times 0 \bigcirc 1 + 0$
45. $1 - 1 \bigcirc 21 - 21$
46. $6 \times 3 \bigcirc 2 \times 9$
47. $4 \times 5 \bigcirc 7 \times 3$
48. $36 \div 6 \bigcirc 2 \times 3$
49. $18 - 2 \bigcirc 2 \times 4$
50. $16 - 6 \bigcirc 2 \times 5$
51. $4 + 13 \bigcirc 6 \times 3$

Complete the number pairs. The members of each pair are prime numbers with their sum shown on the left.

52. **36:** $\{(19, a), (29, b), (31, c)\}$
53. **40:** $\{(37, d), (23, e), (29, f)\}$
54. **60:** $\{(53, g), (47, h), (43, i), (37, j), (31, k)\}$
55. **48:** $\{(43, l), (41, m), (37, n), (31, a), (29, p)\}$
56. **64:** $\{(61, v), (59, w), (53, x), (47, y), (41, z)\}$

## Millions

The abacus shows
**23 thousand thousand.**
This is **23 million.**




### EXERCISES

1. Count by tens of thousands from **ten thousand** to **one hundred thousand.** Name the numbers like this: {10,000, 20,000, 30,000, . . . 100,000}.
2. Count by hundreds of thousands from **one hundred thousand** to **one thousand thousand.** Name the numbers like this: {100,000, 200,000, 300,000, . . . 1,000,000}.

Answer the questions.

3. What is the common name for 1,000,000?
4. How many zeros are there in the compact numeral for **one million?**
5. How many commas do we use in writing the compact numeral for **one million?**

Write the numerals in words.

6. 6,425,000
7. 7,875,000
8. 18,964,000

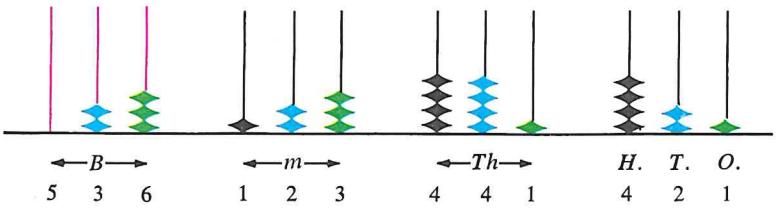
Write compact numerals for

9. eight million, six hundred eighty seven thousand
10. five million, two hundred eighty five thousand, six hundred forty eight
11. four million, three hundred twenty six

Write compact numerals for the numbers in the chart.

| | millions | thousands | ones |
|---|---|---|---|
| 12. | 17 | 263 | 348 |
| 13. | 5 | 692 | 486 |
| 14. | 24 | 509 | 620 |
| 15. | 134 | 689 | 200 |

## Billions




The abacus shows **23 thousand million.** This is **23 billion.**

### EXERCISES

Use words to complete the sentences.

1. A *thousand ones* is called a ___?___.
2. A *thousand thousands* is called a ___?___.
3. A *thousand millions* is called a ___?___.

> In this numeral the ones are shown in blue, the thousands in red, the millions in black, and the billions in blue.
>
> 385,469,738,294

Name the number of billions in

4. 478,296,345,700
5. 38,296,500,834
6. 2,698,346,287
7. 396,285,476

Write compact numerals for

8. three hundred forty six billion, nine hundred twenty seven million, three hundred forty five thousand, nine hundred eighty two
9. seven billion, four hundred twenty nine million, eighty four thousand, three hundred nine

Write compact numerals for the numbers in the chart.

| | billions | millions | thousands | ones |
|---|---|---|---|---|
| 10. | 73 | 469 | 287 | 391 |
| 11. | 27 | 600 | 369 | 489 |
| 12. | 7 | 468 | 354 | |
| 13. | 299 | 300 | 4 | 354 |

## Repeating a factor

$$8 = 2 \times 2 \times 2 = 2^3$$

The 3 in the equations tells how many times the 2 is used as a factor. In the product $2 \times 2 \times 2$ there are 3 factors of 2. This is written more compactly as $2^3$. The 3 is an **exponent.**

### EXERCISES

Name the number of equal factors in each product.

| | | | |
|---|---|---|---|
| **1.** $2 \times 2 \times 2 \times 2$ | **4.** $5 \times 5$ | **7.** $2^4$ | **10.** $5^2$ |
| **2.** $3 \times 3 \times 3$ | **5.** $6 \times 6 \times 6$ | **8.** $3^3$ | **11.** $6^3$ |
| **3.** $4 \times 4 \times 4 \times 4$ | **6.** $8 \times 8$ | **9.** $4^4$ | **12.** $8^2$ |

Solve the equations.

| | | |
|---|---|---|
| **13.** $2^2 = a$ | **17.** $3^2 = e$ | **21.** $4^2 = u$ |
| **14.** $2^3 = b$ | **18.** $3^3 = f$ | **22.** $4^3 = v$ |
| **15.** $2^2 \times 2 = a \times 2 = c$ | **19.** $3^2 \times 3 = e \times 3 = g$ | **23.** $4^2 \times 4 = u \times 4 = w$ |
| **16.** $2^2 \times 2 = 2^d$ | **20.** $3^2 \times 3 = 3^h$ | **24.** $4^2 \times 4 = 4^x$ |

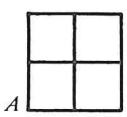
Solve the problems.

**25.** Three boys each have 3 boxes. Each box contains 3 triangles. How many triangles are there in all?

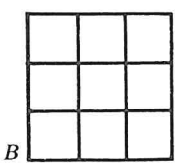
**26.** Each of the triangles has 3 sides. How many sides are there in all?

Study the square regions. Then name the areas, first as a product and then using exponents. For example, the area in *A* is $2 \times 2 = 2^2$ units.

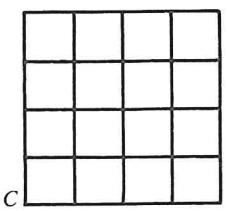
**27.**



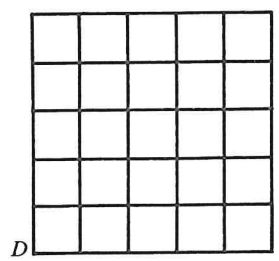

**28.**




**29.**




**30.**



Study the charts.

| 2 | $2 \times 2$ | $2 \times 2 \times 2$ | $2 \times 2 \times 2 \times 2$ | $2 \times 2 \times 2 \times 2 \times 2$ |
|---|---|---|---|---|
| $2^1$ | $2^2$ | $2^3$ | $2^4$ | $2^5$ |
| 2 | 4 | 8 | 16 | 32 |

| 10 | $10 \times 10$ | $10 \times 10 \times 10$ | $10 \times 10 \times 10 \times 10$ | $10 \times 10 \times 10 \times 10 \times 10$ |
|---|---|---|---|---|
| $10^1$ | $10^2$ | $10^3$ | $10^4$ | $10^5$ |
| 10 | 100 | 1000 | 10,000 | 100,000 |

Exponents make it easier to write the product of many equal factors.

## EXERCISES

Copy and complete the charts.

**1.**

| 3 | $3 \times 3$ | $3 \times 3 \times 3$ | $3 \times 3 \times 3 \times 3$ | $3 \times 3 \times 3 \times 3 \times 3$ |
|---|---|---|---|---|
| $3^1$ | $3^2$ | $3^3$ | $c$ | $e$ |
| 3 | $a$ | $b$ | $d$ | $f$ |

**2.**

| 5 | $5 \times 5$ | $5 \times 5 \times 5$ | $5 \times 5 \times 5 \times 5$ | $5 \times 5 \times 5 \times 5 \times 5$ |
|---|---|---|---|---|
| $g$ | $i$ | $k$ | $m$ | $r$ |
| $h$ | $j$ | $l$ | $n$ | $p$ |

Use the equation on the left to help you solve the other equations.

**3.** $4^2 = 16$: $4^3 = 4 \times 4^2 = 4 \times 16 = a$; $4^4 = 4 \times 4^3 = b$

**4.** $6^3 = 216$: $6^4 = 6 \times 6^3 = 6 \times c = d$; $6^5 = 6 \times 6^4 = e$

**5.** $8^2 = 64$: $8^4 = 8^2 \times 8^2 = 64 \times e = f$; $8^3 = 8 \times 8^2 = g$

Solve the equations.

**6.** $2 \times 2 \times 2 \times 2 \times 2 = 2^a$

**7.** $5 \times 5 \times 5 \times 5 = 5^b$

**8.** $7 \times 7 \times 7 = 7^c$

**9.** $9 \times 9 \times 9 \times 9 \times 9 = 9^d$

**10.** $3 \times 3^2 = 3^e$

**11.** $6 \times 6^4 = 6^f$

## Factors of ten in place value

An exponent names a **power** of a factor. For example,

$10^1 = 10$
$10^1$ is called the **first power** of 10.

$10^2 = 10 \times 10$
$10^2$ is called the **second power** of 10.

$10^3 = 10 \times 10 \times 10$
$10^3$ is called the **third power** of 10.

In our system of numeration each place shows a power of 10. For example,

the first four places in our system of numerals are:

| thousands | hundreds | tens | ones |
|---|---|---|---|
| $10^3$ | $10^2$ | $10^1$ | 1 |
| 1000 | 100 | 10 | 1 |

### EXERCISES

Write the compact numeral for

1. $4 \times 10$
2. $(6 \times 10) + 8$
3. $(3 \times 100) + (7 \times 10) + 8$
4. $6 \times 10^2$
5. $(4 \times 10^2) + 3$
6. $(7 \times 10 \times 10 \times 10)$
7. $8 \times 10^3$
8. $(2 \times 10^2) + (7 \times 10)$
9. $(2 \times 10 \times 10 \times 10) + (5 \times 10 \times 10) + (9 \times 10) + 8$
10. $(3 \times 10^3) + (4 \times 10^2) + (7 \times 10) + 6$
11. $(5 \times 10^4) + (3 \times 10^2) + 1$
12. $(9 \times 10^4) + (3 \times 10^3) + (7 \times 10^2) + 9$
13. $(6 \times 10^5) + (4 \times 10^3) + 2$

Complete the table.

14.

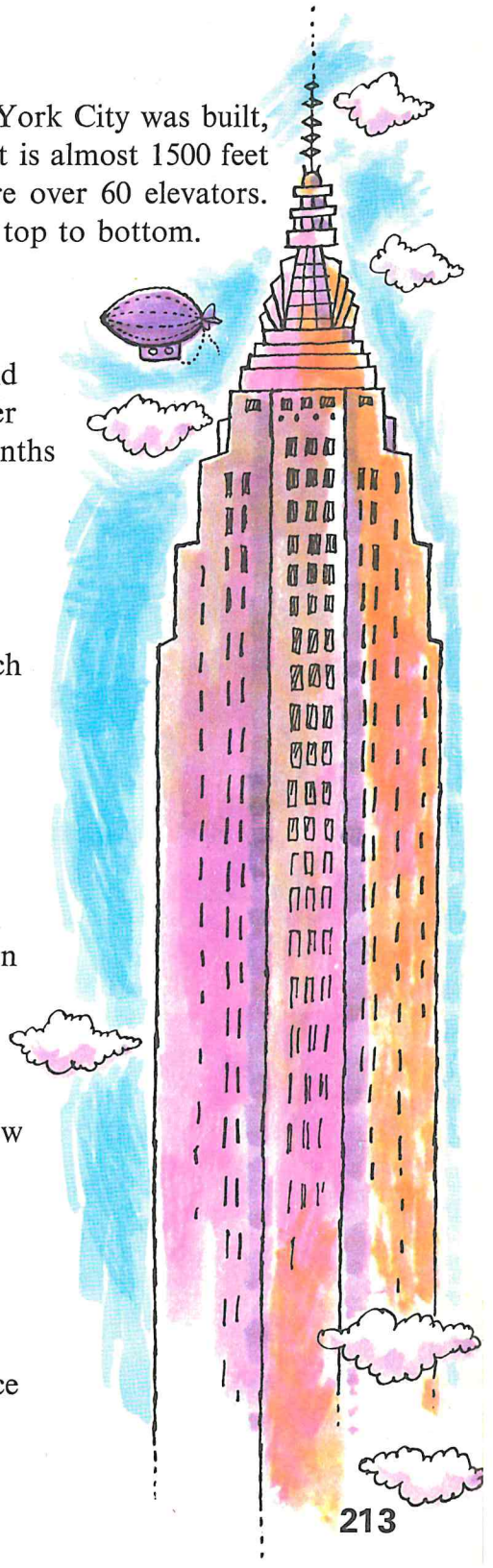
| **powers of 10** | $10^1$ | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ | $10^7$ | $10^8$ | $10^9$ | $10^{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| **compact numeral** | 10 | 100 | *a* | *b* | *c* | *d* | *e* | *f* | *g* | *h* |
| **number of zeros** | 1 | 2 | *i* | *j* | *k* | *l* | *m* | *n* | *r* | *p* |

Write a compact numeral for

15. $(3 \times 10^4) + (4 \times 10^3) + (5 \times 10^2) + (2 \times 10) + 8$
16. $(8 \times 10^5) + (7 \times 10^4) + (4 \times 10^3) + (8 \times 10^2) + (9 \times 10) + 1$
17. $(7 \times 10^4) + (1 \times 10^2) + 5$
18. $(9 \times 10^5) + (8 \times 10^2) + (3 \times 10)$

## The Empire State Building

When the Empire State Building in New York City was built, it was the tallest building in the world. It is almost 1500 feet high and has about 100 floors. There are over 60 elevators. One staircase has almost 1900 steps from top to bottom.



1. It took from the end of July, 1950 until the end of December, 1951 to build the television tower on the Empire State Building. How many months was that?

2. Suppose 1000 men are hired to take care of 100 floors in the Empire State Building. If each floor will have the same number of men, how many men will take care of each floor?

3. There are about 6000 windows in the Empire State Building. If 100 men clean the windows, about how many windows must each man clean to divide the work evenly?

4. If each of 60 elevators can hold 12 persons, how many people can use the elevators at the same time?

5. The Empire State Building is almost 1500 feet high and it has about 100 floors. If the distance between each floor is always the same, about what is this distance?

## Counting on eight fingers

Jerry the Jolly Giant has only 8 fingers instead of 10. He has to count in 8's. Every time he reaches 8, he draws a line to show a set of 8. To count the stones in the picture, his tally marks look like this. 𝍸 𝍸 |||

### EXERCISES

1. What numeral do we use to tell how many stones there are in the picture?

> Jerry counts, "One, two, three, four, five, six, seven, one eight, one eight one, one eight two," and so on.

Use this system to name the numbers shown by the tally marks.

2. 𝍸
3. 𝍸 ||
4. 𝍸 |||
5. 𝍸 ||||
6. 𝍸 |||||
7. 𝍸 𝍸
8. 𝍸 𝍸 |
9. 𝍸 𝍸 ||
10. 𝍸 𝍸 |||

Use Jerry's system to name the number that comes

11. just after one eight five
12. just after three eights two
13. just after four eights seven
14. between one eight one and one eight three
15. between one eight seven and two eights one
16. just before five eights three
17. just before three eights seven
18. just before three eights
19. Draw tally marks to count from *one* to *three eights*.

## Counting on eight fingers

If Jerry used the same digits as we do, he would write these numerals:

The numbers after 7 are $10_{eight}$ and $11_{eight}$.
$10_{eight}$ means *one eight* and $11_{eight}$ means *one eight one.*

### EXERCISES

Using Jerry's system, write compact numerals for the numbers shown here.

**1.** //////

**2.** ////#

**3.** ////#///

**4.** ////#//////

**5.** ////#////#

**6.** ////#////#/

**7.** ////#////#///

**8.** ////#////#//////

**9.** ////#////#////#

Name the missing numerals to compare the way we count with the way Jerry counts.

**10.**

| how we count | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| how Jerry counts | $1_{eight}$ | $2_{eight}$ | $3_{eight}$ | *a* | *b* | *c* | $7_{eight}$ | *d* |

**11.**

| how we count | 9 | 10 | 11 | 12 | 13 | *h* | *i* | *j* |
|---|---|---|---|---|---|---|---|---|
| how Jerry counts | *e* | *f* | *g* | $14_{eight}$ | $15_{eight}$ | $16_{eight}$ | $17_{eight}$ | $20_{eight}$ |

Use words to write the numerals in Jerry's system.

**12.** $15_{eight}$ **13.** $17_{eight}$ **14.** $21_{eight}$ **15.** $31_{eight}$ **16.** $45_{eight}$

In our system we arrange tally marks in sets of ten.
Rearrange the sets of ten into the system that Jerry uses.

**17.** ////#////#/

**18.** ////#////#////#///

**19.** ////#////#////#////#////#////#////

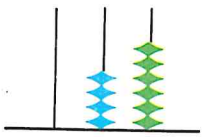
## Comparing base ten and base eight

This chart compares base ten numerals with base eight numerals. Study the chart carefully. Then copy and complete the two columns at the right.

| BASE TEN | | | BASE EIGHT | | |
|---|---|---|---|---|---|
| grouping | compact numeral | numeral in words | grouping | compact numeral | numeral in words |
| • | 1 | one | • | $1_{\text{eight}}$ | one |
| ⁞⁞ | 8 | eight | ⁞⁞ | $10_{\text{eight}}$ | eight |
| ⁞⁞ • | 9 | nine | ⁞⁞ • | *a* | one eight one |
| ⁞⁞ ⁞ | 10 | ten | ⁞⁞ •• | *b* | *c* |
| ⁞⁞ ⁞ ••••• | 15 | fifteen | ⁞⁞ ⁞• | *d* | *e* |
| ⁞⁞ ⁞ ⁞•••• | 16 | sixteen | ⁞⁞ ⁞⁞ | *f* | two eights |
| ⁞⁞ ⁞ ⁞⁞••• | 17 | seventeen | ⁞⁞ • ⁞⁞ | *g* | *h* |
| ⁞⁞ ⁞ ⁞⁞ ••• | 23 | twenty three | ⁞⁞ ⁞• ⁞⁞ | *i* | *j* |
| ⁞⁞ ⁞ ⁞⁞ •••• | 24 | twenty four | ⁞⁞ ⁞⁞ ⁞⁞ | *k* | *m* |
| ⁞⁞ ⁞ ⁞⁞ ••••• | 25 | twenty five | ⁞⁞ ⁞⁞ ⁞⁞ • | *n* | three eights one |
| ⁞⁞ ⁞ ⁞⁞ ⁞⁞ ⁞⁞ •• | 32 | thirty two | ⁞⁞ ⁞⁞ ⁞⁞ ⁞⁞ | $40_{\text{eight}}$ | *p* |
| ⁞⁞ ⁞ ⁞⁞ ⁞⁞ ⁞⁞ ⁞ ⁞⁞ ⁞⁞ ⁞⁞ | 63 | sixty three | ⁞⁞ ⁞⁞ ⁞⁞ ⁞⁞ ⁞⁞ ⁞• ⁞⁞ ⁞⁞ | *t* | *u* |
| ⁞⁞ ⁞ ⁞⁞ ⁞⁞ ⁞⁞ ⁞ ⁞⁞ ⁞⁞ ⁞⁞ | 64 | sixty four | ⁞⁞ ⁞⁞ ⁞⁞ ⁞⁞ ⁞⁞ ⁞⁞ ⁞⁞ ⁞⁞ | *v* | one eight-eights |

## Using a base eight abacus

On the base eight abacus, only seven counters can fit on each rod. Eight counters in one place may be exchanged for one counter in the next place to the left. Study the abacus shown here.

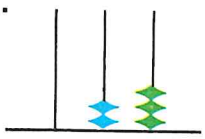


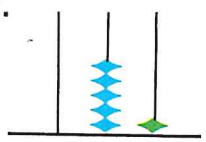
four eights six

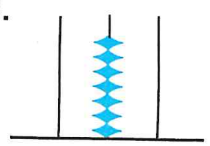
$46_{\text{eight}}$

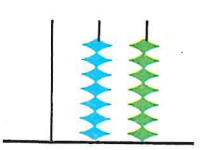
### EXERCISES

Name the number shown on each base eight abacus.

1. 

2. 

3. 

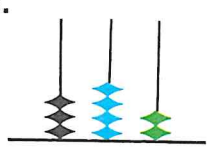
4. 

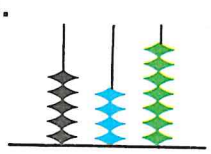
Draw an abacus for each base eight numeral.

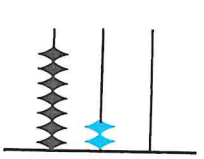
5. $24_{\text{eight}}$
6. $37_{\text{eight}}$
7. $50_{\text{eight}}$
8. $72_{\text{eight}}$
9. four eights three
10. one eight six
11. six eights
12. eight eights

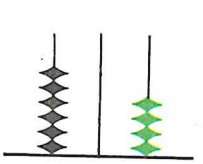
The second place to the left of the ones' place names the number of eight-eights. For example, $342_{\text{eight}}$ means *three eight-eights four eights two.*

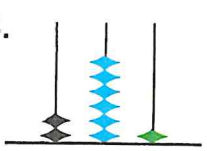
Name the number shown on each base eight abacus.

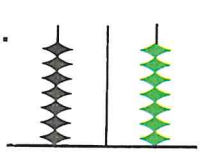
13. 

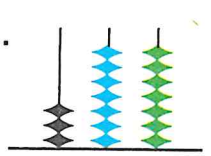
15. 

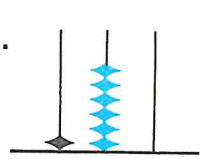
17. 

19. 

14. 

16. 

18. 

20. 

## Number patterns

Find a pattern to help you name the sums.

| | | | | | |
|---|---|---|---|---|---|
| **1.** 36 + 63 | **2.** 27 + 72 | **3.** 18 + 81 | **4.** 54 + 45 | **5.** 346 + 653 | **6.** 176 + 823 |

Find a pattern to help you name the differences.

| | | | | | |
|---|---|---|---|---|---|
| **7.** 83 − 38 | **9.** 74 − 47 | **11.** 91 − 19 | **13.** 61 − 16 | **15.** 82 − 28 | **17.** 64 − 46 |
| **8.** 62 − 26 | **10.** 81 − 18 | **12.** 71 − 17 | **14.** 54 − 45 | **16.** 42 − 24 | **18.** 76 − 67 |

Find a pattern to help you name the differences.

| | | | | | |
|---|---|---|---|---|---|
| **19.** 864 − 468 | **21.** 361 − 163 | **23.** 681 − 186 | **25.** 832 − 238 | **27.** 752 − 257 | **29.** 962 − 269 |
| **20.** 725 − 527 | **22.** 743 − 347 | **24.** 792 − 297 | **26.** 542 − 245 | **28.** 831 − 138 | **30.** 953 − 359 |

Find a pattern to help you name the products.

| | | | | | |
|---|---|---|---|---|---|
| **31.** 37 × 3 | **33.** 37 × 9 | **35.** 37 × 15 | **37.** 74 × 3 | **39.** 74 × 9 | **41.** 74 × 15 |
| **32.** 37 × 6 | **34.** 37 × 12 | **36.** 37 × 18 | **38.** 74 × 6 | **40.** 74 × 12 | **42.** 74 × 18 |

Find a pattern to help you name the products.

| | | | | | |
|---|---|---|---|---|---|
| **43.** 143 × 7 | 143 × 28 | **44.** 125 × 8 | 125 × 16 | **45.** 375 × 8 | 375 × 16 |
| 143 × 14 | 143 × 35 | 125 × 24 | 125 × 32 | 375 × 24 | 375 × 40 |
| 143 × 21 | 143 × 42 | 125 × 40 | 125 × 72 | 375 × 48 | 375 × 64 |

Find a pattern to help you name the quotients.

| | | | | | |
|---|---|---|---|---|---|
| **46.** $7\overline{)21}$ | **47.** $7\overline{)42}$ | **48.** $7\overline{)63}$ | **49.** $7\overline{)84}$ | **50.** $7\overline{)105}$ | **51.** $7\overline{)147}$ |

Complete each number pattern until you have nine digits on the right-hand side of the equation.

**1.**
$$9 + 1 = 10$$
$$99 + 1 = 100$$
$$999 + 1 = 1000$$
$$9999 + 1 = 10000$$

**2.**
$$1^2 = 1$$
$$2^2 = 1 + 3$$
$$3^2 = 1 + 3 + 5$$
$$4^2 = 1 + 3 + 5 + 7$$

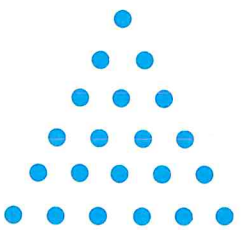
**3.**
$$3 \times 2 = 1 + 2 + 3$$
$$5 \times 2 = 1 + 2 + 3 + 4$$
$$5 \times 3 = 1 + 2 + 3 + 4 + 5$$
$$7 \times 3 = 1 + 2 + 3 + 4 + 5 + 6$$

**4.**
$$8 + 2 = 10$$
$$98 + 2 = 100$$
$$998 + 2 = 1000$$
$$9998 + 2 = 10000$$

Draw dots to form a triangle as we have done.

For each row write an equation to name the total number of dots in the triangle from the top to that row. Continue the pattern until you have ten rows.

**5.**



| Dots per row | Equations for total |
|---|---|
| 1 | $1 = 1$ |
| 2 | $1 + 2 = 3$ |
| 3 | $1 + 2 + 3 = 6$ |
| 4 | $1 + 2 + 3 + 4 = 10$ |
| 5 | ? |
| 6 | ? |

Copy and continue the patterns until you have ten equations in each pattern.

**6.**
$$2^2 - 1^2 = 3$$
$$3^2 - 2^2 = 5$$
$$4^2 - 3^2 = 7$$
$$5^2 - 4^2 = 9$$
$$6^2 - 5^2 = 11$$

**7.**
$$2^2 - 1 = 3 \times 1$$
$$3^2 - 1 = 4 \times 2$$
$$4^2 - 1 = 5 \times 3$$
$$5^2 - 1 = 6 \times 4$$
$$6^2 - 1 = 7 \times 5$$

**8.**
$$2^2 - 1 = 3$$
$$3^2 - 2 = 7$$
$$4^2 - 3 = 13$$
$$5^2 - 4 = 21$$
$$6^2 - 5 = 31$$

**9.**
$$2^2 - 4 = 4 \times 0$$
$$3^2 - 4 = 5 \times 1$$
$$4^2 - 4 = 6 \times 2$$
$$5^2 - 4 = 7 \times 3$$
$$6^2 - 4 = 8 \times 4$$

## CHECKING UP

Name the multiples in each set.

1. $\{14, 21, a, 35, b, 49, 56, c, d\}$
2. $\{24, 32, e, f, 56, 64, g, 80, h\}$
3. $\{36, 42, l, m, 60, n, o, p, 84\}$
4. $\{27, 36, 45, q, r, 72, 81, s, t\}$

Solve the equations. Then name the set of factors.

5. $1 \times 28 = a$, $2 \times 14 = b$, $4 \times 7 = c$. The factors of 28 are set $D$.
6. $1 \times 63 = e$, $3 \times 21 = f$, $9 \times 7 = g$. The factors of 63 are set $H$.
7. $1 \times 81 = i$, $3 \times 27 = j$, $9 \times 9 = k$. The factors of 81 are set $L$.

The first ten multiples of 6 are $\{0, 6, 12, 18, 24, 30, 36, 42, 48, 54\}$.
The first ten multiples of 8 are $\{0, 8, 16, 24, 32, 40, 48, 56, 64, 72\}$.

8. Name the common multiples of 6 and 8, less than 55.

The factors of 18 are $\{1, 2, 3, 6, 9, 18\}$ and the factors of 48 are $\{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$.

9. Name the common factors of 18 and 48.

Name

10. 18 as the product of three primes.
11. 24 as the product of four primes.

Write a compact numeral for

12. 764 million, 325 thousand, 74
13. 999 million, 999 thousand, 91
14. three hundred twenty four million, nine hundred five thousand, two hundred sixty three
15. $(4 \times 10^4) + (3 \times 10^3) + (8 \times 10^2) + (6 \times 10) + 9$
16. $(5 \times 10^6) + (8 \times 10^5) + (0 \times 10^4) + (6 \times 10^3) + (7 \times 10^2) + (2 \times 10) + 4$

Name the numbers in base eight.

17. $9_{\text{ten}}$
18. $12_{\text{ten}}$
19. $15_{\text{ten}}$
20. $17_{\text{ten}}$

Name the numbers in base ten.

21. $42_{\text{eight}}$
22. $27_{\text{eight}}$
23. $35_{\text{eight}}$
24. $25_{\text{eight}}$

The **intersection** of sets $A$ and $B$ is the set that contains all the objects that are in *both* $A$ and $B$. It is written $A \cap B$.

**1.** $A = \{h, a, i, r\}$ $B = \{b, r, u, s, h\}$ $C = \{s, h, a, v, e\}$

Name the intersections.

**a.** $A \cap B$ **b.** $A \cap C$ **c.** $B \cap C$

The product of two factors is a **multiple** of either factor. For example, $3 \times 4 = 12$, so 12 is a multiple of 3 and of 4 and 3 and 4 are factors of 12.

**2.** Name the multiples of **a.** 4, less than 25 **b.** 7, less than 80

**3.** Name the common factors of **a.** 24 and 42 **b.** 20 and 30

A number which has only two factors, itself and 1, is called a **prime number**.

**4.** Name the prime numbers in this set

$\{7, 14, 17, 23, 24, 28, 29, 31, 36, 37, 40\}$

An **exponent** is used to show how many times a number is used as a factor.

$$2^3 = 2 \times 2 \times 2, \quad 2^4 = 2 \times 2 \times 2 \times 2$$

**5.** $2 \times 2 = 2^a$

**6.** $2^2 \times 2 = 2^c$

**7.** $3 \times 3 \times 3 = 3^b$

**8.** $4^2 \times 4 \times 4 = 4^e$

**9.** $3 \times 3^3 = 3^f$

**10.** $5^4 \times 5 = 5^h$

There are **systems of numerals** other than our base ten system. In base eight numerals, $24_{\text{eight}}$ stands for two eights four.

**11.** Write base ten numerals for

**a.** three eights seven

**b.** seven eights four

**c.** five eights

**d.** eight-eights

## LOOKING WAY BACK

Name the sums and differences.

**1.** $436 + 279$

**2.** $34 + 28 + 67 + 94$

**3.** $4839 + 2546$

**4.** $6948 + 287 + 4691 + 4834$

**5.** $789 - 219$

**6.** $438 + 259 + 476 + 283$

**7.** $834 - 265$

**8.** $2709 + 4956 + 2430 + 7321$

**9.** $4863 - 2179$

**10.** $8764 + 2831 + 7029 + 3546 + 8251 + 7349$

**11.** $4739 - 1837$

**12.** $2961 + 4837 + 5629 + 4834 + 5612 + 7914$

Name the products.

**13.** $34 \times 7$

**14.** $69 \times 9$

**15.** $47 \times 5$

**16.** $68 \times 9$

**17.** $384 \times 7$

**18.** $496 \times 2$

**19.** $5768 \times 4$

**20.** $38269 \times 7$

**21.** $63 \times 21$

**22.** $69 \times 28$

**23.** $48 \times 27$

**24.** $39 \times 35$

Name the quotients.

**25.** $6\overline{)78}$ **26.** $7\overline{)91}$ **27.** $7\overline{)105}$ **28.** $9\overline{)153}$ **29.** $8\overline{)184}$ **30.** $6\overline{)234}$

Use the sets for exercises 31 to 42.

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 10\}$ $C = \{1, 3, 5, 7, 9\}$

**31.** $A \cup B$

**32.** $A \cap B$

**33.** $n(A \cup B)$

**34.** $n(A \cap B)$

**35.** $A \cup C$

**36.** $A \cap C$

**37.** $n(A \cup C)$

**38.** $n(A \cap C)$

**39.** $B \cup C$

**40.** $B \cap C$

**41.** $n(B \cup C)$

**42.** $n(B \cap C)$

Name the least and the greatest counting numbers you can show with

**43.** two digits **44.** three digits **45.** four digits **46.** five digits

Name the products.

**47.** $3 \times 4 \times 5$

**48.** $3 \times 4 \times 7$

**49.** $4 \times 5 \times 7$

**50.** $6 \times 9 \times 2$

**51.** $5 \times 8 \times 3$

**52.** $5 \times 7 \times 3$

Name the sums, differences, and products.

**1.** $947283962 + 864397586$

**2.** $380479382 + 287386476$

**3.** $6928473056 + 3280793218$

**4.** $947283962 - 864397586$

**5.** $380479382 - 287386476$

**6.** $9846217308 - 2609384619$

**7.** $473982456 \times 3$

**8.** $296304809 \times 6$

**9.** $8732568419 \times 7$

## Working with prime numbers

**10.** Name all the prime numbers less than 100. There are twenty five primes less than 100.

**11.** 71 is a prime number, so is 17. These numerals have the same digits. Name three other pairs of prime numbers less than 100 with the digits reversed as in 71 and 17.

**12.** A whole number greater than 1 which is not prime is called **composite.** Between 19 and the next prime, 23, there are three composites: 20, 21, 22. Name three pairs of primes between which there are five composites.

## Solve the equations.

**13.** $5 + 7 = a = b \times 6$

**14.** $7 + 11 = c = d \times 6$

**15.** $11 + 13 = e = f \times 6$

**16.** $13 + 17 = g = h \times 6$

**17.** $17 + 19 = i = j \times 6$

**18.** $19 + 23 = k = l \times 6$

**19.** $23 + 25 = m = n \times 6$

**20.** $25 + 29 = r = s \times 6$

*Just For Fun*

Make a code by numbering the letters of the alphabet, in order, from 1 to 26. Write words like this: the last prime before 20, $7 \times 3$, $2^2$, $3 + 1$, the third prime, $(4 \times 3) + 2$. These clues give the letters S, U, D, D, E, N.

## One factor greater than 100

Study this method to name the product $4 \times 638$.

$$\begin{array}{r} 638 \\ \times\ 4 \\ \hline 32 \\ 120 \\ 2400 \\ \hline 2552 \end{array} \qquad \begin{array}{l} \longrightarrow\ 4 \times 8 = 32 \\ \longrightarrow\ 4 \times 30 = 120 \\ \longrightarrow\ 4 \times 600 = 2400 \\ \longrightarrow\ 4 \times 638 = 2552 \end{array}$$

### EXERCISES

Copy and complete the multiplications.

**1.** $\begin{array}{r} 374 \\ \times\ 6 \\ \hline 24 \\ a \\ 1800 \\ \hline 2244 \end{array}$ **2.** $\begin{array}{r} 468 \\ \times\ 8 \\ \hline e \\ 480 \\ 3200 \\ \hline 3744 \end{array}$ **3.** $\begin{array}{r} 736 \\ \times\ 7 \\ \hline 42 \\ 210 \\ m \\ \hline 5152 \end{array}$ **4.** $\begin{array}{r} 984 \\ \times\ 2 \\ \hline 8 \\ 160 \\ 1800 \\ n \end{array}$ **5.** $\begin{array}{r} 572 \\ \times\ 3 \\ \hline 6 \\ 210 \\ r \\ \hline 1716 \end{array}$ **6.** $\begin{array}{r} 279 \\ \times\ 9 \\ \hline 81 \\ s \\ 1800 \\ \hline 2511 \end{array}$

Name the products.

**7.** $\begin{array}{r} 764 \\ \times\ 4 \end{array}$ **9.** $\begin{array}{r} 375 \\ \times\ 8 \end{array}$ **11.** $\begin{array}{r} 573 \\ \times\ 2 \end{array}$ **13.** $\begin{array}{r} 569 \\ \times\ 7 \end{array}$ **15.** $\begin{array}{r} 718 \\ \times\ 3 \end{array}$ **17.** $\begin{array}{r} 308 \\ \times\ 4 \end{array}$

**8.** $\begin{array}{r} 438 \\ \times\ 6 \end{array}$ **10.** $\begin{array}{r} 821 \\ \times\ 7 \end{array}$ **12.** $\begin{array}{r} 689 \\ \times\ 5 \end{array}$ **14.** $\begin{array}{r} 358 \\ \times\ 9 \end{array}$ **16.** $\begin{array}{r} 540 \\ \times\ 8 \end{array}$ **18.** $\begin{array}{r} 768 \\ \times\ 7 \end{array}$

Answer the questions.

**19.** If 658 people paid \$3 each to see a football game, how much money was collected?

**20.** John has a book for pictures of football players. There is room in the book for 275 pictures. Each picture costs 8 cents. How much will it cost to fill the book?

**21.** The school band plays at all the football games. There are 138 members of the band. If each member needs 9 sheets of music, how many sheets of music do they need all together?

Study this method to name the product $4 \times 638$.

$$\begin{array}{r} 638 \\ \times\ 4 \\ \hline 2552 \end{array}$$

$4 \times 8 = 32$ Write 2 in the ones' place.

$4 \times 30 = 120$ $120 + 30 = 150$

Write 5 in the tens' place.

$4 \times 600 = 2400$ $2400 + 100 = 2500$

Write 5 in the hundreds' place and 2 in the thousands' place.

## EXERCISES

Name the products.

| | | | | |
|---|---|---|---|---|
| **1.** $467 \times 7$ | **5.** $696 \times 9$ | **9.** $787 \times 6$ | **13.** $482 \times 7$ | **17.** $736 \times 4$ | **21.** $693 \times 4$ |
| **2.** $782 \times 6$ | **6.** $287 \times 3$ | **10.** $493 \times 7$ | **14.** $298 \times 8$ | **18.** $548 \times 9$ | **22.** $728 \times 2$ |
| **3.** $593 \times 4$ | **7.** $307 \times 5$ | **11.** $847 \times 4$ | **15.** $357 \times 6$ | **19.** $939 \times 3$ | **23.** $546 \times 3$ |
| **4.** $884 \times 5$ | **8.** $905 \times 8$ | **12.** $659 \times 9$ | **16.** $864 \times 5$ | **20.** $918 \times 8$ | **24.** $576 \times 8$ |

Complete the chart.

**25.**

| × | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|
| 274 | *a* | *v* | *c* | *w* | *e* | *x* | *z* |
| 385 | *r* | *s* | *u* | *g* | *h* | *m* | *n* |
| 196 | *k* | *b* | *d* | *f* | *l* | *t* | *i* |

Answer the question.

**26.** At a dress sale Mary tried on 7 dresses each costing \$8.69. If Mary's mother bought all the dresses, what would the total bill be?

## One factor greater than 1000

Study the examples.

| Five step | Short form | Six step | Short form |
|---|---|---|---|
| 3468 × 7 | 3468 × 7 | 83468 × 7 | 83468 × 7 |
| 56 | 24276 | 56 | 584276 |
| 420 | | 420 | |
| 2800 | | 2800 | |
| 21000 | | 21000 | |
| 24276 | | 560000 | |
| | | 584276 | |

### EXERCISES

Use the 5-step method to name the products.

| | | | | |
|---|---|---|---|---|
| **1.** 3568 × 3 | **2.** 4781 × 5 | **3.** 8254 × 6 | **4.** 7193 × 8 | **5.** 3258 × 9 |

Use the 6-step method to name the products.

| | | | | |
|---|---|---|---|---|
| **6.** 37423 × 7 | **7.** 59284 × 5 | **8.** 46973 × 6 | **9.** 73826 × 4 | **10.** 69437 × 8 |

Use the short method to name the products.

| | | | | |
|---|---|---|---|---|
| **11.** 4309 × 4 | **13.** 9080 × 7 | **15.** 6509 × 9 | **17.** 29508 × 7 | **19.** 78250 × 8 |
| **12.** 7038 × 8 | **14.** 8754 × 3 | **16.** 75043 × 8 | **18.** 80952 × 9 | **20.** 90850 × 9 |

Name the products.

| | | | | |
|---|---|---|---|---|
| **21.** 3492 × 7 | **23.** 64308 × 5 | **25.** 38095 × 7 | **27.** 9876 × 9 | **29.** 82157 × 3 |
| **22.** 5249 × 4 | **24.** 72548 × 6 | **26.** 8076 × 8 | **28.** 50369 × 3 | **30.** 73096 × 7 |

Study the equations.

**A**
$$\begin{aligned} 30 \times 20 &= (3 \times 10) \times 20 \\ &= 3 \times (10 \times 20) \\ &= 3 \times 200 \\ &= 600 \end{aligned}$$

**B**
$$\begin{aligned} 30 \times 27 &= 30 \times (20 + 7) \\ &= (30 \times 20) + (30 \times 7) \\ &= 600 + 210 \\ &= 810 \end{aligned}$$

**A** uses the **associative property.** **B** uses the **distributive property.**

## EXERCISES

Name the products.

1. $4 \times 20$ and $40 \times 20$
2. $3 \times 30$ and $30 \times 30$
3. $3 \times 40$ and $30 \times 40$
4. $6 \times 40$ and $60 \times 40$
5. $3 \times 24$ and $30 \times 24$
6. $4 \times 16$ and $40 \times 16$
7. $6 \times 17$ and $60 \times 17$
8. $5 \times 19$ and $50 \times 19$

Complete the number patterns.

9. $30 \times 7 = a$
   $30 \times 20 = e$
   $30 \times 27 = c$

10. $40 \times 2 = a$
    $40 \times 30 = e$
    $40 \times 32 = c$

11. $30 \times 8 = a$
    $30 \times 30 = e$
    $30 \times 38 = c$

12.

| × | 20 | 6 | 26 |
|---|---|---|---|
| 30 | *m* | *n* | *r* |

13.

| × | 30 | 8 | 38 |
|---|---|---|---|
| 40 | *m* | *n* | *r* |

14.

| × | 40 | 3 | 43 |
|---|---|---|---|
| 60 | *m* | *n* | *r* |

Name the products.

15. $27 \times 30$
16. $53 \times 30$
17. $62 \times 40$
18. $57 \times 50$
19. $77 \times 60$
20. $89 \times 90$
21. $64 \times 70$
22. $90 \times 90$
23. $48 \times 90$
24. $69 \times 40$
25. $780 \times 400$
26. $960 \times 700$

Solve the equations.

27. $30 \times 46 = (30 \times 40) + (30 \times 6)$
    $= a + b$
    $= c$

28. $40 \times 64 = (40 \times 60) + (40 \times 4)$
    $= d + e$
    $= f$

## Multiplication

Study the methods for naming the product $32 \times 87$.

| | | |
|---|---|---|
| $2 \times 87 = 174$<br>$30 \times 87 = 2610$<br>$32 \times 87 = 2784$ | $32 \times 87 = (30 + 2) \times 87$<br>$= (30 \times 87) + (2 \times 87)$<br>$= 2610 + 174 = 2784$ | $\begin{array}{r} 87 \\ \times\ 32 \\ \hline 174 \\ 2610 \\ \hline 2784 \end{array}$ |

### EXERCISES

Solve the equations.

**1.** $\begin{array}{rcl} 4 \times 32 & = & 128 \\ 20 \times 32 & = & 640 \\ \hline 24 \times 32 & = & a \end{array}$

**2.** $\begin{array}{rcl} 7 \times 42 & = & 294 \\ 30 \times 42 & = & 1260 \\ \hline 37 \times e & = & c \end{array}$

**3.** $\begin{array}{rcl} 3 \times 57 & = & 171 \\ 40 \times 57 & = & 2280 \\ \hline n \times e & = & r \end{array}$

**4.** $\begin{array}{rcl} 9 \times 62 & = & 558 \\ 30 \times 62 & = & 1860 \\ \hline 39 \times s & = & u \end{array}$

**5.** $\begin{array}{rcl} 4 \times 75 & = & 300 \\ 60 \times 75 & = & r \\ \hline a \times e & = & n \end{array}$

**6.** $\begin{array}{rcl} 6 \times 93 & = & m \\ 50 \times 93 & = & n \\ \hline w \times r & = & s \end{array}$

Name the products.

**7.** $\begin{array}{r} 52 \\ \times\ 37 \\ \hline \end{array}$ **8.** $\begin{array}{r} 81 \\ \times\ 24 \\ \hline \end{array}$ **9.** $\begin{array}{r} 48 \\ \times\ 34 \\ \hline \end{array}$ **10.** $\begin{array}{r} 19 \\ \times\ 65 \\ \hline \end{array}$ **11.** $\begin{array}{r} 94 \\ \times\ 17 \\ \hline \end{array}$ **12.** $\begin{array}{r} 58 \\ \times\ 46 \\ \hline \end{array}$ **13.** $\begin{array}{r} 74 \\ \times\ 73 \\ \hline \end{array}$ **14.** $\begin{array}{r} 92 \\ \times\ 92 \\ \hline \end{array}$ **15.** $\begin{array}{r} 59 \\ \times\ 59 \\ \hline \end{array}$ **16.** $\begin{array}{r} 79 \\ \times\ 97 \\ \hline \end{array}$ **17.** $\begin{array}{r} 85 \\ \times\ 55 \\ \hline \end{array}$ **18.** $\begin{array}{r} 95 \\ \times\ 66 \\ \hline \end{array}$

To rename $29 \times 87$, Susan noticed that because 29 is 1 less than 30, she could name $30 \times 87$ and then subtract $1 \times 87$.

$$\begin{array}{rcl} 30 \times 87 & = & 2610 \\ -\ 1 \times 87 & = & -\ 87 \\ \hline 29 \times 87 & = & 2523 \end{array}$$

Use this method to name the products.

**19.** $\begin{array}{r} 76 \\ \times\ 29 \\ \hline \end{array}$ **20.** $\begin{array}{r} 87 \\ \times\ 39 \\ \hline \end{array}$ **21.** $\begin{array}{r} 54 \\ \times\ 49 \\ \hline \end{array}$ **22.** $\begin{array}{r} 79 \\ \times\ 35 \\ \hline \end{array}$ **23.** $\begin{array}{r} 69 \\ \times\ 81 \\ \hline \end{array}$ **24.** $\begin{array}{r} 99 \\ \times\ 58 \\ \hline \end{array}$

To name the product $43 \times 247$, we may use this method.

$$\begin{array}{r} 247 \\ \times\ 43 \\ \hline 741 \\ 9880 \\ \hline 10621 \end{array} \qquad \begin{array}{rcl} 3 \times 247 & = & 741 \\ 40 \times 247 & = & 9880 \\ \hline 43 \times 247 & = & 10621 \end{array}$$

## EXERCISES

Copy and complete the examples.

**1.** $\begin{array}{r} 249 \\ \times\ 37 \\ \hline 1743 \\ 7470 \\ \hline \end{array}$ **2.** $\begin{array}{r} 153 \\ \times\ 48 \\ \hline 1224 \\ 6120 \\ \hline \end{array}$ **3.** $\begin{array}{r} 378 \\ \times\ 25 \\ \hline 1890 \\ 7560 \\ \hline \end{array}$ **4.** $\begin{array}{r} 467 \\ \times\ 46 \\ \hline 2802 \\ 18680 \\ \hline \end{array}$ **5.** $\begin{array}{r} 284 \\ \times\ 76 \\ \hline 1704 \\ 19880 \\ \hline \end{array}$

Name the missing digits.

**6.** $\begin{array}{r} 516 \\ \times\ 82 \\ \hline 103a \\ 41e80 \\ \hline 4c3n2 \end{array}$ **7.** $\begin{array}{r} 272 \\ \times\ 36 \\ \hline 16e2 \\ 81u0 \\ \hline 9r9s \end{array}$ **8.** $\begin{array}{r} 821 \\ \times\ 54 \\ \hline 3s84 \\ a1050 \\ \hline 4u3v4 \end{array}$ **9.** $\begin{array}{r} 485 \\ \times\ 47 \\ \hline m395 \\ 194n0 \\ \hline 2x7z5 \end{array}$ **10.** $\begin{array}{r} 637 \\ \times\ 29 \\ \hline c733 \\ 1r740 \\ \hline sa473 \end{array}$

Name the products.

**11.** $346 \times 38$ **13.** $472 \times 65$ **15.** $588 \times 37$ **17.** $383 \times 26$ **19.** $296 \times 54$ **21.** $719 \times 78$

**12.** $154 \times 27$ **14.** $293 \times 18$ **16.** $479 \times 67$ **18.** $654 \times 79$ **20.** $187 \times 69$ **22.** $499 \times 99$

Answer the questions.

**23.** There are 34 children in Julie's class. If they each pay \$2.39 for the class outing, how much will they pay in all?

**24.** Shirts are on sale at \$3.59 each. What is the cost
**a.** of 18 shirts? **b.** of 36 shirts? **c.** of 48 shirts?

## Division with remainders

To find the number of sets of 6 that can be formed from a set of 44, we may work like this:

$$6\overline{)44}\quad ?$$

Think of the multiple of 6 which is closest to (but less than) 44.

$$\begin{array}{r} 7 \\ 6\overline{)44} \\ \underline{42} \end{array}$$

Subtract 42 from 44.

$$\begin{array}{r} 7 \\ 6\overline{)44} \\ \underline{42} \\ 2 \end{array}$$

Check that the remainder is less than the divisor.

$$2 < 6$$

A set of 44 may be separated to form 7 sets of 6 with 2 remaining.

When 44 is divided by 6, the **quotient** is 7 and the **remainder** is 2.

### EXERCISES

Copy and complete the examples. Name the quotients.

1. $9\overline{)66}$, $\underline{a}$, $3$
2. $9\overline{)80}$, $\underline{u}$, $8$
3. $8\overline{)58}$, $\underline{c}$, $2$
4. $7\overline{)61}$, $\underline{m}$, $5$
5. $6\overline{)49}$, $\underline{e}$, $1$
6. $9\overline{)61}$, $\underline{v}$, $7$
7. $5\overline{)43}$, $\underline{n}$, $3$
8. $8\overline{)53}$, $\underline{x}$, $5$
9. $8\overline{)69}$, $\underline{r}$, $5$
10. $6\overline{)53}$, $\underline{s}$, $5$

Name the quotients and remainders when

11. 33 is divided by 5; by 7; by 8
12. 34 is divided by 5; by 7; by 8
13. 44 is divided by 6; by 7; by 8
14. 54 is divided by 7; by 8; by 9

Name the quotients and remainders.

15. $4\overline{)26}$
16. $8\overline{)47}$
17. $5\overline{)53}$
18. $2\overline{)19}$
19. $3\overline{)23}$
20. $7\overline{)67}$
21. $5\overline{)37}$
22. $3\overline{)32}$
23. $6\overline{)45}$
24. $6\overline{)58}$
25. $7\overline{)48}$
26. $4\overline{)39}$
27. $7\overline{)39}$
28. $8\overline{)58}$
29. $6\overline{)40}$
30. $9\overline{)84}$
31. $7\overline{)65}$
32. $9\overline{)65}$
33. $8\overline{)69}$
34. $2\overline{)13}$
35. $9\overline{)40}$
36. $9\overline{)94}$
37. $7\overline{)31}$
38. $5\overline{)49}$

Compare the equation with the working form.

$64 - (9 \times 7) = 1$

$$\begin{array}{r} 9 \\ 7\overline{)64} \\ \underline{63} \\ 1 \end{array}$$

Both forms show that when 64 is divided by 7, the quotient is 9 and the remainder is 1.

## EXERCISES

Write the working forms that go with the equations.

1. $33 - (8 \times 4) = 1$
2. $28 - (3 \times 9) = 1$
3. $53 - (6 \times 8) = 5$
4. $31 - (7 \times 4) = 3$
5. $44 - (6 \times 7) = 2$
6. $41 - (6 \times 6) = 5$
7. $35 - (4 \times 8) = 3$
8. $62 - (8 \times 7) = 6$
9. $46 - (9 \times 5) = 1$
10. $67 - (9 \times 7) = 4$
11. $58 - (6 \times 9) = 4$
12. $88 - (9 \times 9) = 7$

Write the equations that go with the working forms.

13. $\begin{array}{r} 5 \\ 4\overline{)23} \\ \underline{20} \\ 3 \end{array}$
15. $\begin{array}{r} 8 \\ 6\overline{)50} \\ \underline{48} \\ 2 \end{array}$
17. $\begin{array}{r} 6 \\ 7\overline{)45} \\ \underline{42} \\ 3 \end{array}$
19. $\begin{array}{r} 3 \\ 9\overline{)30} \\ \underline{27} \\ 3 \end{array}$
21. $\begin{array}{r} 5 \\ 6\overline{)34} \\ \underline{30} \\ 4 \end{array}$
23. $\begin{array}{r} 6 \\ 9\overline{)61} \\ \underline{54} \\ 7 \end{array}$

14. $\begin{array}{r} 7 \\ 5\overline{)37} \\ \underline{35} \\ 2 \end{array}$
16. $\begin{array}{r} 9 \\ 4\overline{)38} \\ \underline{36} \\ 2 \end{array}$
18. $\begin{array}{r} 7 \\ 8\overline{)60} \\ \underline{56} \\ 4 \end{array}$
20. $\begin{array}{r} 9 \\ 7\overline{)68} \\ \underline{63} \\ 5 \end{array}$
22. $\begin{array}{r} 9 \\ 8\overline{)77} \\ \underline{72} \\ 5 \end{array}$
24. $\begin{array}{r} 7 \\ 8\overline{)62} \\ \underline{56} \\ 6 \end{array}$

Solve the problems.

25. For a game, 19 children form teams of 4. How many children will not be able to play during one game?
26. If you put 40 dots into 6 rows of the same size, how many dots will there be in each row and how many dots will be left over?
27. Sixty-five children are going on a class trip by car. If each car holds 7 children, how many cars will be needed?
28. Mrs. Jones washes 42 shirts every week. If each member of her family has a clean shirt every day, how many people are there in her family?

## Two step division

$7\overline{)105}$

Basic division facts alone do not tell us how many sevens are in 105.

$$\begin{array}{r|l} 7\overline{)105} & \\ 70 & 10 \\ \hline 35 & \\ 35 & 5 \\ \hline 0 & 15 \end{array}$$

We can subtract at least 10 sevens from 105.
$105 - (10 \times 7) = 35$

We can subtract 5 more sevens.
$35 - (5 \times 7) = 0$

There are $10 + 5 = 15$ sevens in 105.
The quotient is 15.
We see that $105 - (15 \times 7) = 0$.

### EXERCISES

Write the working forms that go with the equations.

1. $75 - (15 \times 5) = 0$
2. $78 - (13 \times 6) = 0$
3. $96 - (12 \times 8) = 0$
4. $136 - (17 \times 8) = 0$
5. $128 - (16 \times 8) = 0$
6. $126 - (18 \times 7) = 0$

Write the equations that go with the working forms.

7. $4\overline{)52}$

| | |
|---|---|
| 40 | 10 |
| 12 | |
| 12 | 3 |
| 0 | 13 |

9. $5\overline{)95}$

| | |
|---|---|
| 50 | 10 |
| 45 | |
| 45 | 9 |
| 0 | 19 |

11. $7\overline{)112}$

| | |
|---|---|
| 70 | 10 |
| 42 | |
| 42 | 6 |
| 0 | 16 |

13. $9\overline{)162}$

| | |
|---|---|
| 90 | 10 |
| 72 | |
| 72 | 8 |
| 0 | 18 |

8. $3\overline{)48}$

| | |
|---|---|
| 30 | 10 |
| 18 | |
| 18 | 6 |
| 0 | 16 |

10. $8\overline{)104}$

| | |
|---|---|
| 80 | 10 |
| 24 | |
| 24 | 3 |
| 0 | 13 |

12. $6\overline{)108}$

| | |
|---|---|
| 60 | 10 |
| 48 | |
| 48 | 8 |
| 0 | 18 |

14. $8\overline{)160}$

| | |
|---|---|
| 80 | 10 |
| 80 | |
| 80 | 10 |
| 0 | 20 |

Copy and complete the divisions.

15. $4\overline{)76}$

| | |
|---|---|
| 40 | $a$ |
| 36 | |
| 36 | $e$ |
| 0 | $c$ |

16. $6\overline{)90}$

| | |
|---|---|
| 60 | $n$ |
| 30 | |
| 30 | $e$ |
| 0 | $r$ |

17. $8\overline{)96}$

| | |
|---|---|
| 80 | $s$ |
| 16 | |
| 16 | $u$ |
| 0 | $m$ |

18. $9\overline{)135}$

| | |
|---|---|
| 90 | $z$ |
| $v$ | |
| $x$ | $m$ |
| 0 | $n$ |

19. $8\overline{)152}$

| | |
|---|---|
| 80 | $a$ |
| $e$ | |
| $u$ | $r$ |
| 0 | $s$ |

## Checking divisions

We may check our divisions by multiplying.

$$\begin{array}{r|l} 7\overline{)105} & \\ 70 & 10 \\ \hline 35 & \\ 35 & 5 \\ \hline 0 & 15 \end{array}$$

The working form shows that the quotient is 15 and the remainder is 0.

$$\begin{aligned} 10 \times 7 &= 70 \\ 5 \times 7 &= 35 \\ \hline 15 \times 7 &= 105 \end{aligned}$$

The equations show that the product of 15 and 7 is 105.

### EXERCISES

Name the quotients. Then check your work.

| | | | | |
|---|---|---|---|---|
| **1.** $4\overline{)44}$ | **5.** $4\overline{)72}$ | **9.** $8\overline{)144}$ | **13.** $7\overline{)77}$ | **17.** $8\overline{)120}$ |
| **2.** $3\overline{)39}$ | **6.** $8\overline{)112}$ | **10.** $9\overline{)162}$ | **14.** $9\overline{)99}$ | **18.** $6\overline{)120}$ |
| **3.** $5\overline{)65}$ | **7.** $6\overline{)72}$ | **11.** $7\overline{)119}$ | **15.** $5\overline{)90}$ | **19.** $5\overline{)130}$ |
| **4.** $7\overline{)91}$ | **8.** $9\overline{)144}$ | **12.** $9\overline{)153}$ | **16.** $9\overline{)171}$ | **20.** $4\overline{)112}$ |

Solve the problems.

Sam, Jerry, Carol, Ida, Steve, and Missy went to a carnival.

**21.** Jerry spent 75¢ for 5 rides on the ferris wheel. How much did one ride cost?

**22.** Carol, Ida, and Missy each spent 35¢ to ride through the Haunted House. How much did they spend in all?

**23.** Sam and Steve each paid 15¢ for a hot dog, 10¢ for a bottle of pop, and 5¢ for a bag of potato chips. How much did they spend in all?

**24.** A carnival man told them that 162 people ride the Rocket to the Moon each night. If the Rocket takes 9 people at a time, how many times does it go each night?

**25.** The children bought 6 toy monkeys at the Souvenir Booth for 90¢. How much did each monkey cost?

## Using the basic facts in division

Study the methods for naming the quotient $105 \div 7$.

**A**

$$\begin{array}{r|l} 7\overline{)105} & \\ 49 & 7 \\ \hline 56 & \\ 35 & 5 \\ \hline 21 & \\ 21 & 3 \\ \hline 0 & 15 \end{array}$$

**B**

$$\begin{array}{r|l} 7\overline{)105} & \\ 63 & 9 \\ \hline 42 & \\ 42 & 6 \\ \hline 0 & 15 \end{array}$$

**C**

$$\begin{array}{r|l} 7\overline{)105} & \\ 70 & 10 \\ \hline 35 & \\ 35 & 5 \\ \hline 0 & 15 \end{array}$$

Which do you think is the easiest method?

### EXERCISES

Copy and complete the divisions.

1. $\begin{array}{r|l} 4\overline{)68} & \\ 32 & 8 \\ \hline 36 & \\ 36 & 9 \\ \hline 0 & a \end{array}$

3. $\begin{array}{r|l} 5\overline{)80} & \\ 40 & b \\ \hline 40 & \\ 40 & c \\ \hline 0 & x \end{array}$

5. $\begin{array}{r|l} 7\overline{)84} & \\ 63 & e \\ \hline 21 & \\ 21 & r \\ \hline 0 & g \end{array}$

7. $\begin{array}{r|l} 7\overline{)98} & \\ 28 & 4 \\ \hline 70 & \\ 70 & w \\ \hline 0 & x \end{array}$

9. $\begin{array}{r|l} 9\overline{)180} & \\ 90 & e \\ \hline 90 & \\ 90 & 10 \\ \hline 0 & z \end{array}$

2. $\begin{array}{r|l} 8\overline{)112} & \\ 32 & n \\ \hline 80 & \\ 80 & a \\ \hline 0 & p \end{array}$

4. $\begin{array}{r|l} 9\overline{)153} & \\ 81 & c \\ \hline 72 & \\ 36 & r \\ \hline 36 & \\ 36 & 4 \\ \hline 0 & s \end{array}$

6. $\begin{array}{r|l} 6\overline{)114} & \\ 24 & 4 \\ \hline 90 & \\ 60 & n \\ \hline 30 & \\ 30 & u \\ \hline 0 & v \end{array}$

8. $\begin{array}{r|l} 9\overline{)108} & \\ 45 & h \\ \hline 63 & \\ 45 & a \\ \hline 18 & \\ 18 & 2 \\ \hline 0 & s \end{array}$

10. $\begin{array}{r|l} 6\overline{)96} & \\ 24 & k \\ \hline 72 & \\ 36 & e \\ \hline 36 & \\ 36 & 6 \\ \hline 0 & m \end{array}$

Name the quotients.

**11.** $4\overline{)40}$ **13.** $5\overline{)50}$ **15.** $6\overline{)90}$ **17.** $7\overline{)133}$ **19.** $5\overline{)100}$

**12.** $4\overline{)48}$ **14.** $5\overline{)70}$ **16.** $8\overline{)104}$ **18.** $9\overline{)144}$ **20.** $7\overline{)140}$

Complete the sentences.

**21.** A set of 96 can be separated into ___?___ subsets of 8.

**22.** A set of 88 can be separated into ___?___ subsets of 8.

**23.** A set of 102 can be separated into ___?___ subsets of 6.

Study the methods for naming the quotient $175 \div 7$.

**A**

$$\begin{array}{r|l} 7)175 & \\ 70 & 10 \\ \hline 105 & \\ 70 & 10 \\ \hline 35 & \\ 35 & 5 \\ \hline 0 & 25 \end{array}$$

**B**

$$\begin{array}{r|l} 7)175 & \\ 140 & 20 \\ \hline 35 & \\ 35 & 5 \\ \hline 0 & 25 \end{array}$$

Method **B** is easier because it has fewer steps.

## EXERCISES

Copy and complete the divisions.

**1.** $\begin{array}{r|l} 4)92 & \\ 40 & a \\ \hline 52 & \\ 40 & 10 \\ \hline 12 & \\ 12 & r \\ \hline 0 & c \end{array}$

**3.** $\begin{array}{r|l} 5)135 & \\ 50 & n \\ \hline 85 & \\ 50 & 10 \\ \hline 35 & \\ 35 & e \\ \hline 0 & m \end{array}$

**5.** $\begin{array}{r|l} 3)72 & \\ 30 & v \\ \hline 42 & \\ 30 & 10 \\ \hline 12 & \\ 12 & x \\ \hline 0 & z \end{array}$

**7.** $\begin{array}{r|l} 7)182 & \\ 70 & a \\ \hline 112 & \\ 70 & 10 \\ \hline 42 & \\ 42 & c \\ \hline 0 & e \end{array}$

**9.** $\begin{array}{r|l} 8)200 & \\ 80 & m \\ \hline 120 & \\ 80 & 10 \\ \hline 40 & \\ 40 & n \\ \hline 0 & s \end{array}$

**2.** $\begin{array}{r|l} 6)132 & \\ 120 & 20 \\ \hline 12 & \\ 12 & e \\ \hline 0 & s \end{array}$

**4.** $\begin{array}{r|l} 4)112 & \\ 80 & r \\ \hline 32 & \\ 32 & s \\ \hline 0 & u \end{array}$

**6.** $\begin{array}{r|l} 9)234 & \\ 180 & u \\ \hline 54 & \\ 54 & 6 \\ \hline 0 & v \end{array}$

**8.** $\begin{array}{r|l} 8)232 & \\ 160 & w \\ \hline 72 & \\ 72 & 9 \\ \hline 0 & x \end{array}$

**10.** $\begin{array}{r|l} 4)148 & \\ 120 & 30 \\ \hline 28 & \\ 28 & r \\ \hline 0 & z \end{array}$

Name the quotients.

**11.** $4\overline{)92}$ **13.** $7\overline{)154}$ **15.** $5\overline{)140}$ **17.** $8\overline{)184}$ **19.** $6\overline{)144}$

**12.** $7\overline{)203}$ **14.** $9\overline{)189}$ **16.** $8\overline{)216}$ **18.** $6\overline{)180}$ **20.** $9\overline{)261}$

Solve the problems.

**21.** How many students will be on each team if 240 students are separated into 8 teams? 6 teams? 5 teams?

**22.** If a parking lot has 9 rows of the same length, how many cars are in each row if the total number of cars in the lot is 162? 198? 252?

## Division with remainders

Study the example.

$$\begin{array}{r|l} 7\overline{)108} & \\ 70 & 10 \\ \hline 38 & \\ 35 & 5 \\ \hline 3 & 15 \end{array}$$

$10 \times 7 = 70 \qquad 108 - 70 = 38$

$5 \times 7 = 35 \qquad 38 - 35 = 3$

$15 \times 7 = 105 \qquad 108 - (15 \times 7) = 3$

The quotient is 15 and the remainder is 3.

### EXERCISES

Copy and complete the divisions.

**1.** $\begin{array}{r|l} 4\overline{)65} & \\ 40 & 10 \\ \hline 25 & \\ 24 & 6 \\ \hline a & s \end{array}$

**3.** $\begin{array}{r|l} 8\overline{)94} & \\ 80 & 10 \\ \hline 14 & \\ 8 & 1 \\ \hline c & a \end{array}$

**5.** $\begin{array}{r|l} 5\overline{)88} & \\ 50 & 10 \\ \hline 38 & \\ 35 & 7 \\ \hline e & r \end{array}$

**7.** $\begin{array}{r|l} 6\overline{)81} & \\ 60 & 10 \\ \hline 21 & \\ 18 & 3 \\ \hline n & m \end{array}$

**9.** $\begin{array}{r|l} 7\overline{)114} & \\ 70 & 10 \\ \hline 44 & \\ 42 & 6 \\ \hline v & x \end{array}$

**2.** $\begin{array}{r|l} 8\overline{)153} & \\ 80 & 10 \\ \hline 73 & \\ 72 & 9 \\ \hline z & a \end{array}$

**4.** $\begin{array}{r|l} 9\overline{)125} & \\ 90 & 10 \\ \hline 35 & \\ 27 & 3 \\ \hline m & n \end{array}$

**6.** $\begin{array}{r|l} 6\overline{)143} & \\ 120 & 20 \\ \hline 23 & \\ 18 & 3 \\ \hline a & x \end{array}$

**8.** $\begin{array}{r|l} 9\overline{)247} & \\ 180 & 20 \\ \hline 67 & \\ 63 & 7 \\ \hline u & r \end{array}$

**10.** $\begin{array}{r|l} 5\overline{)172} & \\ 150 & 30 \\ \hline 22 & \\ 20 & 4 \\ \hline s & e \end{array}$

Write each of these in the working form.

**11.** $125 - (15 \times 8) = 5$

**12.** $73 - (18 \times 4) = 1$

**13.** $115 - (16 \times 7) = 3$

**14.** $121 - (13 \times 9) = 4$

**15.** $110 - (18 \times 6) = 2$

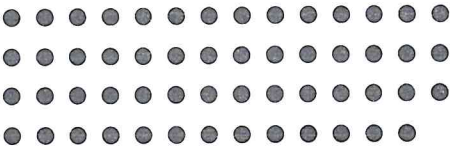
**16.** $222 - (24 \times 9) = 6$

Name the quotients and remainders.

**17.** $5\overline{)93}$ **19.** $7\overline{)99}$ **21.** $8\overline{)117}$ **23.** $3\overline{)68}$ **25.** $6\overline{)100}$ **27.** $9\overline{)175}$

**18.** $4\overline{)105}$ **20.** $6\overline{)154}$ **22.** $5\overline{)118}$ **24.** $8\overline{)194}$ **26.** $7\overline{)178}$ **28.** $9\overline{)300}$

*29. Write a multiplication or a division equation to represent the number of dots.



Copy and complete the divisions.

**1.** $8\overline{)112}$

| | |
|---|---|
| 80 | 10 |
| 32 | |
| 32 | 4 |
| 0 | $a$ |

**3.** $5\overline{)85}$

| | |
|---|---|
| 50 | 10 |
| 35 | |
| 35 | $e$ |
| 0 | $c$ |

**5.** $6\overline{)116}$

| | |
|---|---|
| 60 | 10 |
| 56 | |
| 54 | 9 |
| $n$ | $e$ |

**7.** $7\overline{)125}$

| | |
|---|---|
| 70 | 10 |
| 55 | |
| 49 | 7 |
| $r$ | $s$ |

**9.** $9\overline{)140}$

| | |
|---|---|
| 90 | 10 |
| 50 | |
| 45 | $v$ |
| $u$ | $x$ |

**2.** $6\overline{)144}$

| | |
|---|---|
| 120 | 20 |
| 24 | |
| 24 | 4 |
| 0 | $z$ |

**4.** $8\overline{)196}$

| | |
|---|---|
| 160 | 20 |
| 36 | |
| 32 | 4 |
| $m$ | $n$ |

**6.** $7\overline{)192}$

| | |
|---|---|
| 140 | $a$ |
| 52 | |
| 49 | $u$ |
| $s$ | $r$ |

**8.** $9\overline{)205}$

| | |
|---|---|
| 180 | $s$ |
| 25 | |
| 18 | $t$ |
| $u$ | $v$ |

**10.** $4\overline{)144}$

| | |
|---|---|
| 120 | $w$ |
| 24 | |
| 24 | $x$ |
| 0 | $a$ |

Name the quotients and remainders.

**11.** $4\overline{)36}$ **14.** $7\overline{)38}$ **17.** $5\overline{)65}$ **20.** $7\overline{)91}$ **23.** $9\overline{)115}$ **26.** $9\overline{)108}$

**12.** $6\overline{)72}$ **15.** $6\overline{)75}$ **18.** $5\overline{)68}$ **21.** $7\overline{)96}$ **24.** $9\overline{)129}$ **27.** $8\overline{)100}$

**13.** $3\overline{)40}$ **16.** $3\overline{)41}$ **19.** $3\overline{)42}$ **22.** $8\overline{)96}$ **25.** $9\overline{)132}$ **28.** $9\overline{)135}$

Complete each sentence. Write $<$, $>$, or $=$ to replace the ◯.

**29.** $8 - 4 \bigcirc 8 \div 2$

**30.** $(7 \times 5) + (14 \times 5) \bigcirc 20 \times 5$

**31.** $112 - (18 \times 6) \bigcirc 4$

**32.** $56 \div 8 \bigcirc 54 \div 9$

**33.** $144 \div 9 \bigcirc 144 \div 8$

**34.** $85 \div 5 \bigcirc 4 \times (2 + 2)$

**35.** $9 + 9 + 9 \bigcirc 9 \times 3$

**36.** $24 \div 2 \bigcirc 8 \times 3$

Name a multiple of 10 to make each sentence true.

**37.** $3 \times a < 34$

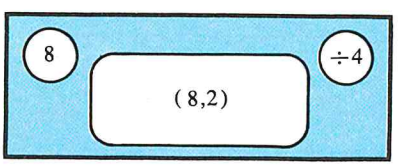
**38.** $7 \times b > 6000$

**39.** $(2 + 3) \times 100 = 5 \times c$

**40.** $(10 + 7) \times d > 1500$

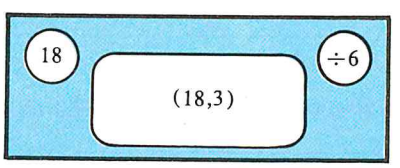
Study each TV and complete the number pairs in the set.

**41.**




{(8, 2), (4, 1), (16, $a$), (24, $b$), (40, $c$), (64, $d$), (36, $e$)}

**42.**




{(18, 3), (6, 1), (12, $f$), (60, $g$), (48, $h$), (84, $i$), (120, $j$)}

## Function rules

> A **function** is a set of ordered pairs together with a rule. If we know the first member of a pair, the rule tells us how to find the second member.

The rule for this function is *add 5*: $\{(1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$.
If $m$ stands for the first member of any pair and $n$ for the second member of the pair, then any number pair can be written $(m, n)$.
We can write an equation for the rule, like this: $m + 5 = n$.
The first members, $m$, belong to the set $1, 2, 3, 4, 5, \ldots$.
We can use the equation, $m + 5 = n$, to find the second members, $n$, like this: $\{(1 + 5), (2 + 5), (3 + 5), (4 + 5), (5 + 5), \ldots\}$.

### EXERCISES

Use the rule to complete the number pairs in each set.

1. $m + 3 = n$: $\{(2, a), (4, b), (7, c), (15, d), (29, e), (816, f)\}$
2. $m - 3 = n$: $\{(7, g), (14, h), (23, i), (87, j), (301, k), (496, l)\}$
3. $3 \times m = n$: $\{(2, x), (6, y), (7, p), (18, q), (21, r), (46, s)\}$
4. $m \div 3 = n$: $\{(6, t), (21, u), (12, v), (42, w), (54, x), (96, z)\}$
5. $m + 6 = n$: $\{(13, a), (18, b), (46, c), (71, d), (89, e), (109, f)\}$
6. $m - 6 = n$: $\{(6, g), (16, h), (27, i), (41, j), (38, k), (104, l)\}$
7. $5 \times m = n$: $\{(2, x), (4, y), (6, p), (16, q), (18, r), (48, s)\}$
8. $m \div 4 = n$: $\{(4, t), (8, u), (16, v), (36, w), (68, x), (168, z)\}$
9. $m \times 4 = n$: $\{(4, a), (5, b), (7, c), (9, d), (12, e), (24, f)\}$

Use the rule $3 \times m = n$ to complete this chart.

10.

| $m$ | 2 | 4 | 9 | 12 | 16 | $r$ | $s$ | $u$ | 26 | $x$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 6 | $a$ | $c$ | $e$ | $f$ | 21 | 48 | 54 | $v$ | 72 |

Use the rule $m \div 6 = n$ to complete this chart.

11.

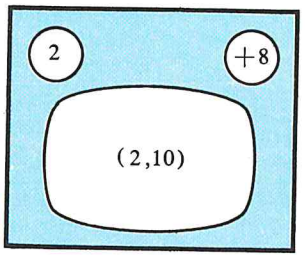
| $m$ | 6 | 18 | 36 | 24 | 54 | 48 | 66 | 72 | $v$ | $x$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 1 | $a$ | $c$ | $e$ | $f$ | $r$ | $s$ | $u$ | 7 | 15 |

Galen said, "For any number you name, I can name one that is 5 greater." Galen was using the function rule "$m + 5 = n$." The number pairs are $\{(1, 6)\ (2, 7)\ (3, 8)\ (4, 9)\ (5, 10), \ldots\}$

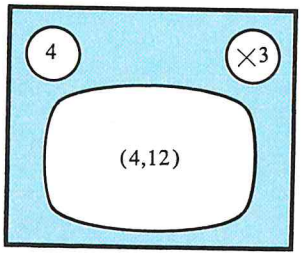
## EXERCISES

Write equations to give the rules for the functions.

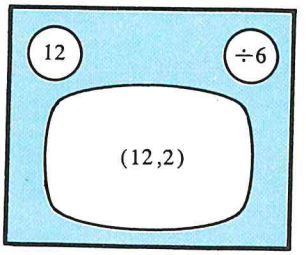
**1.**




**2.**




**3.**




Use the rule to write each function as a set of ordered pairs. The numbers in the set are the first members of the pairs.

**4.** $m + 6 = n$: $\{1, 2, 12, 18, 34, 46, 57\}$

**5.** $m + 17 = n$: $\{2, 6, 18, 21, 48, 67, 385\}$

**6.** $5 \times m = n$: $\{4, 3, 18, 16, 27, 39, 428\}$

**7.** $m \div 7 = n$: $\{21, 42, 63, 126, 182, 245, 364\}$

**8.** $m + 12 = n$: $\{2, 6, 18, 42, 56, 38, 29\}$

**9.** $m - 12 = n$: $\{14, 12, 30, 26, 48, 39, 452\}$

**10.** $6 \times m = n$: $\{5, 7, 19, 21, 38, 46, 248\}$

**11.** $m \div 8 = n$: $\{24, 16, 40, 384, 464, 584, 736\}$

Solve the problems.

**12.** Betty said, "I have 4 more pennies than Jackie. Together we have 40 pennies." How many pennies did Jackie have?

**13.** Hank and Heather shared 60 cents. Heather got twice as much as Hank. How much did Hank get?

***14.** Sarah had 10 pennies more than Mary. Mary had 12 pennies. How many pennies did the girls have in all?

***15.** Together Bob and Bill had 36 pennies. Bob had 6 pennies more than Bill. How many pennies did Bill have?

## High mountains

| | | |
|---|---|---|
| Asia | Mt. Everest | 29,000 |
| South America | Mt. Aconcagua | 22,800 |
| North America | Mt. McKinley | 20,300 |
| Africa | Mt. Kilimanjaro | 19,300 |
| Europe | Mt. Elborus | 18,500 |
| Antarctica | Mt. Vinson | 16,900 |
| Australia | Mt. Kosciusko | 7300 |

### EXERCISES

1. Mt. Everest is the highest mountain in the world. Mt. McKinley is the highest mountain in North America. How much higher than Mt. McKinley is Mt. Everest?
2. Mt. Vinson is the highest mountain in Antarctica and Mt. Kosciusko is the highest in Australia. Is the sum of the heights of these two mountains greater than or less than the height of Mt. Everest?
3. How much higher than Mt. Vinson is Mt. Kilimanjaro?
4. How much higher than Mt. Elborus is Mt. McKinley?
5. **a.** We can say that the height of Mt. Everest is **about 30,000 feet** and that the height of Mt. Kilimanjaro is **about 20,000 feet.** Use these heights to find **about how much** higher Mt. Everest is than Mt. Kilimanjaro.

   **b.** Using the heights at the top of the page, how much higher than Mt. Kilimanjaro is Mt. Everest?
6. Make a list of other famous mountains and their heights. Can you find other interesting number facts about mountains?

## Sharpshooting

Answer set **A** as quickly as you can. Then see if you can answer **B** even more quickly.

**A**

1. There are 513 girls and 546 boys in the school. How many more boys than girls are there?
2. What is two thirds of 15?
3. $378 + 4 + 6 + 9 + 8 + 2 = n$
4. $(3 \times 7) + 2 = n$
5. Write the next fractions, $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{a}{c}, \frac{d}{e}, \frac{f}{g}.\right\}$
6. $476 \div 7 = n$
7. What is the difference between $(8 \times 8)$ and $(7 \times 9)$?
8. If a dozen buns cost 72¢, what is the cost per bun?
9. $(4 \times 5) + (3 \times 9) = n$
10. Use the rule $n = (2 \times m) + 5$ to complete these ordered pairs: $\{(1, 7), (2, 9), (3, x), (4, y)\}$.
11. Name the product of 7 and 13.
12. $72 - 12 = n$

**B**

1. Name the sum of 64,829 and 35,423.
2. What is the cost of 75 ping pong balls at 8¢ each?
3. $1291 - 468 = n$
4. $(5 \times 8) - 3 = n$
5. $\frac{3}{4} = \frac{n}{40}$
6. Use the rule $n = (3 \times m) + 2$ to name $n$ when $m$ is 5.
7. Name the number 14 greater than 147.
8. $\frac{16}{24} = \frac{n}{3}$
9. What is the area of a rectangular region 6 inches long and 15 inches wide?
10. If it costs \$163 to travel by airplane and \$129 to travel by train, which is cheaper? By how much?
11. What is the cost of 6 ties at 75¢ each?
12. $894 + 7 + 8 + 6 + 9 = n$

### Be Careful!

Bob said, "I can take 1 from 10 and get zero."
Helen said, "I can take 1 from 1000 and get zero."

1. Using an idea like this, what do you get when you take zero from 10? zero from 1000?
2. Are you subtracting numbers or are you taking away digits?
3. Is place value involved?

## Statements

| A | B |
|---|---|
| George likes rice. | George does not like rice. |
| $4 + 8 = 12$ | $4 + 8 \neq 12$ |
| $11 > 10$ | $11 \not> 10$ |
| $12 < 16$ | $12 \not< 16$ |

The sign $\neq$ means **is not equal to.**
The sign $\not>$ means **is not greater than.**
The sign $\not<$ means **is not less than.**

The statements on the right disagree with those on the left.

### EXERCISES

Pair the statements that disagree.

1. An ostrich has 8 legs.
2. $6 \neq 4 + 2$
3. Karen likes mathematics.
4. $3 + 3 = 6$
5. A triangle has 3 sides.
6. My teacher is Mr. Brown.

a. $6 = 4 + 2$
b. My teacher is not Mr. Brown.
c. Karen does not like mathematics.
d. $6 \neq 3 + 3$
e. A triangle does not have 3 sides.
f. An ostrich does not have 8 legs.

Make a statement to disagree with

7. It is raining.
8. He lives in a cave.
9. $3 + 5 = 8$
10. $6 < 9$
11. We are going to school.
12. $14 > 12$

Pair the statements that disagree.

13. Nobody lives in caves.
14. No numbers are even.
15. Everybody lives in caves.
16. No one likes to go to the movies.
17. Everybody likes mathematics.
18. All stones are round.
19. No stones are round.
20. Everybody likes candy.
21. All birds fly.

a. Some people live in caves.
b. Sally likes to go to the movies.
c. You do not live in a cave.
d. Two is an even number.
e. Fred does not like candy.
f. Bob does not like mathematics.
g. Some stones are round.
h. This stone is not round.
i. An ostrich cannot fly.

| | | | | |
|---|---|---|---|---|
| Today is Thursday. | $p = 4$ | $q = 4$ | $r < 4$ | $s = 5$ |
| Today is Friday. | $p < 4$ | $q > 4$ | $r > 4$ | $s = 4$ |

The statements in each pair disagree. Both statements in any pair cannot be true at the same time.

## EXERCISES

Pair the statements that disagree.

**1.** $3 + 7 = 10$
**2.** No number can be even.
**3.** $17 < 18$
**4.** 4 is a factor of 12.
**5.** $3 \times 5 = 16$
**6.** 18 is another name for $2 \times 9$.
**7.** There are 5 tens in 50.
**8.** $24 = 6 \times 5$

**a.** 2 is an even number.
**b.** $24 = 6 \times 4$
**c.** $17 > 18$
**d.** 18 is not equal to $9 \times 2$.
**e.** 12 is not a multiple of 4.
**f.** $15 = 3 \times 5$
**g.** $3 + 7 > 10$
**h.** There are 7 tens in fifty.

**9–16.** List the eight true statements in exercises 1 to 8.

Make a true statement to disagree with

**17.** $4 + 6 > 10$
**18.** $7 \times 3 = 4 \times 6$
**19.** $83 = 30 + 8$
**20.** Half of 16 is 4.
**21.** $3 < 2$
**22.** 1 and 0 are the only factors of 0.
**23.** $\{1, 2, 3\}$ is a subset of $\{4, 5, 6\}$.
**24.** There are 25 hours in every day.
**25.** $72 \div 8 = 8$

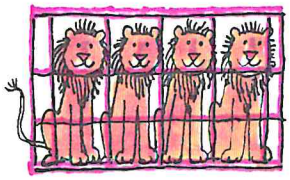
Make a false statement to disagree with

**26.** $4 + 6 = 10$
**27.** $19 < 5 \times 4$
**28.** There are 7 days in a week.
**29.** $7864 > 7863$
**30.** One third of 12 is 4.
**31.** $4 \times 3 = 2 \times 6$
**32.** The product of 7 and 5 is 35.
**33.** A square is a kind of rectangle.

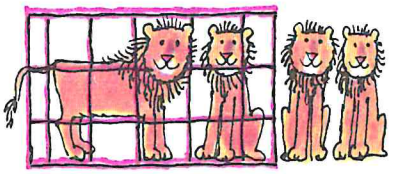
Complete the sentence.

**34.** If there are 2 numbers $p$ and $q$, $p$ must be equal to $q$, less than $q$, or ___?___ than $q$.

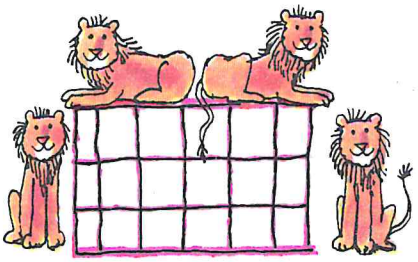
## All, some, none



**All** of the lions are in the cage.



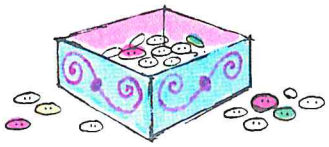
**Some** of the lions are in the cage.



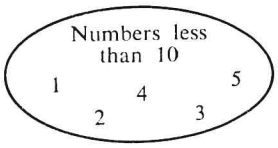
**None** of the lions are in the cage.

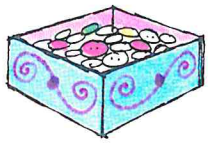
### EXERCISES

Complete each sentence with *all*, *some*, or *none*.

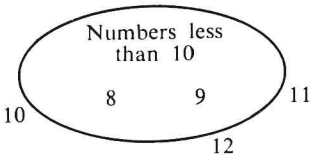


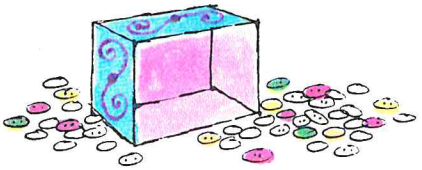
**1.** ___?___ of the buttons are in the box.



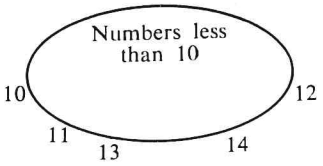



**2.** ___?___ of the buttons are in the box.

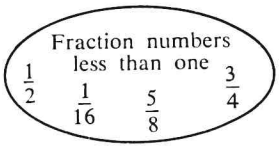


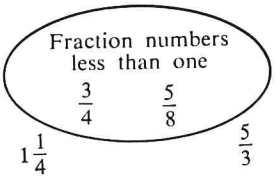
**3.** ___?___ of the buttons are in the box.



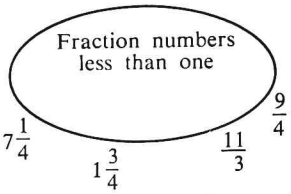

**4.** ___?___ of the numbers 1, 2, 3, 4, and 5 are less than 10.




**5.** ___?___ of the numbers 8, 9, 10, 11 and 12 are less than 10.




**6.** ___?___ of the numbers 10, 11, 12, 13, and 14 are less than 10.




**7.** ___?___ of the numbers $\frac{1}{16}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ are less than 1.

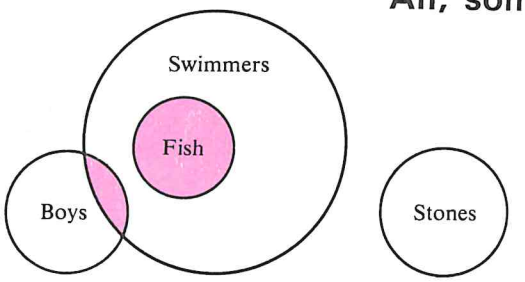
**8.** ___?___ of the numbers $\frac{3}{4}$, $\frac{5}{8}$, $\frac{5}{3}$, and $1\frac{1}{4}$ are less than 1.

**9.** ___?___ of the numbers $\frac{9}{4}$, $1\frac{3}{4}$, $7\frac{1}{4}$, and $\frac{11}{3}$ are less than 1.

## All, some, none

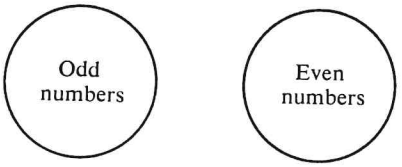
This diagram shows:

**All** fish swim.
**Some** boys swim.
**Some** boys do not swim.
**No** stones swim.

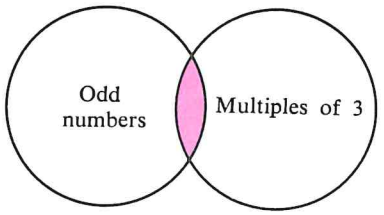



## EXERCISES

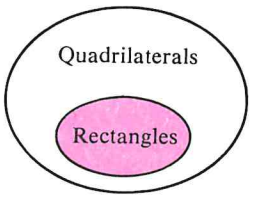
Study the diagrams. Write *T* or *F* to show whether each statement is *true* or *false*



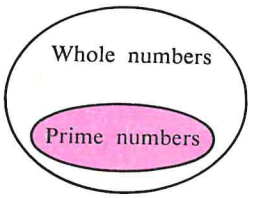

1. No even numbers are odd.
2. No odd numbers are even.
3. Some odd numbers are even.



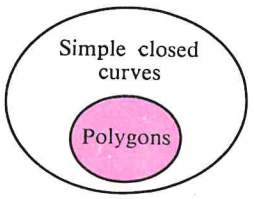

4. Some multiples of 3 are odd numbers.
5. Some odd numbers are multiples of 3.
6. All odd numbers are multiples of 3.




7. All rectangles are quadrilaterals.
8. Some rectangles are not quadrilaterals.




9. All prime numbers are whole numbers.
10. No prime numbers are whole numbers.



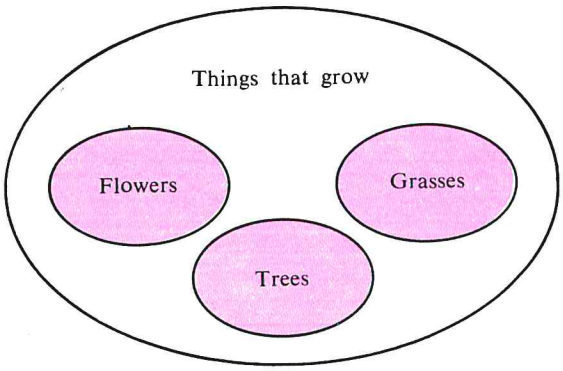

11. All polygons are simple closed curves.
12. Some polygons are not simple closed curves.

## Sets and subsets

This diagram shows:

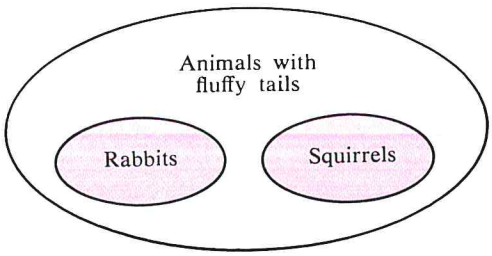
**All** flowers grow.
**All** grasses grow.
**All** trees grow.

The diagram also shows that there *may be* other things that grow that are not flowers, grasses, or trees.

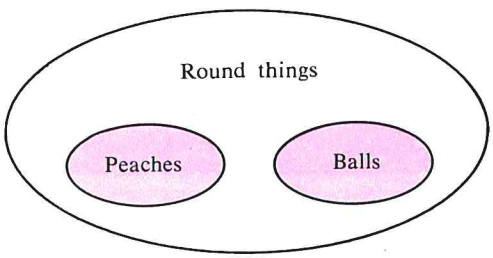



## EXERCISES

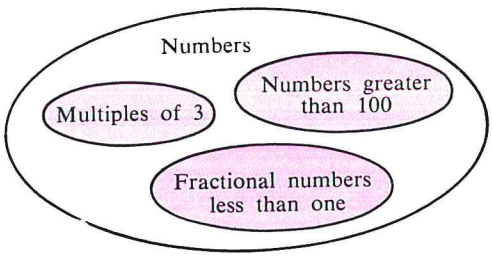
Study the diagrams. Write *T* or *F* to show whether each statement is *true* or *false*.



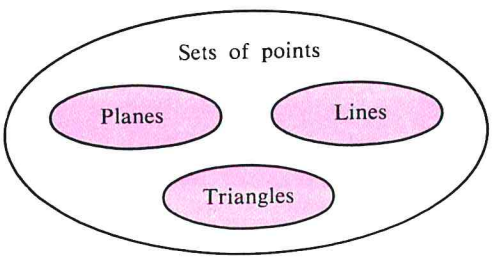

1. All rabbits have fluffy tails.
2. All squirrels have fluffy tails.
3. All animals with fluffy tails must be rabbits.
4. All animals with fluffy tails must be rabbits or squirrels.




5. All peaches are round.
6. All balls are round.
7. All round things must be peaches or balls.
8. All round things must be balls.




9. Any multiple of 3 is a number.
10. Any number is less than 100.
11. Any number must be a multiple of 3, a fractional number less than 1, or a number greater than 100.

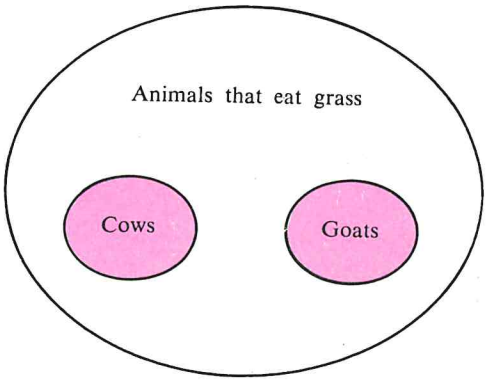



12. Any line is a set of points.
13. Any plane is a set of points.
14. Any set of points is a line or a triangle.
15. Any set of points is a plane or a line.

The diagram shows:

All cows and all goats eat grass.

Animals that eat grass may be cows or goats or some other kind of animal.



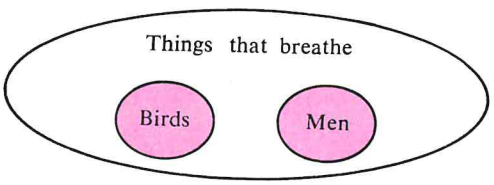

We can also say:

**If** an animal is a cow or a goat,
**then** it eats grass.

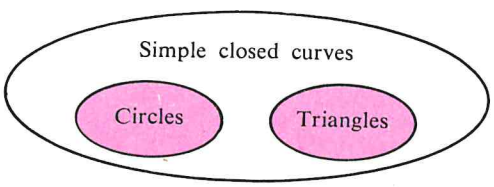
**If** an animal eats grass,
**then** it may be a cow or a goat
or it may be some other kind of animal.

## EXERCISES

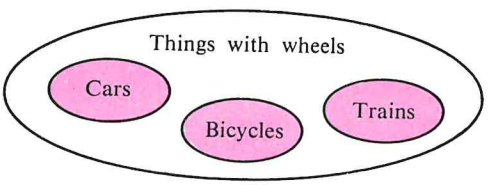
Study the diagrams. Write *T* or *F* to show whether each statement is *true* or *false*.



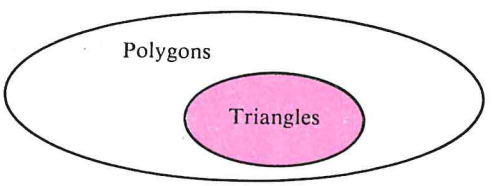

1. All birds breathe.
2. If it is a bird, then it breathes.
3. If it breathes, then it must be a bird or a man.
4. If it breathes, then it must be a bird.




5. If it is a circle, then it is a simple closed curve.
6. If it is a simple closed curve, then it is a circle.
7. If it is a triangle, then it is a simple closed curve.

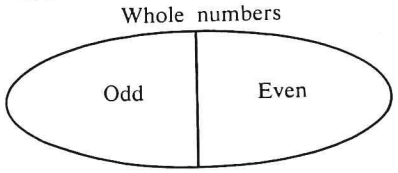



8. All cars have wheels.
9. If it is a car, then it has wheels.
10. If it is a bicycle, then it has wheels.
11. If it has wheels, then it must be a car.

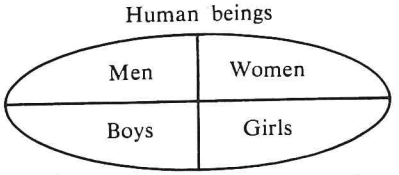



12. If it is a triangle, then it is a polygon.
13. If it is a polygon, then it is a triangle.
14. If it is not a polygon, then it is not a triangle.

## Sets and subsets



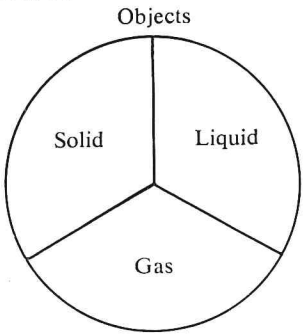

The diagram shows that a whole number must be either even or odd.



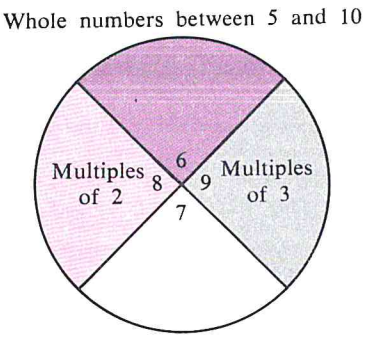

The diagram shows that a human being must be either a man, a woman, a boy, or a girl.

## EXERCISES

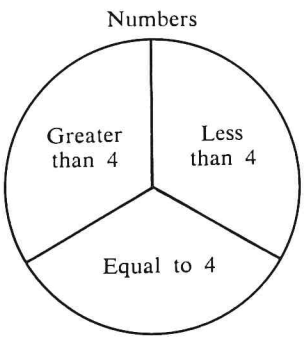
Study the diagrams. Write *T* or *F* to show whether each statement is *true* or *false*.




1. An object must be either a solid, a liquid, or a gas.
2. If an object is not a liquid or a gas, then it must be a solid.
3. If an object is not a gas, then it must be either a solid or a liquid.




4. A number between 5 and 10 must be either 7 or a multiple of 2 or of 3.
5. If a number between 5 and 10 is not a multiple of 2 or of 3, then it must be 7.
6. If a number between 5 and 10 is not 7 and is not a multiple of 2, then it is not a multiple of 3.



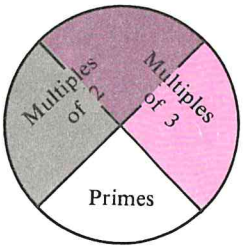

7. A number must be greater than 4, less than 4, or equal to 4.
8. If a number is not equal to 4 or less than 4, then it must be greater than 4.
9. If a number is not greater than 4 and not equal to 4, then it is not less than 4.

The diagram shows that

if a whole number between 3 and 20 is a prime,

then it cannot be a multiple of 2 or of 3.

Whole numbers between 3 and 20




## EXERCISES

Use the diagram above to help you decide whether each number is prime.

**1.** 12 **2.** 13 **3.** 15 **4.** 16 **5.** 17 **6.** 19

Use the diagrams to help you answer the questions.

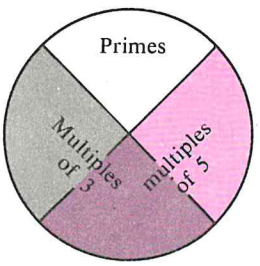
**7.** If an odd number between 6 and 30 is not a multiple of 3 or of 5, is it a prime?

**8.** Is 23 a prime?

**9.** Is 27 a prime?

**10.** Is 29 a prime?

Odd numbers between 6 and 30




**11.** If a number does not leave a remainder of 1, of 2, or of 3 when divided by 4, is it a multiple of 4?

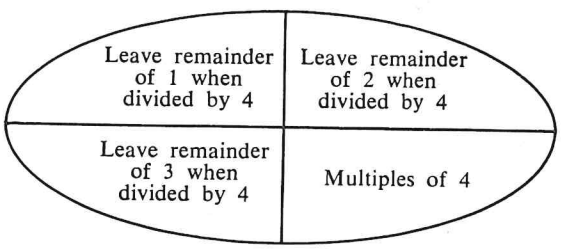
**12.** Is 25 a multiple of 4?

**13.** Is 26 a multiple of 4?

**14.** Is 27 a multiple of 4?

**15.** Is 28 a multiple of 4?

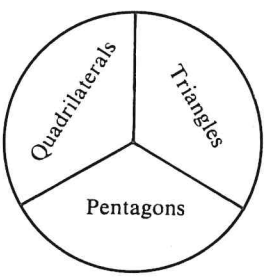
Whole numbers


Leave remainder of 1 when divided by 4
Leave remainder of 2 when divided by 4
Leave remainder of 3 when divided by 4
Multiples of 4

**16.** If a polygon has fewer than 6 sides and is not a triangle or a pentagon, is it a quadrilateral?

**17.** If a polygon is not a triangle, a quadrilateral, or a pentagon, what is the least number of sides it must have?

Polygons with fewer than 6 sides



## Division

We may use this method to complete the division $378 \div 27$.

$$
\begin{array}{r|l}
27\overline{)378} & \\
243 & 9 \\
\hline
135 & \\
135 & 5 \\
\hline
0 & 14
\end{array}
\qquad
\begin{aligned}
&\longrightarrow \quad 9 \times 27 = 243 \\
&\longrightarrow \quad 378 - 243 = 135 \\
&\longrightarrow \quad 5 \times 27 = 135 \\
&\longrightarrow \quad 135 - 135 = 0
\end{aligned}
$$

$9 + 5 = 14$, so $378 \div 27 = 14$

### EXERCISES

Copy and complete the divisions.

**1.**
$$
\begin{array}{r|l}
26\overline{)338} & \\
260 & 10 \\
\hline
78 & \\
78 & a \\
\hline
0 & e
\end{array}
$$

**2.**
$$
\begin{array}{r|l}
43\overline{)344} & \\
258 & 6 \\
\hline
86 & \\
86 & c \\
\hline
0 & r
\end{array}
$$

**3.**
$$
\begin{array}{r|l}
36\overline{)468} & \\
288 & 8 \\
\hline
180 & \\
180 & e \\
\hline
0 & s
\end{array}
$$

**4.**
$$
\begin{array}{r|l}
47\overline{)564} & \\
470 & 10 \\
\hline
94 & \\
94 & u \\
\hline
0 & v
\end{array}
$$

**5.**
$$
\begin{array}{r|l}
46\overline{)598} & \\
a & 10 \\
\hline
c & \\
e & 3 \\
\hline
0 & 13
\end{array}
$$

**6.**
$$
\begin{array}{r|l}
54\overline{)756} & \\
n & 10 \\
\hline
r & \\
s & 4 \\
\hline
0 & 14
\end{array}
$$

**7.**
$$
\begin{array}{r|l}
32\overline{)512} & \\
u & 10 \\
\hline
v & \\
x & 6 \\
\hline
0 & 16
\end{array}
$$

**8.**
$$
\begin{array}{r|l}
45\overline{)540} & \\
z & 10 \\
\hline
a & \\
e & 2 \\
\hline
0 & 12
\end{array}
$$

Name the quotients.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **9.** | $25\overline{)375}$ | **13.** | $27\overline{)405}$ | **17.** | $36\overline{)432}$ | **21.** | $44\overline{)704}$ | **25.** | $82\overline{)984}$ |
| **10.** | $34\overline{)612}$ | **14.** | $56\overline{)952}$ | **18.** | $64\overline{)832}$ | **22.** | $28\overline{)420}$ | **26.** | $49\overline{)833}$ |
| **11.** | $24\overline{)456}$ | **15.** | $37\overline{)481}$ | **19.** | $48\overline{)720}$ | **23.** | $53\overline{)636}$ | **27.** | $62\overline{)806}$ |
| **12.** | $74\overline{)814}$ | **16.** | $28\overline{)448}$ | **20.** | $87\overline{)870}$ | **24.** | $42\overline{)756}$ | **28.** | $63\overline{)819}$ |

Solve the problem.

**29.** If the 69 children in the two fourth grades are making 828 name cards for Parents' Night, how many cards must each child make?

In this division there is a remainder.
$390 - (14 \times 27) = 12$

$$\begin{array}{r|l} 27\overline{)390} & \\ 270 & 10 \\ \hline 120 & \\ 108 & 4 \\ \hline 12 & 14 \end{array}$$

The quotient is 14 and the remainder is 12. $12 < 27$.

## EXERCISES

Copy and complete the divisions.

**1.** $26\overline{)350}$

| | |
|---|---|
| 260 | 10 |
| 90 | |
| 78 | $a$ |
| $e$ | $c$ |

**2.** $43\overline{)450}$

| | |
|---|---|
| 258 | 6 |
| 192 | |
| 172 | $n$ |
| $e$ | $r$ |

**3.** $36\overline{)480}$

| | |
|---|---|
| 288 | 8 |
| 192 | |
| 180 | $s$ |
| $u$ | $v$ |

**4.** $47\overline{)580}$

| | |
|---|---|
| 470 | 10 |
| 110 | |
| 94 | $x$ |
| $z$ | $a$ |

**5.** $46\overline{)517}$

| | |
|---|---|
| $a$ | 10 |
| $c$ | |
| $e$ | 1 |
| $n$ | 11 |

**6.** $54\overline{)775}$

| | |
|---|---|
| $e$ | 10 |
| $r$ | |
| $s$ | 4 |
| $u$ | 14 |

**7.** $32\overline{)526}$

| | |
|---|---|
| $v$ | 10 |
| $x$ | |
| $z$ | 6 |
| $c$ | 16 |

**8.** $45\overline{)554}$

| | |
|---|---|
| $m$ | 10 |
| $n$ | |
| $a$ | 2 |
| $e$ | 12 |

Name the quotients and remainders.

**9.** $25\overline{)390}$ **11.** $27\overline{)420}$ **13.** $36\overline{)440}$ **15.** $44\overline{)710}$ **17.** $69\overline{)584}$

**10.** $34\overline{)640}$ **12.** $56\overline{)965}$ **14.** $64\overline{)850}$ **16.** $28\overline{)440}$ **18.** $75\overline{)950}$

Solve the equations.

**19.** $567 - (17 \times 33) = a$
**20.** $632 - (11 \times 56) = c$
**21.** $685 - (13 \times 52) = e$
**22.** $569 - (15 \times 37) = n$
**23.** $583 - (14 \times 41) = m$
**24.** $990 - (17 \times 58) = p$
**25.** $450 - (18 \times 24) = r$
**26.** $999 - (12 \times 83) = s$
**27.** $440 - (16 \times 27) = u$
**28.** $613 - (14 \times 43) = v$
**29.** $740 - (13 \times 56) = w$
**30.** $680 - (15 \times 45) = z$

## Division

In naming $621 \div 27$, Bobby noticed

$10 \times 27 = 270$
$20 \times 27 = 2 \times (10 \times 27) = 540$
$30 \times 27 = 3 \times (10 \times 27) = 810$

He knew that the quotient must be between 20 and 30, since $540 < 621$ and $810 > 621$.
His final quotient was 23.

$$\begin{array}{r|l} 27\overline{)621} & \\ 540 & 20 \\ \hline 81 & \\ 81 & 3 \\ \hline 0 & 23 \end{array}$$

### EXERCISES

Copy and complete the divisions.

**1.**
$$\begin{array}{r|l} 23\overline{)782} & \\ 690 & 30 \\ \hline 92 & \\ 92 & a \\ \hline 0 & e \end{array}$$

**2.**
$$\begin{array}{r|l} 43\overline{)903} & \\ 860 & 20 \\ \hline 43 & \\ 43 & c \\ \hline 0 & n \end{array}$$

**3.**
$$\begin{array}{r|l} 38\overline{)874} & \\ 760 & 20 \\ \hline 114 & \\ 114 & r \\ \hline 0 & s \end{array}$$

**4.**
$$\begin{array}{r|l} 28\overline{)896} & \\ 840 & 30 \\ \hline 56 & \\ 56 & u \\ \hline 0 & v \end{array}$$

**5.**
$$\begin{array}{r|l} 43\overline{)903} & \\ a & 20 \\ \hline e & \\ c & 1 \\ \hline 0 & 21 \end{array}$$

**6.**
$$\begin{array}{r|l} 26\overline{)832} & \\ n & 30 \\ \hline e & \\ r & 2 \\ \hline 0 & 32 \end{array}$$

**7.**
$$\begin{array}{r|l} 34\overline{)850} & \\ s & 20 \\ \hline u & \\ z & 5 \\ \hline 0 & 25 \end{array}$$

**8.**
$$\begin{array}{r|l} 28\overline{)868} & \\ a & 30 \\ \hline x & \\ e & 1 \\ \hline 0 & 31 \end{array}$$

Name the quotients.

**9.** $33\overline{)858}$ **11.** $24\overline{)768}$ **13.** $36\overline{)864}$ **15.** $42\overline{)966}$ **17.** $27\overline{)702}$

**10.** $47\overline{)846}$ **12.** $83\overline{)996}$ **14.** $21\overline{)735}$ **16.** $41\overline{)820}$ **18.** $37\overline{)962}$

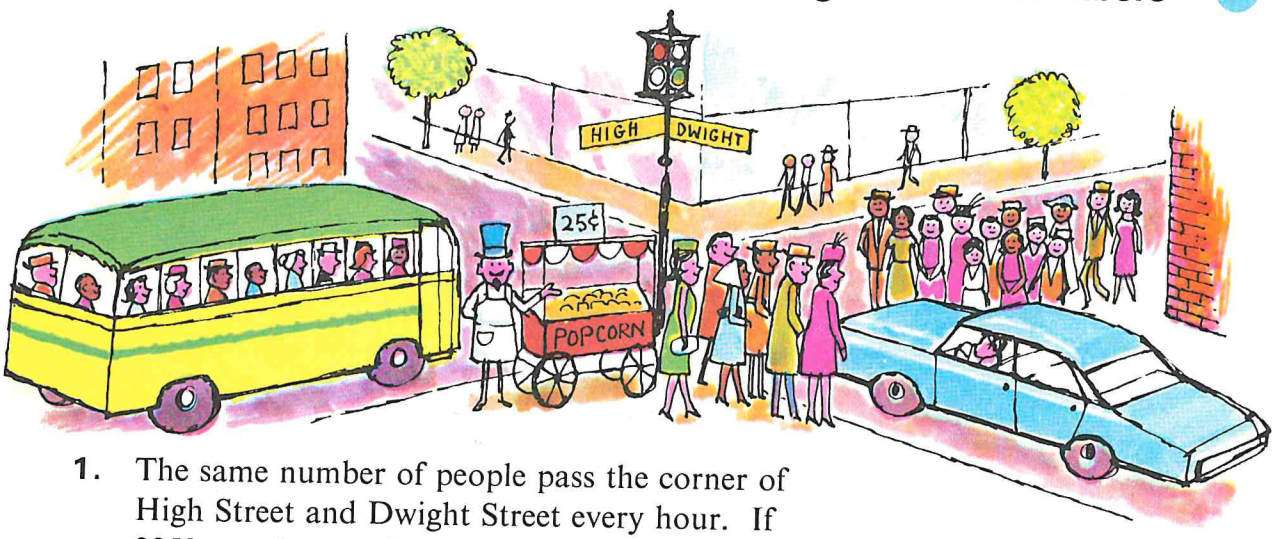
Name the quotients and remainders.

**19.** $33\overline{)860}$ **23.** $36\overline{)876}$ **27.** $27\overline{)726}$ **31.** $19\overline{)486}$ **35.** $27\overline{)683}$

**20.** $31\overline{)987}$ **24.** $43\overline{)975}$ **28.** $22\overline{)957}$ **32.** $61\overline{)978}$ **36.** $32\overline{)846}$

**21.** $24\overline{)775}$ **25.** $42\overline{)981}$ **29.** $24\overline{)783}$ **33.** $52\overline{)989}$ **37.** $44\overline{)628}$

**22.** $28\overline{)593}$ **26.** $38\overline{)961}$ **30.** $25\overline{)351}$ **34.** $23\overline{)641}$ **38.** $71\overline{)897}$




1. The same number of people pass the corner of High Street and Dwight Street every hour. If 2250 people pass the corner in 1 hour, how many people pass the corner in 6 hours?

2. If a box of popcorn costs 25¢, how many boxes can be bought for $7.25?

3. If each bus can carry 48 people, how many buses are needed to carry 384 people?

4. If there are 365 days in 1 year, how many days are in 27 years? (Do not count the extra days for leap years.)

5. If there are 250 words on each page of a book and there are 85 pages in the book, how many words are there in the whole book?

6. If your heart beats 4500 times an hour, how many times does it beat in 8 hours?

7. If each box contains 24 toys, how many boxes are needed to give a toy to each of 480 children?

8. A crate of oranges costs $5.40. If each orange costs 9¢, how many oranges does the crate contain?

9. If there are 12 apples in one bag, how many apples are there in 7 bags?

## CHECKING UP

Name the products.

1. $247 \times 16$
2. $4829 \times 45$
3. $953 \times 98$
4. $836 \times 76$
5. $906 \times 54$
6. $4238 \times 67$

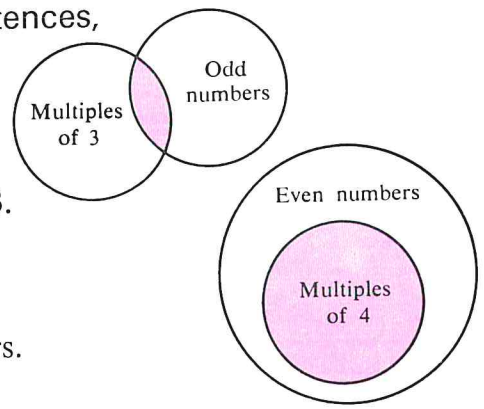
Name the quotients and remainders.

7. $6\overline{)448}$
8. $7\overline{)382}$
9. $4\overline{)998}$
10. $16\overline{)384}$
11. $24\overline{)386}$
12. $18\overline{)910}$
13. $43\overline{)586}$
14. $39\overline{)895}$
15. $73\overline{)792}$
16. $46\overline{)835}$

Use the rule to complete the number pairs in each set.

17. $m + 7 = n$: $\{(4, 11), (7, a), (11, b), (18, c), (23, d), (49, e)\}$
18. $m - 15 = n$: $\{(25, 10), (37, f), (47, g), (15, h), (51, i), (90, j)\}$
19. $m \times 4 = n$: $\{(5, 20), (7, k), (0, l), (18, x), (24, y), (32, z)\}$
20. $m \div 5 = n$: $\{(15, 3), (30, p), (75, q), (90, r), (200, s), (5, t)\}$

Study the diagrams. Then complete the sentences, using *all*, *some*, or *no*.




21. ___?___ multiples of 3 are odd numbers.
22. ___?___ odd numbers are multiples of 3.
23. ___?___ odd numbers are not multiples of 3.
24. ___?___ multiples of 4 are even numbers.
25. ___?___ even numbers are multiples of 4.
26. ___?___ multiples of 4 are not even numbers.

Solve the problem.

27. What is the cost of 75 bats if each bat costs \$2.75?

Complete each sentence. Write $<$ $>$ or $=$ to replace the ◯.

28. $123 \div 3 \bigcirc 5 \times 8$
29. $846 \div 9 \bigcirc 2 \times 48$
30. $345 \times 16 \bigcirc 421 \times 12$
31. $166 - 67 \bigcirc 9 \times 11$
32. $17 \times 68 \bigcirc 34 \times 34$
33. $321 + 849 \bigcirc 42 \times 45$
34. $29 \times 29 \bigcirc 31 \times 27$
35. $12 \times (13 + 27) \bigcirc 12 \times 40$
36. $387 \div 9 \bigcirc 3 \times 14$
37. $191 - 119 \bigcirc 171 - 117$

To name the product $4 \times 638$, we may use the method shown here.

$$\begin{array}{r} 638 \\ \times\ 4 \\ \hline 2400 \\ 120 \\ 32 \\ \hline 2552 \end{array}$$

Copy and complete the multiplications.

**1.**
$$\begin{array}{r} 673 \\ \times\ 3 \\ \hline 1800 \\ 210 \\ 9 \\ \hline \end{array}$$

**2.**
$$\begin{array}{r} 542 \\ \times\ 4 \\ \hline 2000 \\ 160 \\ 8 \\ \hline \end{array}$$

**3.**
$$\begin{array}{r} 489 \\ \times\ 6 \\ \hline 2400 \\ 480 \\ 54 \\ \hline \end{array}$$

**4.**
$$\begin{array}{r} 394 \\ \times\ 5 \\ \hline 1500 \\ 450 \\ 20 \\ \hline \end{array}$$

**5.**
$$\begin{array}{r} 672 \\ \times\ 8 \\ \hline 4800 \\ 560 \\ e \\ \hline \end{array}$$

Name the products.

**6.** $648 \times 9 = a$ **7.** $836 \times 7 = b$ **8.** $754 \times 6 = c$

To help us multiply we can use doubles. For example, if 246 is a factor, we can use the doubles

$1 \times 246 = 246$
$2 \times 246 = 492$
$4 \times 246 = 984$
$8 \times 246 = 1968$

We multiply $27 \times 246$ like this:

$1 \times 246 = 246$
$2 \times 246 = 492$
$4 \times 246 = 984$
so $7 \times 246 = 1722$
$10 \times 246 = 2460$
$10 \times 246 = 2460$
so $27 \times 246 = 6642$

Use this method to name:

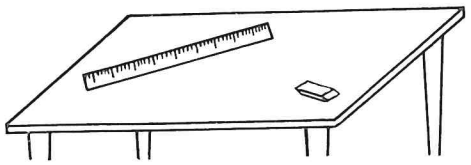
**9.** $13 \times 246$ **11.** $19 \times 564$ **13.** $25 \times 564$ **15.** $19 \times 378$
**10.** $15 \times 246$ **12.** $23 \times 564$ **14.** $25 \times 378$ **16.** $22 \times 378$

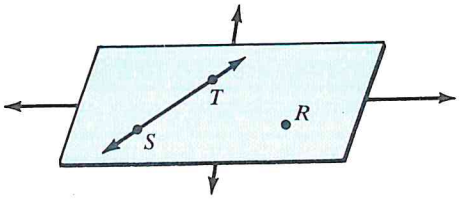
*Just For Fun*

Can you see the trick in this problem?

**17.** There is a ladder on the side of a ship. The rungs on the ladder are 1 foot apart. If there are 8 rungs showing at low tide, how many rungs will be showing when the tide rises 4 feet?

## Points, lines, and planes

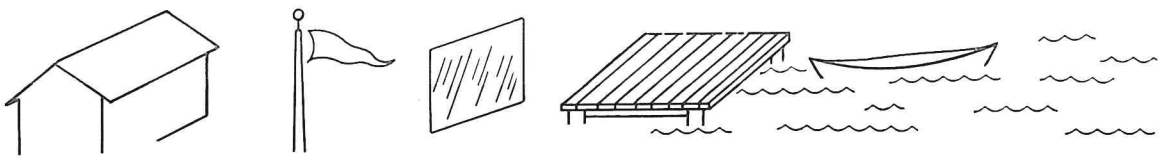





A **plane** is a set of points in a flat surface. The **line** $ST$ and the **point** $R$ are in the blue plane. The arrows show that the plane goes on and on in all directions.

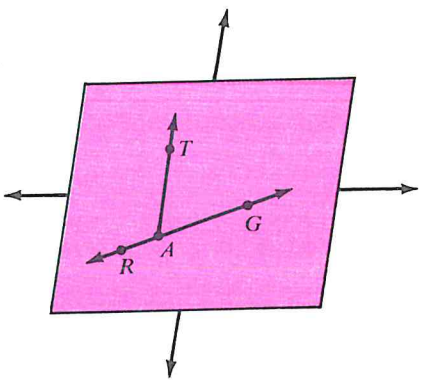
### EXERCISES

Which of the objects in this picture suggest a plane?



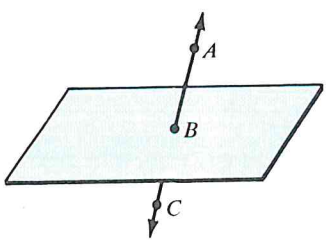
Use the diagrams to help you.

Ray $AT$ and line $RG$ are in the red plane.




1. Name the point common to $\overrightarrow{AT}$ and $\overleftrightarrow{RG}$.
2. Name two other rays in the plane.
3. Name two angles in the plane.
4. Name the vertices of these angles.
5. $\overline{RA}$ is a line segment. Name two other line segments in the plane.

Only one point in line $AC$ is in the green plane. Point $B$ is the intersection of $\overleftrightarrow{AC}$ and the plane.

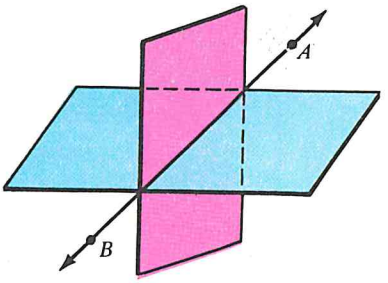



6. Will $\overleftrightarrow{AC}$ and the green plane have any other points but $B$ in common?

Complete the sentence.

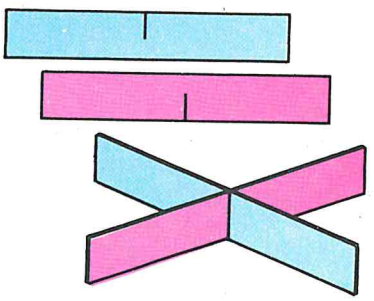
7. The intersection of a plane and a line which is not completely in the plane is a ___?___.

## Working with intersecting planes




The intersection of the blue plane and the red plane is $\overleftrightarrow{AB}$.

## EXERCISES

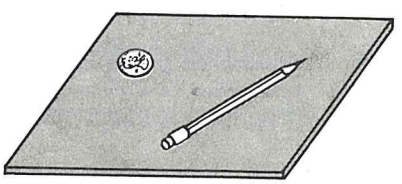


Cut two pieces of cardboard halfway through. Place them together as in the diagram.

1. If you think of these as planes, can you see the line segment formed by the intersection?

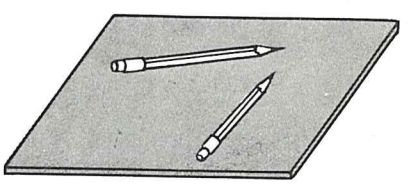
Look at the walls and ceiling of your classroom.

2. What is the intersection of the wall and the ceiling?
3. Can we think of the wall and the ceiling as parts of planes?
4. Find other examples of intersecting planes in your classroom.



Put a penny and a pencil on your desk as in the diagram. Think of them as a point and a line.

5. How many planes can contain *both* the penny and the pencil?



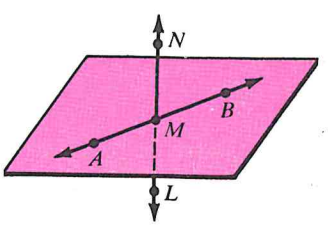
Place two pencils on your desk and think of these as lines.

6. How many planes can contain *both* pencils?

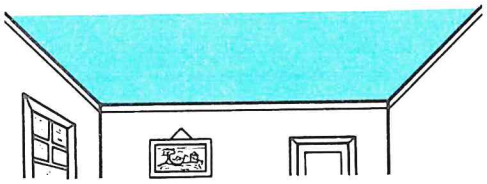
Line $AB$ is in the red plane, but line $NL$ is not.

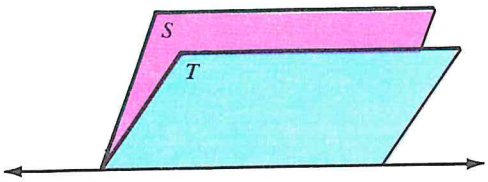
7. Name the intersection of $\overleftrightarrow{NL}$ and the red plane.
8. Name the intersection of $\overleftrightarrow{NL}$ and $\overleftrightarrow{AB}$.

*9. Do you think we can draw a plane which contains both $\overleftrightarrow{AB}$ and $\overleftrightarrow{NL}$?

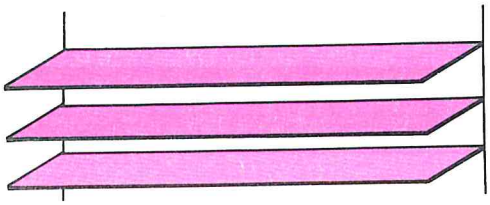


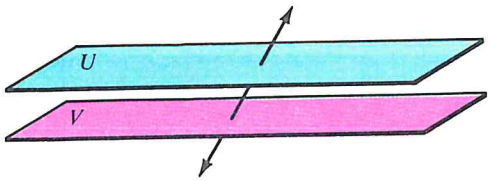
## Working with planes






These planes intersect to form an **angle.**

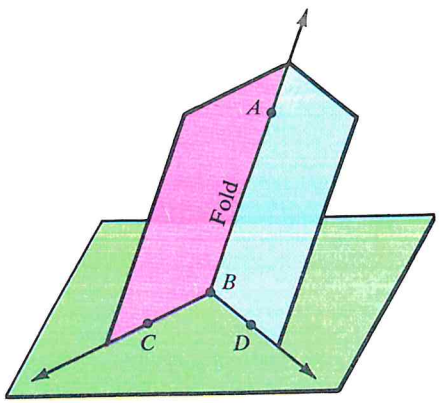





These planes do not intersect to form an angle.
They are **parallel planes.**

## EXERCISES

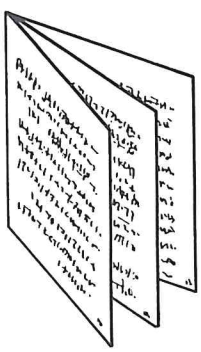
Fold a piece of paper and place it on your desk like this.




1. Can you see three planes?
2. Name the line
   a. common to the red and blue planes
   b. common to the red and green planes
   c. common to the blue and green planes
3. Name the point common to all three planes.

Discuss the questions.

4. What are examples in your classroom of angles formed by planes?
5. What are examples in your classroom of parallel planes?
6. Can three or more planes intersect in a line?
7. Think of the pages of your book as a set of planes. Do all these planes intersect in the same line?

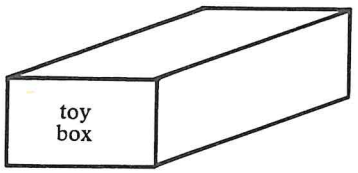


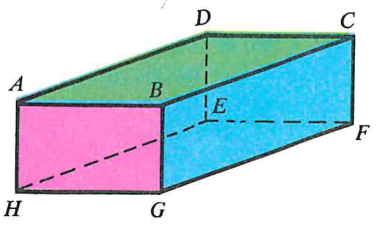
Complete the sentences.

8. Planes that do not meet are ___?___.
9. If two planes meet, they meet in a ___?___.

## Putting planes together

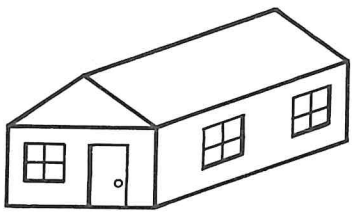
Study the diagrams.

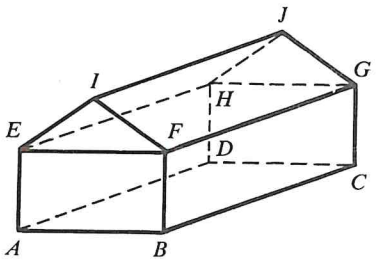



1. How many planes are needed to form a box?
2. Two planes intersect in a line. How many lines are formed by the planes of a box?
3. Three planes meet at a point (called a corner point or a **vertex**). How many corner points are in a box?
4. How many pairs of parallel planes are in a box?

Study the diagrams.





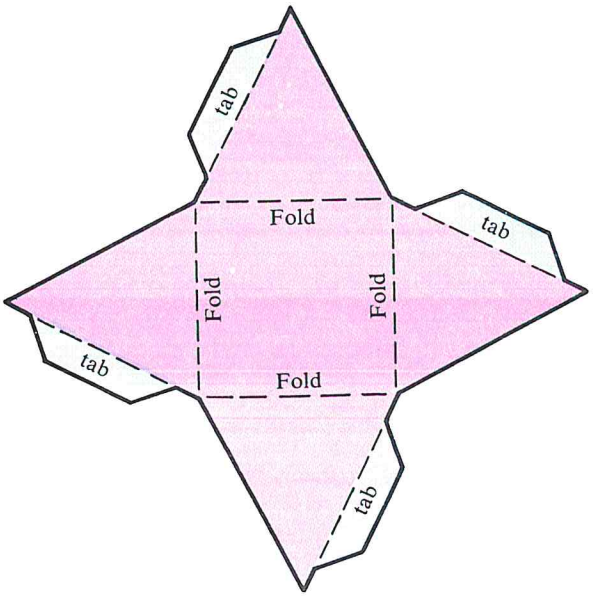

We can name a plane with three points not on the same line.
For example, $ABF$ names the plane at the front of the house.

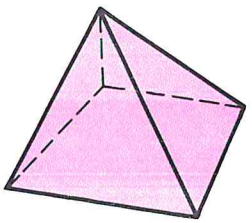
5. Name two planes which intersect in the line $FB$.
6. Name three planes which intersect in the line $FG$.
7. Name the plane parallel to plane $ABC$.
8. Name the plane parallel to plane $DCG$.
9. What point do the planes $AEF$, $FGC$, and $ADC$ have in common?
10. Name four lines which intersect in the point $G$.

*11. Could $\overleftrightarrow{IJ}$ and $\overleftrightarrow{EF}$ be contained in the same plane?

*12. Name four planes which have only the point $F$ in common.

## Space figures



This is a pattern for a **simple closed surface** called a **pyramid.**

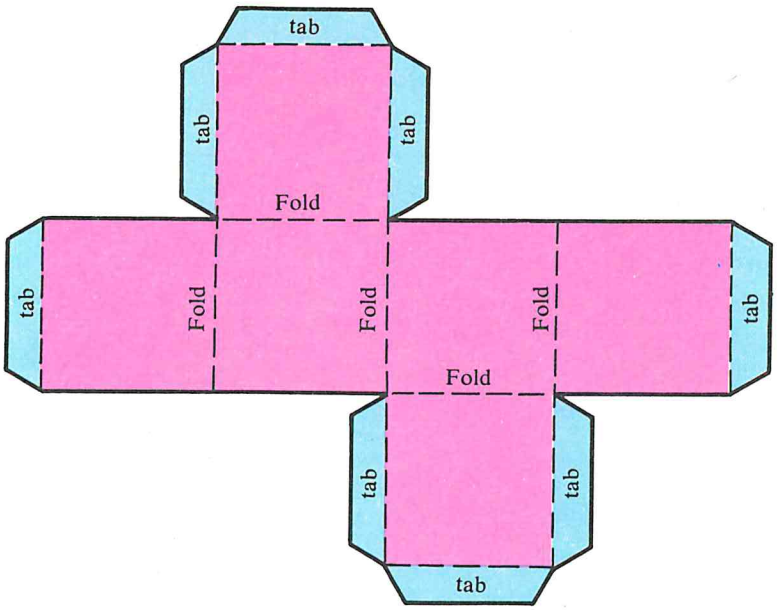
### EXERCISES

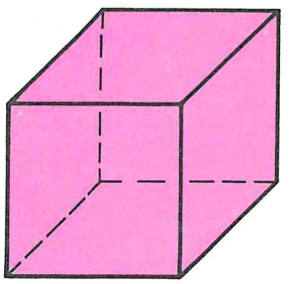
Copy the first figure on a piece of cardboard or heavy paper.
Cut out the pattern and fold along the line segments.
When you have pasted the tabs, your model should look like the second figure.

> The regions of the planes are called **faces** or **sides.**
> The line segments are called **edges.**
> The corner points are called **vertices.**

Study your model of a pyramid.

1. How many planes form a pyramid?
2. How many edges are in a pyramid?
3. How many vertices are in a pyramid?
4. How many faces meet at each corner?
5. What is the shape of each face of the pyramid?
6. What is the length of each edge in your model?



This is a pattern for another **simple closed surface** called a **cube.**

## EXERCISES

Copy the first figure on a piece of cardboard or heavy paper. Cut out your model and fold it as shown. When you have pasted the tabs, your model should look like the second figure.

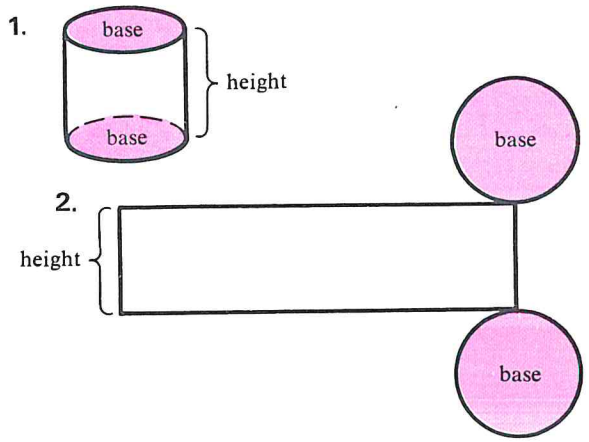
Study your model of a cube.

1. How many planes are needed to form a cube?
2. How many faces are there? How many pairs of faces are parallel?
3. How many edges are there? How many pairs of edges are parallel?
4. How many vertices are there? How many planes meet at each vertex?
5. What is the shape of the faces?
6. What is the measure of each edge? Are all of the edges congruent?

## Cylinders

A can shows another kind of simple closed surface called a **cylinder.** The **bases** are usually circular regions. The **height** is the shortest distance between the bases.

The curved surface can be thought of as a rectangular region rolled to fit the bases.

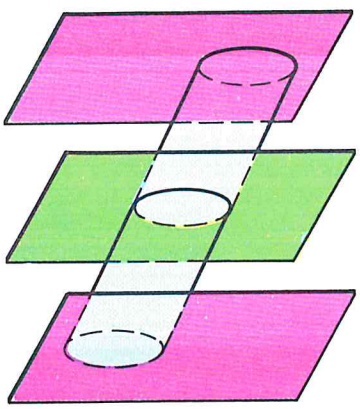



### EXERCISES

Answer the questions about the cylinder.

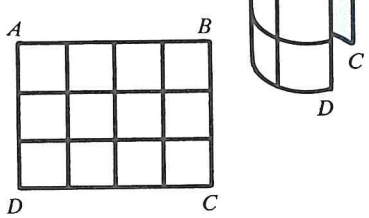
1. How is a cylinder different from a cube or a pyramid?
2. What are some examples of cylinders?
3. Is the curved surface of a cylinder part of a plane?
4. If you set the curved surface of a cylinder on your desk, in how many directions can the cylinder roll?

Study the drawing.

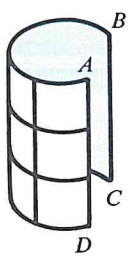


5. Is each base part of a plane?
6. Are the planes of the bases parallel?
7. The green plane is parallel to the red planes and intersects the cylinder. What shape is the intersection of the green plane and the cylinder?

Study the rectangular region.

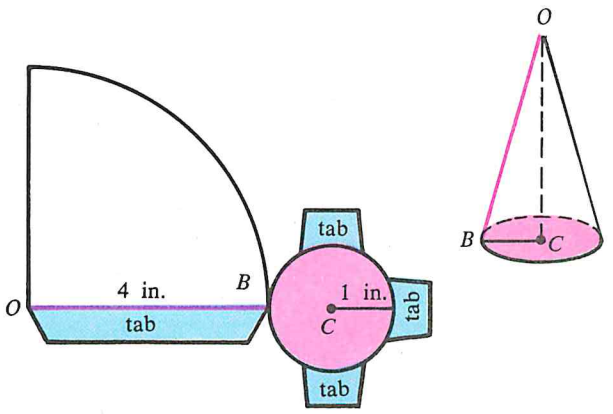



8. What is the area of the rectangular region $ABCD$?
9. If you rolled this rectangle to form the curved surface of a cylinder, what would be the area of the curved surface of the cylinder?
10. To find the total area of the surface of this cylinder, what would you have to include in addition to the area of $ABCD$?



This is a pattern for another simple closed surface called a **cone.** The **base** is a circular region.

$\overline{BC}$ is a radius of the base.
The length of $\overline{OC}$ is the **height** of the cone.

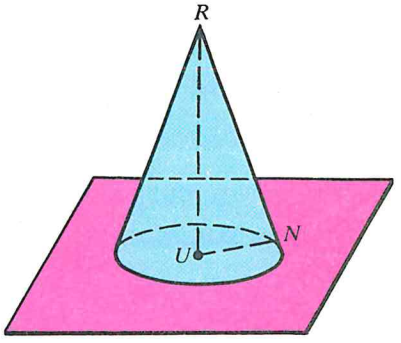



## EXERCISES

Draw the first figure on a piece of cardboard or heavy paper. Make sure $\angle AOB$ is a right angle. When you have cut the circular region and pasted the tabs, your model should look like the second figure.

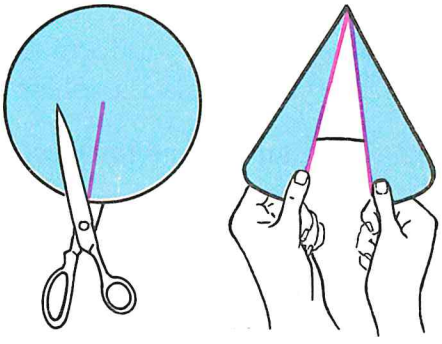
1. How is a cone different from a cube?
2. How is a cone different from a cylinder? How are they alike?

Study the diagram and answer the questions.




3. Does the base of the cone lie in a plane?
4. Can the entire surface of a cone be in a plane?
5. Does $\overline{RU}$ form a right angle with $\overline{UN}$?
6. If a plane parallel to the red plane intersects the cone, what is the shape of the intersection?
7. If you set a cone on its side on your desk and roll it, what kind of path does it take?

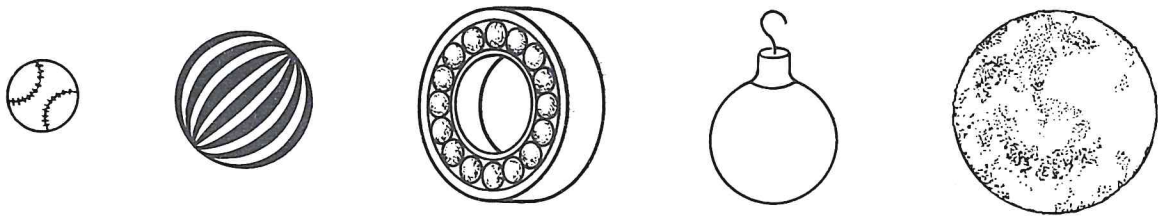
Cut out a circular region from a piece of paper. Cut along a radius and form a cone by sliding one end under the other.



8. What happens to the height of the cone as the diameter of the base becomes less?

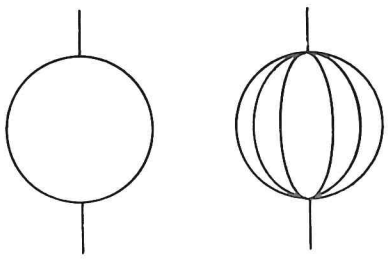
*9. What does the radius of the circle become after the cone is formed?

## Spheres



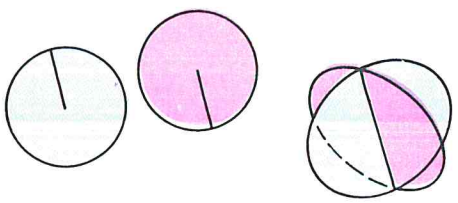
All these figures show a kind of simple closed surface called a **sphere.**

### EXERCISES



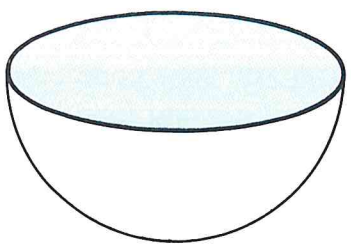
1. If you tie a circle of wire to a string and spin the string, what figure will you form?

2. Does a sphere have a point, like the center of a circle, which is the same distance from every point on the sphere?



3. On a piece of cardboard draw two circles with the same radius. Cut out the circular regions. Cut along a radius of each circle and fit the regions together, as in the diagram. If you fit a number of circular regions together like this, what shape do you begin to form?

4. If you cut across a sphere, what curve do you make?



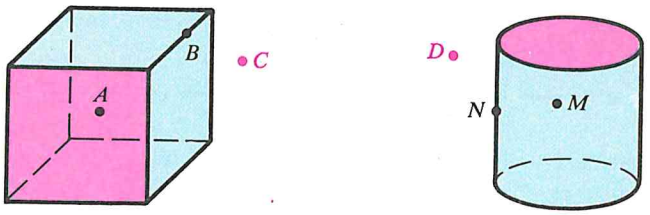
5. What is the intersection of a sphere and a plane? (This is another way to ask question 4.)

*6. How can you form the largest circle when you intersect a sphere and a plane?

7. Can a plane intersect a sphere in a point?

8. Can two spheres intersect
   **a.** in a point? **b.** in a circle? **c.** in a sphere?

9. Explain each of your answers for exercise 8.

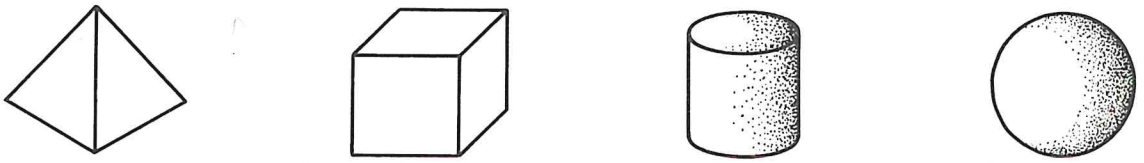



A simple closed surface separates the points in space into three sets:
the points **inside** the surface (for example, $A$ and $M$),
the points **on** the surface (for example, $B$ and $N$),
the points **outside** the surface (for example, $C$ and $D$).

## EXERCISES

Answer the questions.

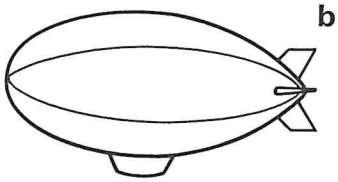
**1.** Is it possible to get inside these simple closed figures without intersecting the surface?



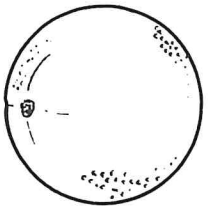
**2.** A man at a fair said, "I have something no one has ever seen before." He showed the people a coconut. They laughed, but then he said, "No one has ever seen the *inside* of this coconut." Was he right?

**3.** What is interesting about the space inside these simple closed surfaces?

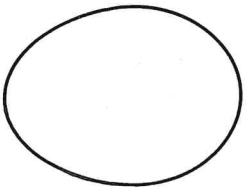
**a.**



**b.**



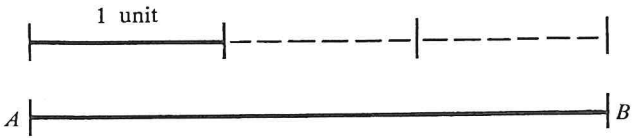
**c.**



Complete the sentence.

**4.** We cannot join a point outside a simple closed surface and a point inside the surface without ___?___ the surface.

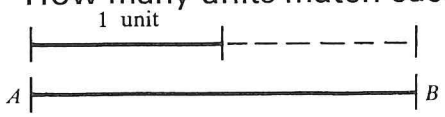
## Measuring line segments



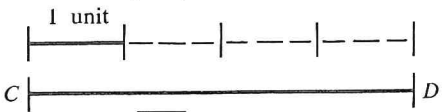

To measure the length of line segments we compare a standard **unit of length,** such as an inch, with the line segment. The measure is the number of units. $m(\overline{AB}) = 3$

### EXERCISES

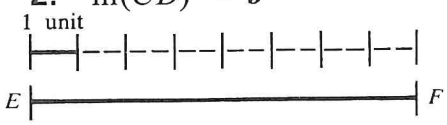
How many units match each line segment?




1. $m(\overline{AB}) = a$

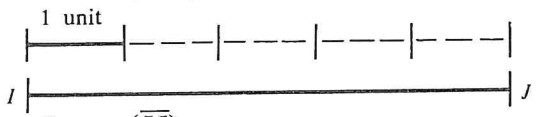



2. $m(\overline{CD}) = b$

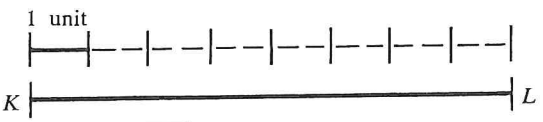



3. $m(\overline{EF}) = c$

4. $m(\overline{GH}) = d$




5. $m(\overline{IJ}) = e$



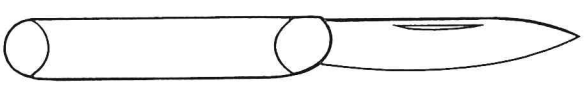

6. $m(\overline{KL}) = f$

Use your ruler to measure the length of each object in inches.

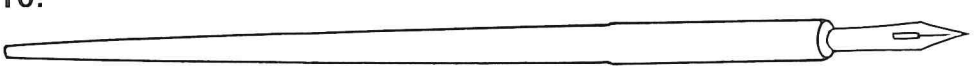
7.

8.

9.

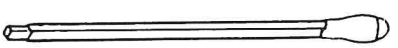


10.

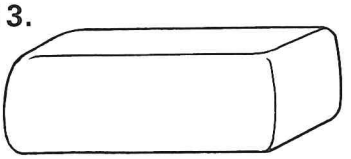


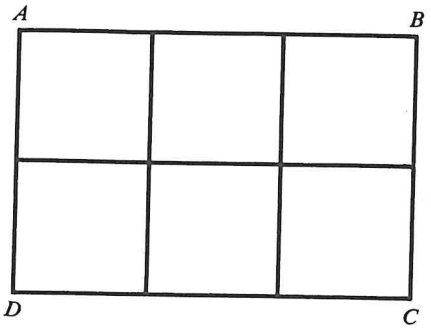
11.

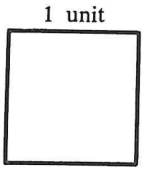
12.



13.





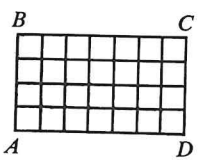

To measure the area of regions, we compare a standard **unit of area** with the region.

There are 2 rows. Each row contains 3 units of area.

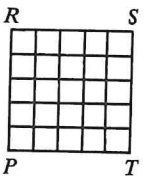
$$m(ABCD) = 2 \times 3 = 6$$

## EXERCISES

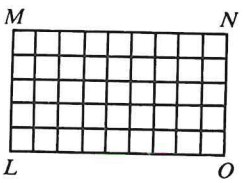
How many square units match each rectangular region?




1. $m(ABCD) = a$




2. $m(PRST) = b$

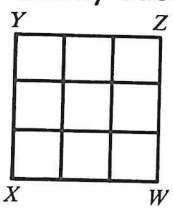


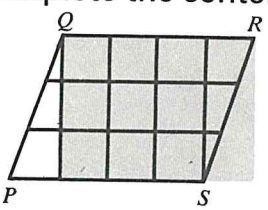

3. $m(LMNO) = c$

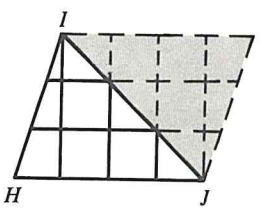
Find the answers.

4. Draw a rectangle with sides of 3 inches and 4 inches. How many **square inches** match your rectangular region?
5. Draw a rectangle with sides of 5 inches and 6 inches. How many **square inches** match your rectangular region?

Study each figure. Then complete the sentences.



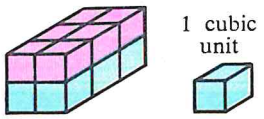



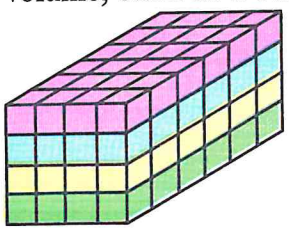

6. The area of $XYZW$ is ___?___ square units.
7. The area of $PQRS$ is ___?___ square units.
8. The area of $HIJ$ is ___?___ square units.

## Finding volume

To measure the **volume** of the space inside a simple closed surface, we compare a standard unit of **volume,** such as a cubic inch, with the space.



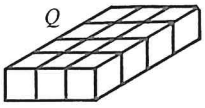

There are 2 layers. Each layer has $(2 \times 3)$ cubic units. The **volume** is 12 cubic units.



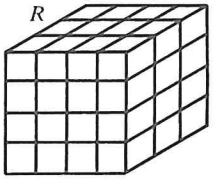
There are 4 layers. Each layer has $(4 \times 6)$ cubic units. The **volume** is 96 cubic units.

### EXERCISES

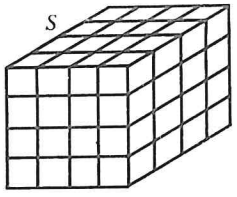
How many cubic units match each simple closed surface?



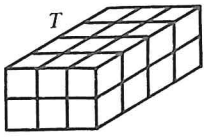

**1.** $m(Q) = a$



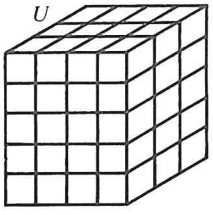

**4.** $m(R) = b$



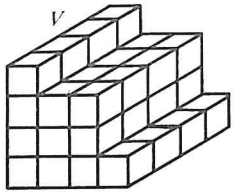

**7.** $m(S) = c$



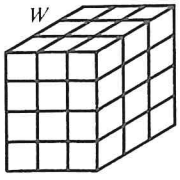

**2.** $m(T) = d$



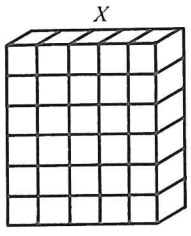

**5.** $m(U) = e$



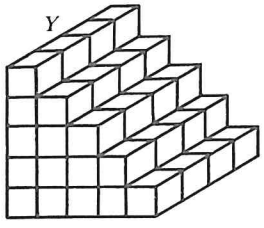

**8.** $m(V) = f$




**3.** $m(W) = g$




**6.** $m(X) = h$



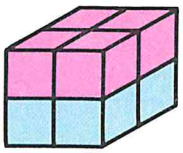

**9.** $m(Y) = i$

A

$m(A) = 1$

The volume of $A$ is 1 cubic unit.

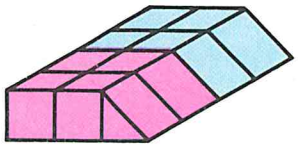
B



$m(B) = 8$

The volume of $B$ is 8 cubic units.

C



$m(C) = 10$

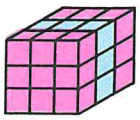
The volume of $C$ is 10 cubic units.

## EXERCISES

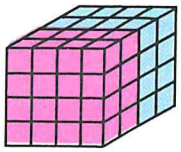
How many cubic units match each cube?

L

M



N

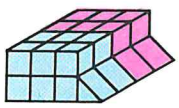


**1.** $m(L) = a$ **2.** $m(M) = b$ **3.** $m(N) = c$

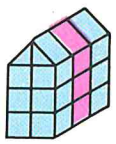
How many cubic units match each simple closed surface?

F

J



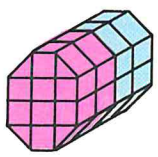
G



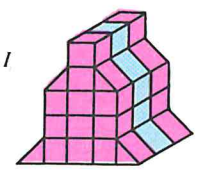
**4.** $m(F) = a$ **6.** $m(J) = c$ **8.** $m(G) = e$

H

K



I

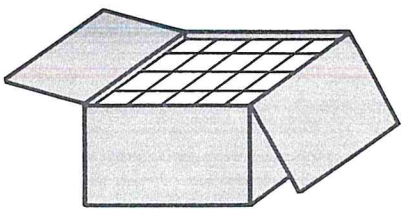


**5.** $m(H) = b$ **7.** $m(K) = d$ **9.** $m(I) = f$

## Using sets and numbers

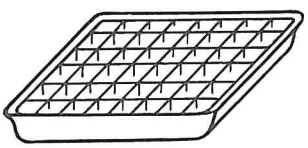
1. Mrs. Jones' kitchen floor is covered with tiles. If there are 10 rows each containing 12 tiles, how many tiles are there in all?

2. Tom collects postage stamps. Each page in his book can hold 8 rows with 7 stamps per row. How many stamps can each page hold?

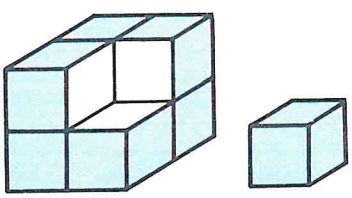


3. Name the greatest number of blocks that will fit in a container that can hold 3 layers of blocks, with 5 rows per layer and 4 blocks per row.

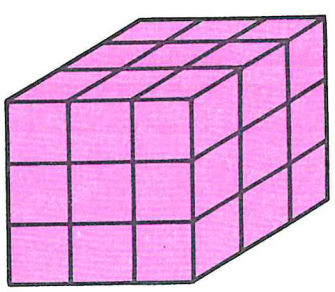
4. Name the greatest number of ice cubes that can fit in a container that can hold 9 layers of ice cubes, with 8 rows per layer and 6 ice cubes per row.



5. How many square tiles, 4 inches per side, can fit in a rectangular floor space that is 60 inches long and 40 inches wide?



6. Jimmy placed 8 blocks together to form a cube and painted the outside blue. Name the number of blocks with
   - **a.** 6 faces blue
   - **b.** 5 faces blue
   - **c.** 4 faces blue
   - **d.** 3 faces blue
   - **e.** fewer than 3 faces blue



7. Jimmy placed 27 blocks together to form a cube and painted the outside red. Name the number of blocks with
   - **a.** 6 faces red
   - **b.** 5 faces red
   - **c.** 4 faces red
   - **d.** 3 faces red
   - **e.** 2 faces red
   - **f.** 1 face red
   - **g.** no faces red

Name the products.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | $36 \times 3$ | **3.** | $69 \times 9$ | **5.** | $46 \times 25$ | **7.** | $38 \times 79$ | **9.** | $471 \times 8$ | **11.** | $496 \times 24$ |
| **2.** | $42 \times 8$ | **4.** | $54 \times 12$ | **6.** | $55 \times 37$ | **8.** | $136 \times 5$ | **10.** | $471 \times 18$ | **12.** | $849 \times 48$ |

Name the quotients and remainders.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **13.** | $4\overline{)64}$ | **15.** | $4\overline{)38}$ | **17.** | $8\overline{)96}$ | **19.** | $7\overline{)168}$ | **21.** | $9\overline{)252}$ | **23.** | $6\overline{)327}$ |
| **14.** | $3\overline{)69}$ | **16.** | $3\overline{)22}$ | **18.** | $8\overline{)76}$ | **20.** | $7\overline{)135}$ | **22.** | $6\overline{)276}$ | **24.** | $9\overline{)855}$ |

Complete each sentence. Write $=$, $<$, or $>$ to replace the ◯.

**25.** $15 \times 15 \bigcirc 14 \times 16$

**26.** $432 + 678 \bigcirc 478 + 632$

**27.** $1000 - 666 \bigcirc 3 \times 111$

**28.** $4^2 \bigcirc 2^2 + 3^2$

**29.** $576 \div 4 \bigcirc 432 \div 3$

**30.** $24 \times 8 \bigcirc 768 \div 3$

**31.** $6^2 + 7^2 \bigcirc 5^2 + 8^2$

**32.** $12 \times (17 + 13) \bigcirc (12 \times 17) + (12 \times 13)$

**33.** $4^2 + 6^2 \bigcirc (4 + 6)^2$

**34.** $3^2 \times 9^2 \bigcirc 4^2 \times 8^2$

Name the sums.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **35.** | 45<br>26<br>37<br>+ 52 | **36.** | 124<br>39<br>68<br>+ 43 | **37.** | 258<br>74<br>191<br>+ 17 | **38.** | 3541<br>287<br>4508<br>+ 9256 | **39.** | 2857<br>4921<br>7848<br>+ 6785 | **40.** | 1234<br>5678<br>9123<br>+ 4567 |

Solve the problems.

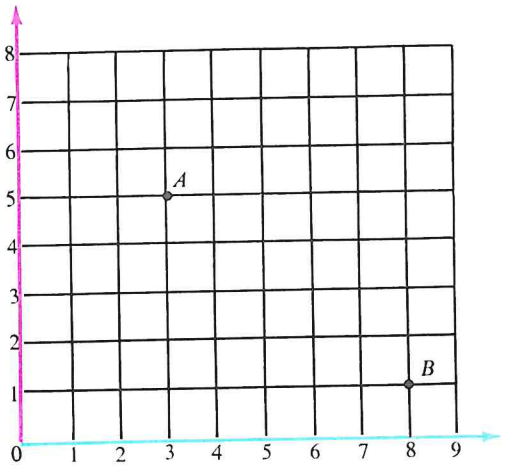
**41.** Nancy bought 8 candy bars for 7¢ each and Joan bought 9 balloons for 6¢ each. How much did the girls spend together?

**42.** A box of a dozen ping pong balls costs \$1.05. A box of a dozen golf balls costs \$4.35. If a man purchased 4 boxes of ping pong balls and 2 boxes of golf balls, how much did he have to pay?

**43.** Al sells newspapers for 7¢ each. If he sold 48 papers on Monday, 56 on Tuesday, and 54 on Wednesday, how much money did he collect in the 3 days?

## Number planes

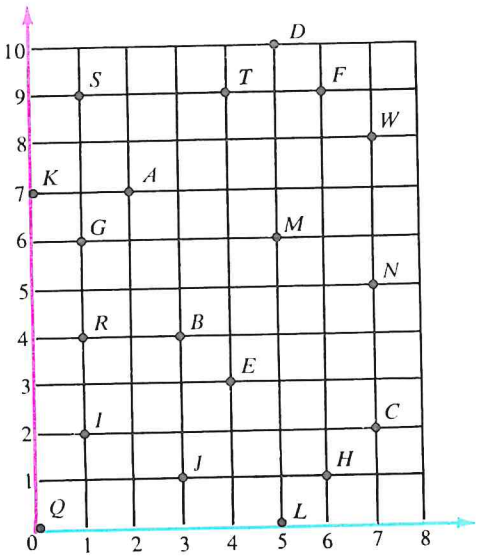
Do you remember how to name points on a number plane? The point $A$ is 3 units along the blue number line and 5 units along the red number line. We write the number pair $(3, 5)$ to name the point $A$. Point $B$ is named by the number pair $(8, 1)$.




## EXERCISES

Study the number plane. Use number pairs to name the points.

**1.** $A$ **2.** $B$ **3.** $D$ **4.** $F$ **5.** $H$ **6.** $I$ **7.** $K$ **8.** $M$




Use letters to name the points.

**9.** $(7, 2)$ **10.** $(4, 3)$ **11.** $(1, 6)$ **12.** $(3, 1)$ **13.** $(5, 0)$ **14.** $(0, 0)$ **15.** $(7, 5)$ **16.** $(1, 4)$ **17.** $(6, 1)$ **18.** $(1, 9)$ **19.** $(3, 4)$ **20.** $(7, 8)$

Use half an inch as a unit to draw a number plane with 11 lines in each direction. Label each set of lines from 0 to 10.

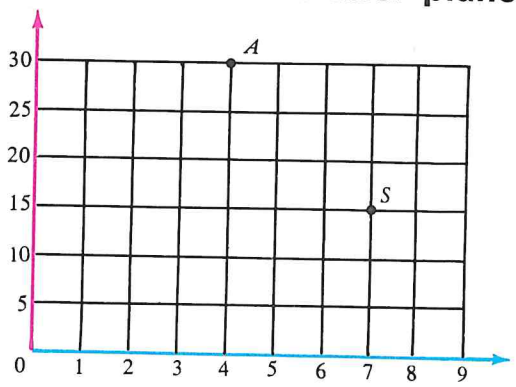
**21.** Use the letters to label the points.

$A$: $(5, 8)$ $B$: $(1, 6)$ $C$: $(2, 6)$ $D$: $(2, 0)$ $E$: $(4, 0)$ $F$: $(4, 3)$ $G$: $(6, 3)$ $H$: $(6, 0)$ $I$: $(8, 0)$ $J$: $(8, 6)$ $K$: $(9, 6)$ $L$: $(5, 8)$

**22.** Join the points in order with straight lines. What does the shape remind you of?

## Number planes

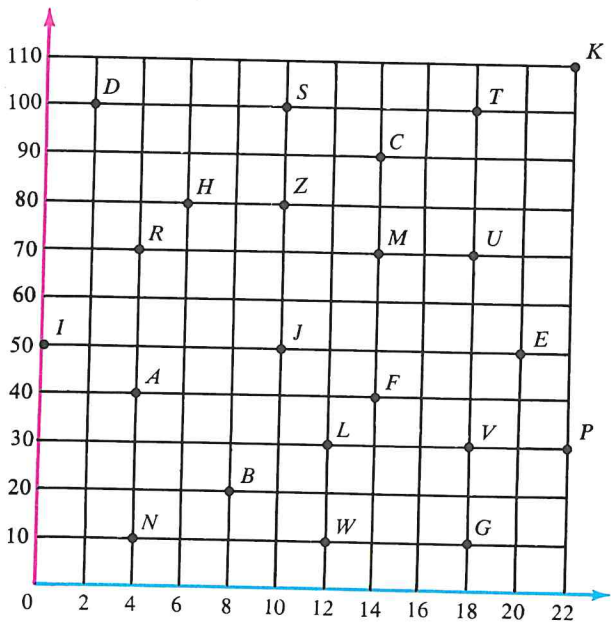
Usually we count by ones on a number line. However we may count by other numbers. In this drawing we count by ones on the blue number line and by fives on the red number line. The point (4, 30) is labeled $A$; the point (7, 15) is labeled $S$.




## EXERCISES

Use number pairs to name the points.

| | | | | | |
|---|---|---|---|---|---|
| **1.** | $A$ | **5.** | $G$ | **9.** | $R$ |
| **2.** | $C$ | **6.** | $I$ | **10.** | $T$ |
| **3.** | $D$ | **7.** | $J$ | **11.** | $V$ |
| **4.** | $F$ | **8.** | $L$ | **12.** | $W$ |




Use letters to name the points.

| | | | |
|---|---|---|---|
| **13.** | (8, 20) | **18.** | (4, 10) |
| **14.** | (6, 80) | **19.** | (2, 100) |
| **15.** | (14, 70) | **20.** | (10, 100) |
| **16.** | (20, 50) | **21.** | (10, 80) |
| **17.** | (22, 110) | **22.** | (18, 70) |

Draw a number plane with 13 lines going up and 13 lines going across. Number upwards by fives from 0 to 60 and across by ones from 0 to 12.

**23.** Use the letters to label the points.

| | | | | |
|---|---|---|---|---|
| $A$: (3, 0) | $D$: (0, 20) | $G$: (5, 50) | $J$: (7, 35) | $M$: (7, 20) |
| $B$: (5, 5) | $E$: (2, 30) | $H$: (6, 55) | $K$: (10, 30) | $N$: (7, 5) |
| $C$: (5, 20) | $F$: (5, 35) | $I$: (7, 50) | $L$: (12, 20) | $O$: (9, 0) |

**24.** Join the points in order with straight lines. What have you drawn?

## Collecting data

Tom is the captain of an 8 man football team called the Bulldogs. One of his jobs is to find out how much each of the players weighs. He records the weights like this

| Player | Weight (in pounds) | Player | Weight (in pounds) |
|---|---|---|---|
| Jim | 75 | Ron | 70 |
| Tom | 85 | Bob | 65 |
| Dan | 60 | Ray | 90 |
| Pat | 75 | Hal | 80 |

Facts such as these are called **data**.

### EXERCISES

Use Tom's data to answer the questions.

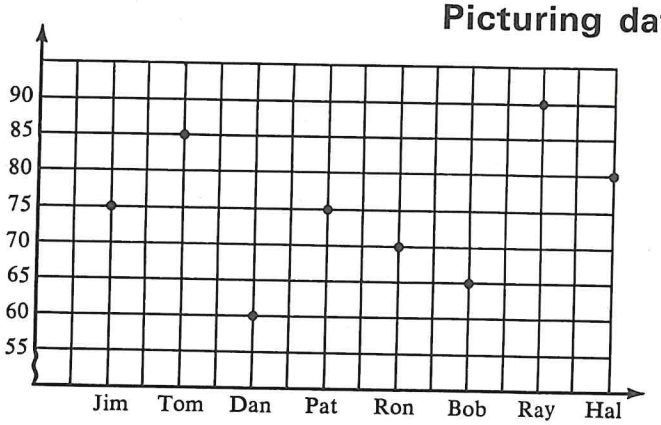
1. What is the weight of the heaviest player?
2. What is the weight of the lightest player?
3. Who are the 5 heaviest players? the 3 lightest?
4. What is the difference in weight between the heaviest player and the lightest player?

Use the data to answer the questions.

| Student | Height (in inches) | Student | Height (in inches) |
|---|---|---|---|
| Tim | 60 | Ted | 65 |
| Mary | 55 | Les | 70 |
| Bill | 65 | Lois | 55 |
| Ann | 50 | Jean | 60 |

5. What is the height of the tallest student? the shortest student?
6. Who are the 5 tallest students? the 3 shortest?
7. What is the difference in height between the tallest student and the shortest student?

This is a **graph** of Tom's data. A graph is an easy way of picturing data.



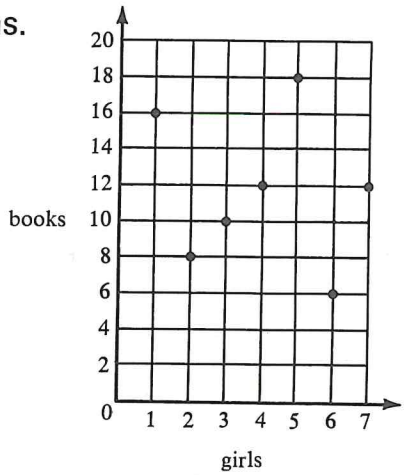

## EXERCISES

Use Tom's graph to complete the pairs in this set.

1. {(Jim, 75), (Tom, 85), (Dan, *a*), (Pat, *b*), (Ron, *c*), (Bob, *d*), (Ray, *e*), (Hal, *f*)}

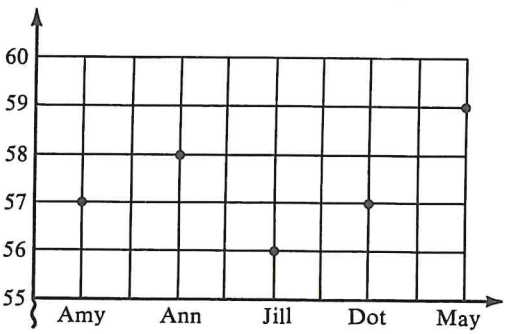
Study the graphs. Then answer the questions.

Seven girls collected data on the number of books they read in a month. They recorded their data on this graph.



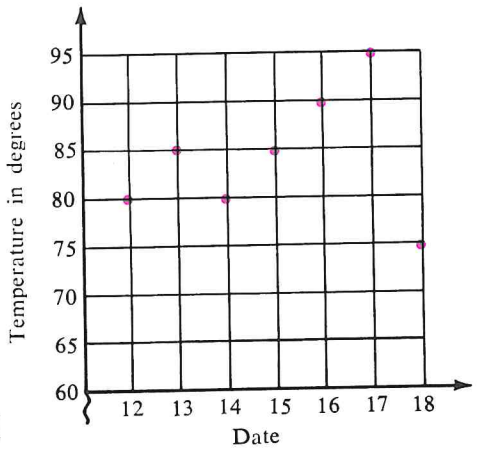

2. Which girl read the most books? How many did she read?
3. Who read the fewest books?
4. Did any girls read the same number of books?
5. Form ordered pairs like (1, 16) from the data on the graph.

This graph shows the heights in inches of the girls in Amy's club.




6. Form pairs like (Amy, 57) from the graph.
7. What is the height of the tallest girl? of the shortest girl?

## Graphing data

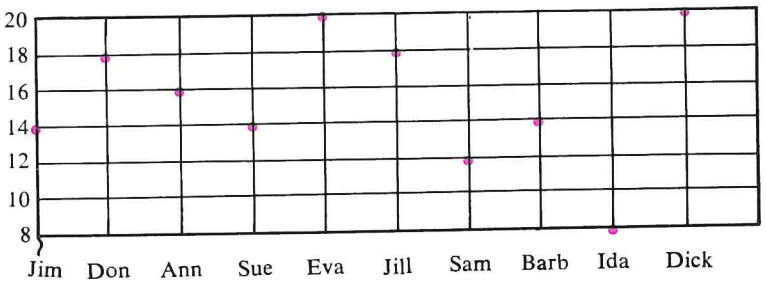



Study the graphs.

Here is a graph showing the highest temperature reached during each day of one week in June.

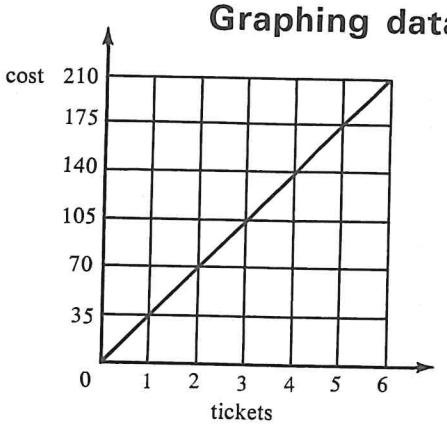
1. What temperature was reached on June 13?
2. What was the highest temperature reached during the week? the lowest?
3. List the ordered pairs formed from the day of the month and the temperature on that day. For example, the first pair would be (12, 80).
4. What is the difference between the highest temperature and the lowest temperature?
5. What unit was used along the number line for temperature?

Ten children in Mr. Jones class recorded their spelling test scores on a graph.




6. Who scored highest?
7. Who scored lowest?
8. Who had scores of 14?
9. What was Don's score?
10. What was the difference between the highest and lowest scores?
11. List the ordered pairs giving the student and the spelling score.
12. If a score of 16 or better is needed to pass the test, who did not pass?

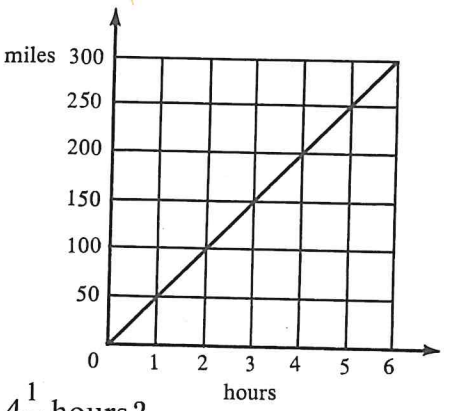
Sometimes the points on a graph are joined to form a curve. Sally made a graph of the cost of tickets for the school play. From the graph we see that 1 ticket costs 35 cents, 2 tickets cost 70 cents, and so on.




## EXERCISES

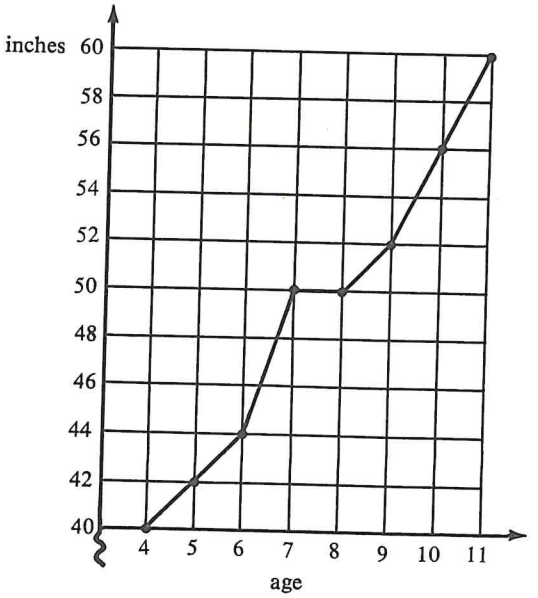
### Study the graphs.

Mr. Allen made a graph of the distance he would travel at 50 miles per hour.




1. How far would Mr. Allen travel in 5 hours?
2. If Mr. Allen wanted to go 150 miles, how long would it take him?
3. List seven ordered pairs from the graph.
4. How far could Mr. Allen go in $2\frac{1}{2}$ hours? in $4\frac{1}{2}$ hours?
   Can you get this information by estimating from the graph?

On every birthday Carol's mother measured Carol's height. She kept her data on a graph like this.

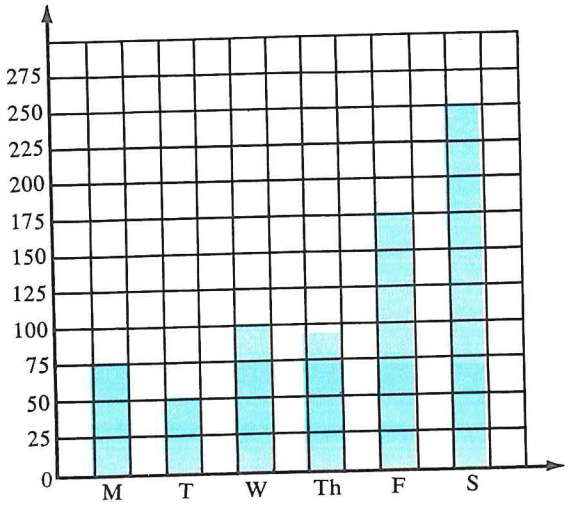



5. How tall was Carol when she was 4?
6. How tall was Carol when she was 8?
7. In what two years were her heights the same?
8. How many inches had Carol grown in the 8 years?
9. List eight ordered pairs from the graph.

## Bar graphs

Sometimes we show data on a **bar graph.**

Mr. Wells made a bar graph to record the number of customers who visited his toy store for a week in May.




## EXERCISES

Study Mr. Wells' bar graph.

1. How many people visited the store on Monday? on Friday?
2. On which day did he have the fewest customers? the most customers?
3. Can you *estimate* how many customers he had on Thursday?
4. Would Mr. Wells need an assistant early in the week or later in the week?
5. List the ordered pairs from the bar graph. The first would be (Monday, 75).

Joan kept a record of how long she watched television each day. Her data looked like this.

Monday: $1\frac{1}{2}$ hours

Tuesday: 2 hours

Wednesday: 1 hour

Thursday: $1\frac{1}{2}$ hours

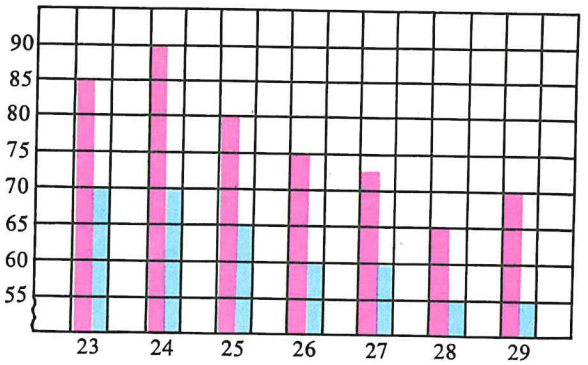
Friday: 3 hours

Saturday: $3\frac{1}{2}$ hours

Sunday: 3 hours

6. To make a bar graph of her data, what would be good units for Joan to use on the number lines?
7. Make a bar graph of Joan's data.
8. How many hours did Joan watch television this week?

A **bar graph** can be used to show more than one set of data. The graph here pictures the highest and the lowest temperatures for each day of a week in October.

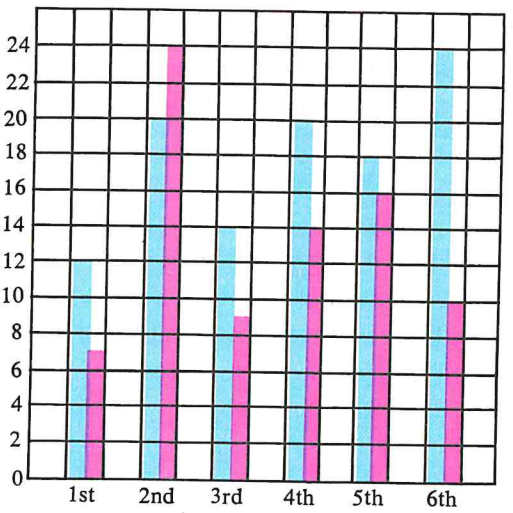



## EXERCISES

Study the temperature bar graph.

1. On what day was it the warmest? the coolest?
2. What was the highest temperature on the 25th? the 27th? the 29th?
3. What was the lowest temperature on the 24th? the 25th? the 28th?
4. List the ordered pairs showing the day and the lowest temperature.

Tom used a double bar graph to present data about the scores of the Bulldogs football games. The Bulldogs' scores are in blue and the other teams' are in red. Can you tell the scores for each of the games?




Study the football bar graph.

5. How many games did the Bulldogs win? lose?
6. What was the score of the game 3?
7. Form six ordered pairs, giving the Bulldogs' score first.
8. In which game did the Bulldogs score the most points? How many?
9. In which game were the most points scored all together? How many?
10. In which game did the Bulldogs win by the greatest number of points? What was the difference between the scores for that game?

## Finding averages

The table shows the number of eggs Sam and Jim gathered each day for a week. Both boys gathered the same number of eggs that week. Jim gathered 5 eggs every day. Sam gathered different amounts. We can say they each gathered an **average** of 5 eggs per day. $35 \div 7 = 5$

An **average** tells you what each amount would be if all the amounts were the same.

Gathering Eggs

| Day | Sam | Jim |
|---|---|---|
| Sun | 4 | 5 |
| Mon | 5 | 5 |
| Tues | 7 | 5 |
| Wed | 4 | 5 |
| Thur | 4 | 5 |
| Fri | 6 | 5 |
| Sat | 5 | 5 |
| Total | 35 | 35 |

## EXERCISES

Jack and Fred play on the same baseball team. They recorded the number of hits they got for each of four games.

Data

| Game | Jack's Hits | Fred's Hits |
|---|---|---|
| 1 | 3 | 2 |
| 2 | 2 | 2 |
| 3 | 1 | 2 |
| 4 | 2 | 2 |
| Total | 8 | 8 |

1. How many hits did each boy get all together?
2. How many hits did Fred get per game?
3. How many hits did Jack get **on the average?**

Jean, Carol and Fran recorded how long they watched television each day for a week. Here is their data.

Hours Watching Television

| Day | Jean | Carol | Fran |
|---|---|---|---|
| M | 2 | 3 | 1 |
| T | 2 | 3 | 3 |
| W | 3 | 3 | 2 |
| Th | 2 | 3 | 4 |
| F | 3 | 3 | 1 |
| Sa | 5 | 3 | 5 |
| Su | 4 | 3 | 5 |
| Total | | | |

4. How many hours during the week did Carol watch television? how many hours per day?
5. How many hours did Jean watch television during the week? how many hours a day **on the average**?
6. How many hours did Fran watch television during the week? how many hours a day **on the average**?

| Test | Mary | Joe | Sally |
|---|---|---|---|
| 1 | 16 | — | 14 |
| 2 | 15 | 16 | 14 |
| 3 | 17 | 15 | 14 |
| 4 | 13 | 17 | 14 |
| 5 | 14 | 16 | 14 |
| Total | 75 | 64 | 70 |

## Finding averages

Mary and Sally took 5 spelling tests and Joe took 4 spelling tests. Mary's **average score** is $75 \div 5 = 15$.
Joe's **average score** is $64 \div 4 = 16$.
Sally had the same score of 14 on every test, so her **average score** is 14.

## EXERCISES

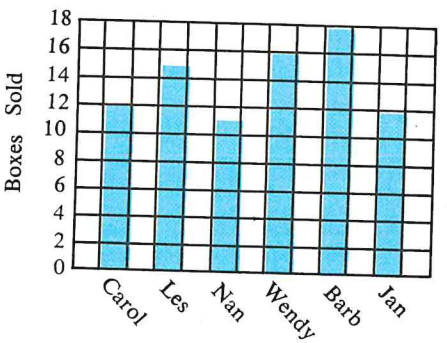
Name the average of each set of numbers.

1. {12, 15, 17, 16}
2. {25, 31, 24, 20}
3. {8, 9, 6, 4, 7, 8}
4. {16, 2, 3, 7, 6, 8, 5, 9}

Find the answers.

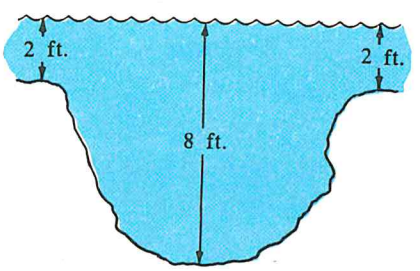
5. If Karl had scores of 9, 7, and 8 on three arithmetic tests, what was his average score?

6. The bar graph shows the number of boxes of cookies sold by 6 Brownies. What is the average number of boxes sold by the girls?




7. Use the bar graph for exercises 5 to 10 on page 279 to name
   a. the average number of points scored by the Bulldogs.
   b. the average number of points scored by the opponents.

8. Johnny is told that the average depth of the river is 4 feet. If Johnny is 5 feet tall, is it safe for him to walk across the river?




9. Sandy's teacher said that the average number of days per year that a student is absent from school is 8. Does this mean that every student is absent 8 days a year?

10. Find examples of averages in newspapers and magazines and bring them to class.

## More about data

Steve collected data about the people in his row. The average height of the students in Steve's row is 57 inches. The average weight of the students in Steve's row is 80 pounds. Steve said, "The heights are between 54 and 60 inches. The **range** of heights is 6 inches. The weights are between 69 and 92 pounds. The **range** of weights is 23 pounds."

| | Height (inches) | Weight (pounds) |
|---|---|---|
| Lois | 55 | 76 |
| Ida | 60 | 92 |
| Frank | 58 | 81 |
| Steve | 58 | 82 |
| Ann | 54 | 69 |
| Total | 285 | 400 |

$285 \div 5 = 57$ $\quad$ $400 \div 5 = 80$

$60 - 54 = 6$ $\quad$ $92 - 69 = 23$

The **range** of a set of data is the difference between the greatest and the least amount.

### EXERCISES

1. If the highest score on a history test is 96 and the lowest score is 72, what is the range of scores?

2. If the heights of most of the men in the world are between 3 feet 6 inches and 7 feet 4 inches, what is the range of heights?

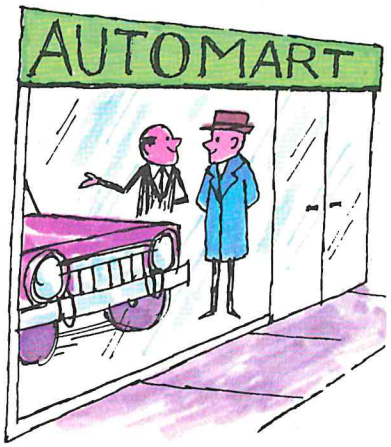
3. The weather man said, "The temperature range today was 27 degrees." If the lowest temperature for the day was 65 degrees, what was the highest temperature?

4. Six girls recorded the amounts of money they earned in a week.
   a. Name the range of the money earned.
   b. How much did the girls earn all together?
   c. How much did each girl earn on the average?
   d. Which girl earned the most?
   e. Which girl earned the average amount?

| | Money Earned (cents) |
|---|---|
| Cathy | 65 |
| Dana | 105 |
| Edith | 80 |
| Gale | 90 |
| Irene | 125 |
| Tina | 75 |

5. Jim, Hank, and Sean wanted to earn some money by shining shoes. They decided to share their earnings equally. If Jim earned \$1.55, Hank earned \$1.75 and Sean earned \$1.35, how much did each boy get? What was the range?

1. An automobile manufacturer stated that, on the average, one of his cars is sold every 6 seconds. This means about 14,000 of these cars are sold every day, about 100,000 are sold a week, and about 5,200,000 are sold a year.




2. On the average, a baseball player gets 1 hit every 4 times he comes to bat.

3. The height of most men in the United States ranges from 5 feet to 6 feet 4 inches; the height of most women ranges from 4 feet 10 inches to 6 feet. Most men weigh between 122 and 205 pounds; most women weigh between 102 and 180 pounds.

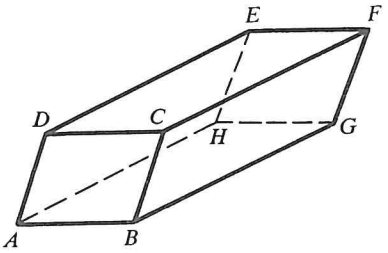
4. The average human body temperature is about 99 degrees.

*Just For Fun*

1. Name the average height and the range of heights of the children in your class room.
2. Make a bar graph of the data you collect for exercise 1.
3. Collect other data about the members of your class. Name the average and the range. You may collect data about such things as the number of people in each family and the number of books each child reads in a week.

## CHECKING UP

Study the space figure.

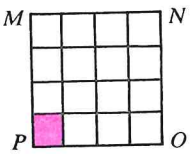



1. Name a plane parallel to plane *EFG*.
2. Name the intersection of planes *DEF* and *CBG*.
3. Name three planes which have the vertex *D* in common.

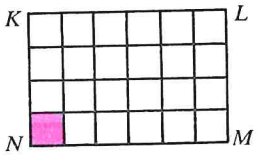
Complete the sentences.

4. The intersection of a plane and a line not in the plane is a ___?___.
5. The intersection of two planes is a ___?___.

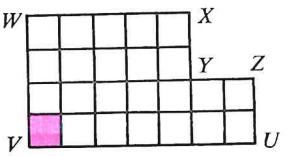
Use the small square units to name the measures.




6. m(*MNOP*) = ***a***

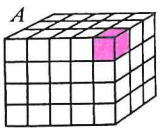



7. m(*KLMN*) = ***b***

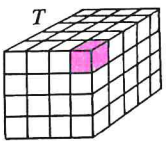



8. m(*UVWXYZ*) = ***c***

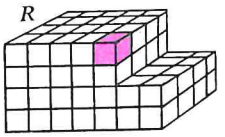
Use the small cubic units to name the measures.




9. m(*A*) = ***a***



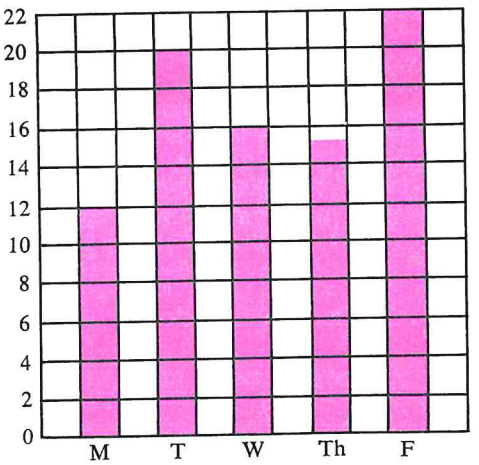

10. m(*T*) = ***b***




11. m(*R*) = ***c***

Study the bar graph.

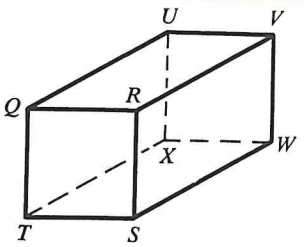
The graph shows the number of students absent from school during one week.




12. How many were absent on Tuesday? on Thursday?
13. On what day were 16 students absent?
14. During the week the number of absences ranged from ___?___ to ___?___.
15. Name the average number of students absent per day.

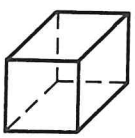
We can think of a **plane** as a set of points in a flat surface. If two planes intersect, they form an **angle.** If they do not intersect, they are **parallel.** Parts of planes, regions, can be joined to form **simple closed surfaces.**

1. Name the intersection of the planes $QRS$ and $QUX$.
2. Name three sets of parallel planes.
3. How many planes, edges, and vertices form this figure?



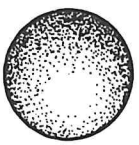

The special simple closed surfaces we studied were:

**pyramid**


**cube**

**cylinder**

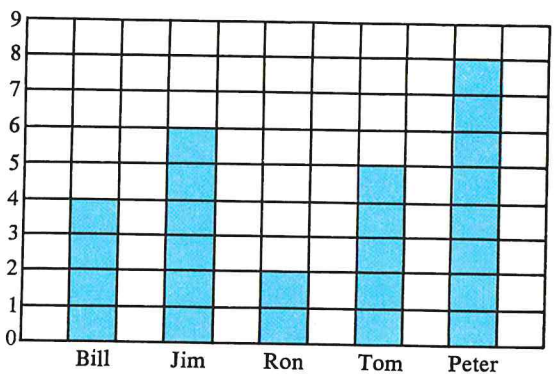
**cone**


**sphere**

4. Name the number of faces of **a.** a pyramid **b.** a cube.
5. Name the figure formed by the base of **a.** a cylinder **b.** a cone.

A **number pair** names points on a **number plane.** We can use the number plane to show data. We can learn interesting things about data by finding the **average** and **range** of the set of numbers.

Five boys played in a basketball game. They made a graph of the number of points each player scored.




6. Which player scored the most?
7. How many points were scored in all?
8. What was the average number of points scored?
9. What was the range of points?

## LOOKING WAY BACK

Name the sums and differences.

| | | | | |
|---|---|---|---|---|
| **1.** $23 + 67$ | **3.** $52 + 79 + 83$ | **5.** $348 + 269 + 752$ | **7.** $4328 + 6945$ | **9.** $64728 + 49291$ |
| **2.** $52 - 39$ | **4.** $426 - 258$ | **6.** $911 - 644$ | **8.** $5432 - 3579$ | **10.** $21728 - 14654$ |

Name the products, quotients, and remainders.

| | | | | |
|---|---|---|---|---|
| **11.** $78 \times 7$ | **13.** $468 \times 5$ | **15.** $39 \times 27$ | **17.** $54 \times 82$ | **19.** $247 \times 39$ |
| **12.** $3\overline{)84}$ | **14.** $5\overline{)72}$ | **16.** $6\overline{)98}$ | **18.** $8\overline{)264}$ | **20.** $9\overline{)560}$ |

Write the compact numerals for each number.

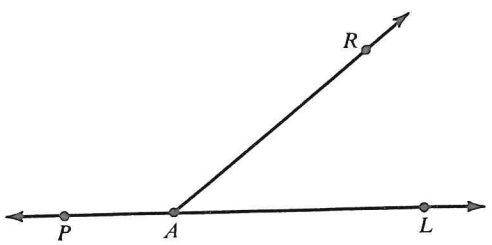
**21.** three thousand, five hundred nineteen

**22.** two hundred fifty six thousand

**23.** 4 million, 752 thousand, twenty seven

**24.** sixteen million, seventy two thousand, five hundred five

Study the figure.




**25.** Name the line.

**26.** Name three line segments.

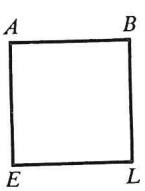
**27.** Name three rays.

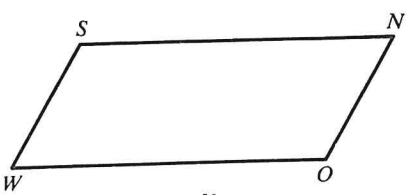
**28.** Name two angles.

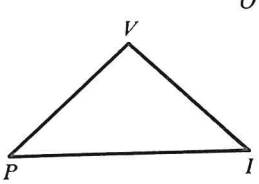
**29.** Name the vertex of each angle.

**30.** Name the ray common to each angle.

Study the figures.



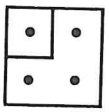
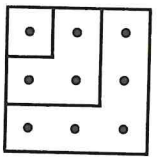
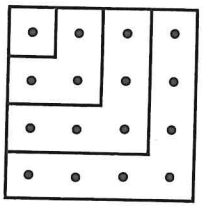

**31.** Name the triangle.

**32.** Name the square.

**33.** Name the parallelogram.

**34.** A simple closed curve whose sides are line segments with common end points is called a ___?___.

## EXTRA FOR EXPERTS

**1.** Study the pattern and continue it through the next six squares.
Can you see why $\{1, 4, 9, 16, \ldots\}$ are called the **square numbers?**

Copy and complete the patterns.

**2.**

$1^2 = 1$
$2^2 = 4$ $\quad 2^2 - 1^2 = 3$
$3^2 = 9$ $\quad 3^2 - 2^2 = h$
$4^2 = a$ $\quad 4^2 - 3^2 = i$
$5^2 = b$ $\quad 5^2 - 4^2 = j$
$6^2 = c$ $\quad 6^2 - 5^2 = k$
$7^2 = d$ $\quad 7^2 - 6^2 = l$
$8^2 = e$ $\quad 8^2 - 7^2 = m$
$9^2 = f$ $\quad 9^2 - 8^2 = n$
$10^2 = g$ $\quad 10^2 - 9^2 = p$

**3.**

$1^3 = 1$
$2^3 = 8$ $\quad 2^3 - 1^3 = 7$
$3^3 = a$ $\quad 3^3 - 2^3 = i$
$4^3 = b$ $\quad 4^3 - 3^3 = j$
$5^3 = c$ $\quad 5^3 - 4^3 = k$
$6^3 = d$ $\quad 6^3 - 5^3 = l$
$7^3 = e$ $\quad 7^3 - 6^3 = m$
$8^3 = f$ $\quad 8^3 - 7^3 = n$
$9^3 = g$ $\quad 9^3 - 8^3 = z$
$10^3 = h$ $\quad 10^3 - 9^3 = p$

**4.** Do you see the pattern of odd numbers in exercise 2?

Copy and continue the patterns until each has eight equations.

**5.**
$(8 \times 1) + 1 = 9$
$(8 \times 12) + 2 = 98$
$(8 \times 123) + 3 = 987$

**6.**
$(9 \times 1) + 2 = 11$
$(9 \times 12) + 3 = 111$
$(9 \times 123) + 4 = 1111$

Study the method for naming this sum.

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$
$$= (10 + 1) + (9 + 2) + (8 + 3) + (7 + 4) + (6 + 5)$$
$$= 5 \times 11 = 55$$

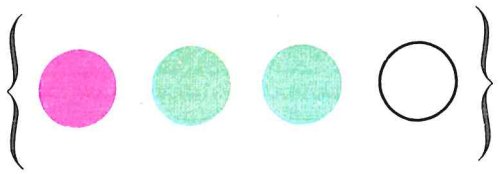
Use the method to name

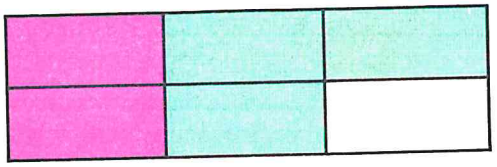
**7.** $10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20 + 21 + 22 + 23$

**8.** $20 + 21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29$

**9.** the sum of all the whole numbers from 1 to 100

## Joining equivalent subsets





$\frac{1}{4}$ of the circles is red.

$\frac{2}{4}$ of the circles are green.

$\frac{1}{4} + \frac{2}{4}$ of the circles are red or green.

$\frac{3}{4}$ of the circles are red or green.

$\frac{2}{6}$ of the region is red.

$\frac{3}{6}$ of the region is green.

$\frac{2}{6} + \frac{3}{6}$ of the region is red or green.

$\frac{5}{6}$ of the region is red or green.

## EXERCISES

Use the sets to solve the equations.

{★★★☆}

1. $\frac{2}{4} + \frac{1}{4} = \frac{a}{4}$

{★★●●◇}

2. $\frac{2}{5} + \frac{2}{5} = \frac{m}{5}$

{☆●●}

3. $\frac{1}{3} + \frac{1}{3} = \frac{c}{3}$

{◆○○○}

4. $\frac{1}{4} + \frac{3}{4} = \frac{n}{4}$

{■▲▲○○}

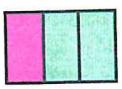
5. $\frac{1}{5} + \frac{2}{5} = \frac{e}{5}$

{●●■■■◇}

6. $\frac{2}{6} + \frac{3}{6} = \frac{r}{6}$

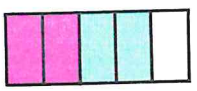
Use the shaded regions to solve the equations.

7. $\frac{2}{4} + \frac{2}{4} = \frac{a}{4}$



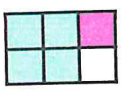
8. $\frac{1}{3} + \frac{2}{3} = \frac{n}{3}$

9. $\frac{3}{6} + \frac{2}{6} = \frac{m}{6}$

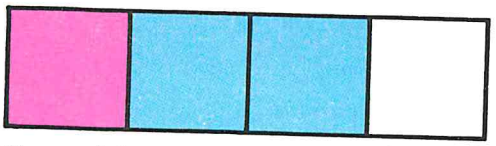


10. $\frac{2}{5} + \frac{2}{5} = \frac{e}{5}$

11. $\frac{1}{4} + \frac{1}{4} = \frac{c}{4}$



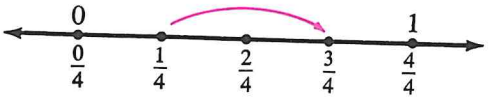
12. $\frac{4}{6} + \frac{1}{6} = \frac{r}{6}$



The rod has 4 parts of the same size. $\frac{1}{4}$ is red and $\frac{2}{4}$ are blue.

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

## Adding fractional numbers




This number line has 4 parts of the same length. The move shows

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

## EXERCISES

Use the rods to solve the equations.

1. $\frac{2}{4} + \frac{1}{4} = a$

2. $\frac{2}{5} + \frac{2}{5} = d$

3. $\frac{3}{6} + \frac{1}{6} = g$

4. $\frac{1}{3} + \frac{1}{3} = b$

5. $\frac{3}{4} + \frac{1}{4} = e$

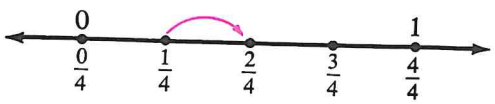
6. $\frac{2}{5} + \frac{3}{5} = h$

7. $\frac{1}{5} + \frac{3}{5} = c$

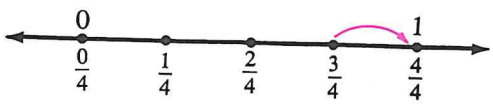
8. $\frac{2}{6} + \frac{2}{6} = f$

9. $\frac{2}{6} + \frac{3}{6} = i$

Use the number lines to solve the equations.

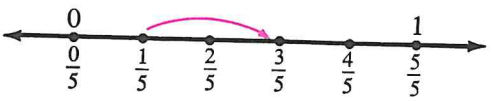



10. $\frac{1}{4} + \frac{1}{4} = a$




11. $\frac{3}{4} + \frac{1}{4} = c$

12. $\frac{2}{3} + \frac{1}{3} = b$




13. $\frac{1}{5} + \frac{2}{5} = d$

## Removing subsets

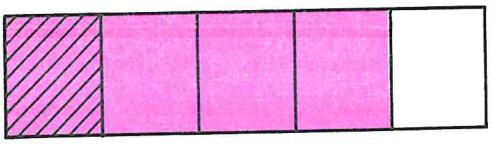
{● ● ▲ □ □}

$\frac{3}{5}$ of the objects are blue.

$\frac{2}{5}$ of the objects are blue circles.

$\left(\frac{3}{5} - \frac{2}{5}\right)$ of the objects are blue triangles.

$$\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$$



$\frac{4}{5}$ of the region is red.

$\frac{1}{5}$ of the region is red and shaded.

$\left(\frac{4}{5} - \frac{1}{5}\right)$ of the region is red and not shaded.

$$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

## EXERCISES

Use the sets to help you solve the equations.

{● ▲ ©}

1. $\frac{2}{3} - \frac{1}{3} = \frac{a}{3}$

{◆ ● ● △}

3. $\frac{3}{4} - \frac{1}{4} = \frac{c}{4}$

{● ● ■ ©}

5. $\frac{3}{4} - \frac{2}{4} = \frac{e}{4}$

{© © © ◆ ○}

7. $\frac{4}{5} - \frac{3}{5} = \frac{i}{5}$

{● ● ■ ■ ■ ☆}

2. $\frac{5}{6} - \frac{2}{6} = \frac{m}{6}$

{★ ★ ★ ● ● ◇}

4. $\frac{5}{6} - \frac{3}{6} = \frac{n}{6}$

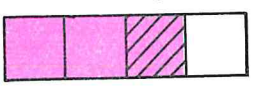
{▼ ▼ ▼ ▼ ● ☆}

6. $\frac{5}{6} - \frac{4}{6} = \frac{r}{6}$

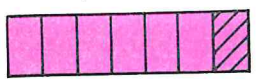
{★ ★ ★ ● □ □}

8. $\frac{4}{6} - \frac{3}{6} = \frac{s}{6}$

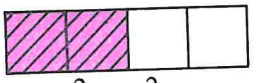
Use the regions to help you solve the equations.



9. $\frac{3}{4} - \frac{2}{4} = \frac{a}{4}$

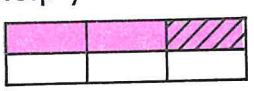


10. $\frac{7}{7} - \frac{1}{7} = \frac{n}{7}$

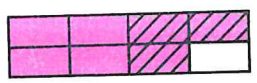


11. $\frac{2}{4} - \frac{2}{4} = a$

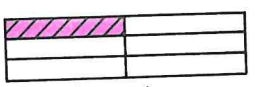
12. $1 - \frac{2}{2} = e$



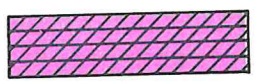
13. $\frac{3}{6} - \frac{1}{6} = \frac{c}{6}$



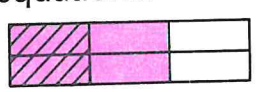
14. $\frac{7}{8} - \frac{3}{8} = \frac{r}{8}$



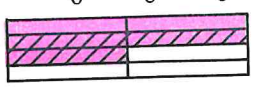
15. $\frac{1}{6} - \frac{1}{6} = b$



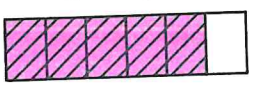
16. $1 - \frac{4}{4} = f$



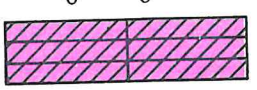
17. $\frac{4}{6} - \frac{2}{6} = \frac{e}{6}$



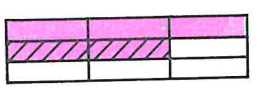
18. $\frac{5}{8} - \frac{3}{8} = \frac{s}{8}$



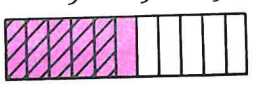
19. $\frac{5}{6} - \frac{5}{6} = c$



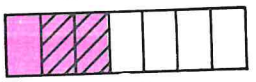
20. $1 - \frac{6}{6} = g$



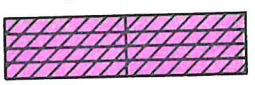
21. $\frac{5}{9} - \frac{2}{9} = \frac{n}{9}$



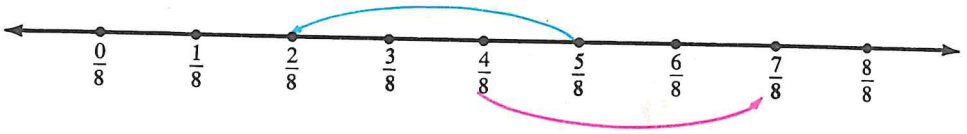
22. $\frac{6}{11} - \frac{5}{11} = \frac{h}{11}$



23. $\frac{3}{7} - \frac{2}{7} = d$



24. $1 - \frac{8}{8} = h$

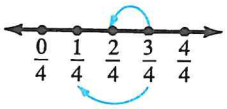



The red arrow shows the addition $\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$.

The blue arrow shows the subtraction $\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$.

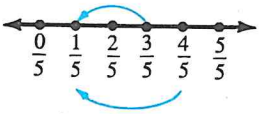
## EXERCISES

Use the number lines to solve the equations.



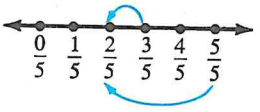

**1.** $\frac{3}{4} - \frac{1}{4} = \frac{a}{4}$

$\frac{3}{4} - \frac{2}{4} = \frac{c}{4}$



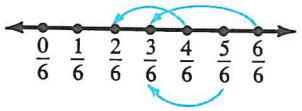

**3.** $\frac{3}{5} - \frac{2}{5} = \frac{a}{5}$

$\frac{4}{5} - \frac{3}{5} = \frac{c}{5}$




**5.** $\frac{3}{5} - \frac{1}{5} = \frac{a}{5}$

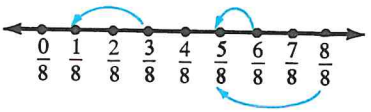
$1 - \frac{3}{5} = \frac{c}{5}$




**2.** $\frac{4}{6} - \frac{2}{6} = \frac{a}{6}$

$\frac{5}{6} - \frac{2}{6} = \frac{c}{6}$

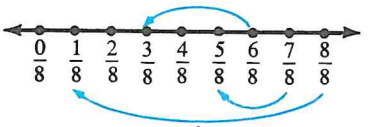
$1 - \frac{3}{6} = \frac{e}{6}$




**4.** $\frac{3}{8} - \frac{2}{8} = \frac{a}{8}$

$\frac{6}{8} - \frac{1}{8} = \frac{c}{8}$

$1 - \frac{3}{8} = \frac{e}{8}$




**6.** $\frac{6}{8} - \frac{3}{8} = \frac{a}{8}$

$\frac{7}{8} - \frac{2}{8} = \frac{c}{8}$

$1 - \frac{7}{8} = \frac{e}{8}$

Write the numerals to complete this chart of moves on a fractional number line.

**7.**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **start** | $\frac{3}{4}$ | $\frac{7}{8}$ | $\frac{6}{7}$ | $\frac{3}{8}$ | $\frac{5}{8}$ | $\frac{5}{6}$ | $\frac{4}{6}$ | $\frac{7}{8}$ | 1 | 1 |
| **move to the left** | $\frac{1}{4}$ | $\frac{2}{8}$ | $\frac{1}{7}$ | $\frac{2}{8}$ | $\frac{4}{8}$ | $\frac{2}{6}$ | $\frac{r}{6}$ | $\frac{s}{8}$ | $\frac{2}{3}$ | $\frac{3}{4}$ |
| **stop** | $\frac{a}{4}$ | $\frac{c}{8}$ | $\frac{e}{7}$ | $\frac{i}{8}$ | $\frac{m}{8}$ | $\frac{n}{6}$ | $\frac{3}{6}$ | $\frac{3}{8}$ | $\frac{u}{3}$ | $\frac{v}{4}$ |

## Adding and subtracting fractional numbers

Compare the equations.

addition

$$\frac{3}{8} + \frac{4}{8} = (3 + 4) \times \frac{1}{8} = \frac{7}{8}$$

$$\frac{3}{8} + \frac{4}{8} = \frac{3 + 4}{8} = \frac{7}{8}$$

subtraction

$$\frac{7}{8} - \frac{4}{8} = (7 - 4) \times \frac{1}{8} = \frac{3}{8}$$

$$\frac{7}{8} - \frac{4}{8} = \frac{7 - 4}{8} = \frac{3}{8}$$

### EXERCISES

Copy and solve the equations.

1. $\frac{2}{6} + \frac{3}{6} = \frac{2 + a}{6} = \frac{c}{6}$
2. $\frac{1}{8} + \frac{4}{8} = \frac{1 + a}{8} = \frac{c}{8}$
3. $\frac{3}{5} + \frac{1}{5} = \frac{3 + a}{5} = \frac{c}{5}$
4. $\frac{6}{8} + \frac{1}{8} = \frac{6 + a}{8} = \frac{c}{8}$
5. $\frac{3}{6} - \frac{1}{6} = \frac{3 - a}{6} = \frac{c}{6}$
6. $\frac{4}{5} - \frac{1}{5} = \frac{4 - a}{5} = \frac{c}{5}$
7. $\frac{7}{8} - \frac{1}{8} = \frac{7 - a}{8} = \frac{c}{8}$
8. $\frac{4}{9} - \frac{2}{9} = \frac{4 - a}{9} = \frac{c}{9}$

Solve the equations.

9. $\frac{1}{4} + \frac{2}{4} = \frac{a}{4}$
10. $\frac{3}{6} + \frac{3}{6} = \frac{n}{6}$
11. $\frac{3}{8} + \frac{4}{8} = \frac{r}{8}$
12. $\frac{3}{5} - \frac{2}{5} = \frac{s}{5}$
13. $\frac{4}{8} + \frac{2}{8} = \frac{x}{8}$
14. $\frac{7}{8} - \frac{3}{8} = \frac{z}{8}$
15. $\frac{1}{3} + \frac{1}{3} = a$
16. $1 - \frac{6}{8} = e$
17. $1 + \frac{1}{6} = f$
18. $1 - \frac{1}{5} = g$
19. $\frac{4}{6} + \frac{2}{6} = k$
20. $1 - \frac{3}{6} = l$

Solve the problems.

21. Walter gave $\frac{1}{4}$ of his candy bar to Jerry and $\frac{1}{4}$ to Henry. How much of his candy bar did he give away?
22. Mike mowed $\frac{1}{5}$ of his lawn in the morning and $\frac{2}{5}$ in the afternoon. How much of the lawn did he mow in all?
23. Beatrice polished $\frac{3}{8}$ of the floor on one day and $\frac{5}{8}$ the next day. How much more did she polish on the second day?

## Using the distributive property

In Chapter 4 you often used the distributive property like this:

$$\begin{aligned}(3 \times 8) + (4 \times 8) &= (3 + 4) \times 8 \\ &= 7 \times 8 \\ &= 56\end{aligned}$$

We shall also use the distributive property with fractional numbers like this:

$$\begin{aligned}\left(3 \times \tfrac{1}{8}\right) + \left(4 \times \tfrac{1}{8}\right) &= (3 + 4) \times \tfrac{1}{8} \\ &= 7 \times \tfrac{1}{8} = \tfrac{7}{8}\end{aligned}$$

### EXERCISES

Use the distributive property to solve the equations.

1. $(5 \times 8) + (3 \times 8) = a \times 8 = b$
2. $(7 \times 6) + (4 \times 6) = c \times 6 = d$
3. $(8 \times 3) + (6 \times 3) = e \times 3 = f$
4. $(4 \times 7) + (2 \times 7) = g \times 7 = h$
5. $\left(1 \times \frac{1}{5}\right) + \left(3 \times \frac{1}{5}\right) = a \times \frac{1}{5} = b$
6. $\left(2 \times \frac{1}{7}\right) + \left(4 \times \frac{1}{7}\right) = c \times \frac{1}{7} = d$
7. $\left(2 \times \frac{1}{8}\right) + \left(5 \times \frac{1}{8}\right) = e \times \frac{1}{8} = f$
8. $\left(3 \times \frac{1}{6}\right) + \left(2 \times \frac{1}{6}\right) = g \times \frac{1}{6} = h$
9. $\left(5 \times \frac{1}{7}\right) - \left(1 \times \frac{1}{7}\right) = j \times \frac{1}{7} = k$
10. $\left(6 \times \frac{1}{8}\right) - \left(2 \times \frac{1}{8}\right) = l \times \frac{1}{8} = m$
11. $\left(6 \times \frac{1}{10}\right) - \left(2 \times \frac{1}{10}\right) = v \times p = q$
12. $\left(8 \times \frac{1}{9}\right) - \left(5 \times \frac{1}{9}\right) = r \times s = t$

Solve the equations.

13. $\frac{5}{8} + \frac{2}{8} = \frac{5 + 2}{8} = \frac{a}{8}$
14. $\frac{2}{7} + \frac{3}{7} = \frac{2 + 3}{7} = \frac{c}{7}$
15. $\frac{3}{8} + \frac{4}{8} = \frac{3 + e}{8} = \frac{m}{8}$
16. $\frac{5}{12} + \frac{2}{12} = \frac{5 + n}{12} = \frac{r}{12}$
17. $\frac{5}{6} - \frac{1}{6} = \frac{5 - 1}{6} = \frac{a}{6}$
18. $\frac{7}{12} - \frac{3}{12} = \frac{7 - 3}{12} = \frac{c}{12}$
19. $\frac{9}{10} - \frac{4}{10} = \frac{9 - e}{10} = \frac{m}{10}$
20. $\frac{8}{10} - \frac{7}{10} = \frac{8 - n}{10} = \frac{r}{10}$

## Short forms

Study the equations and the short forms.

| Equation | Short form | Equation | Short form |
|---|---|---|---|
| $\frac{3}{8} + \frac{5}{8} = \frac{3+5}{8}$<br>$= \frac{8}{8}$<br>$= 1$ | $\frac{3}{8}$<br>$+ \frac{5}{8}$<br>$\frac{8}{8} = 1$ | $\frac{5}{8} - \frac{3}{8} = \frac{5-3}{8}$<br>$= \frac{2}{8}$<br>$= \frac{1}{4}$ | $\frac{5}{8}$<br>$- \frac{3}{8}$<br>$\frac{2}{8} = \frac{1}{4}$ |

### EXERCISES

Name the sums and differences.

1. $\frac{2}{5} + \frac{2}{5}$
2. $\frac{3}{5} + \frac{1}{5}$
3. $\frac{3}{7} + \frac{2}{7}$
4. $\frac{2}{9} + \frac{5}{9}$
5. $\frac{4}{9} + \frac{1}{9}$
6. $\frac{2}{7} + \frac{4}{7}$
7. $\frac{3}{5} - \frac{1}{5}$
8. $\frac{5}{7} - \frac{3}{7}$
9. $\frac{6}{7} - \frac{5}{7}$
10. $\frac{5}{9} + \frac{4}{9}$
11. $\frac{6}{7} - \frac{2}{7}$
12. $\frac{3}{8} + \frac{1}{8}$
13. $\frac{1}{4} + \frac{1}{4}$
14. $\frac{1}{8} + \frac{6}{8}$
15. $\frac{3}{10} + \frac{5}{10}$
16. $\frac{4}{9} + \frac{2}{9}$
17. $\frac{4}{8} + \frac{3}{8}$
18. $\frac{8}{8} - \frac{3}{8}$
19. $\frac{6}{8} - \frac{1}{8}$
20. $\frac{9}{10} - \frac{5}{10}$
21. $\frac{7}{9} - \frac{4}{9}$

Complete the number charts.

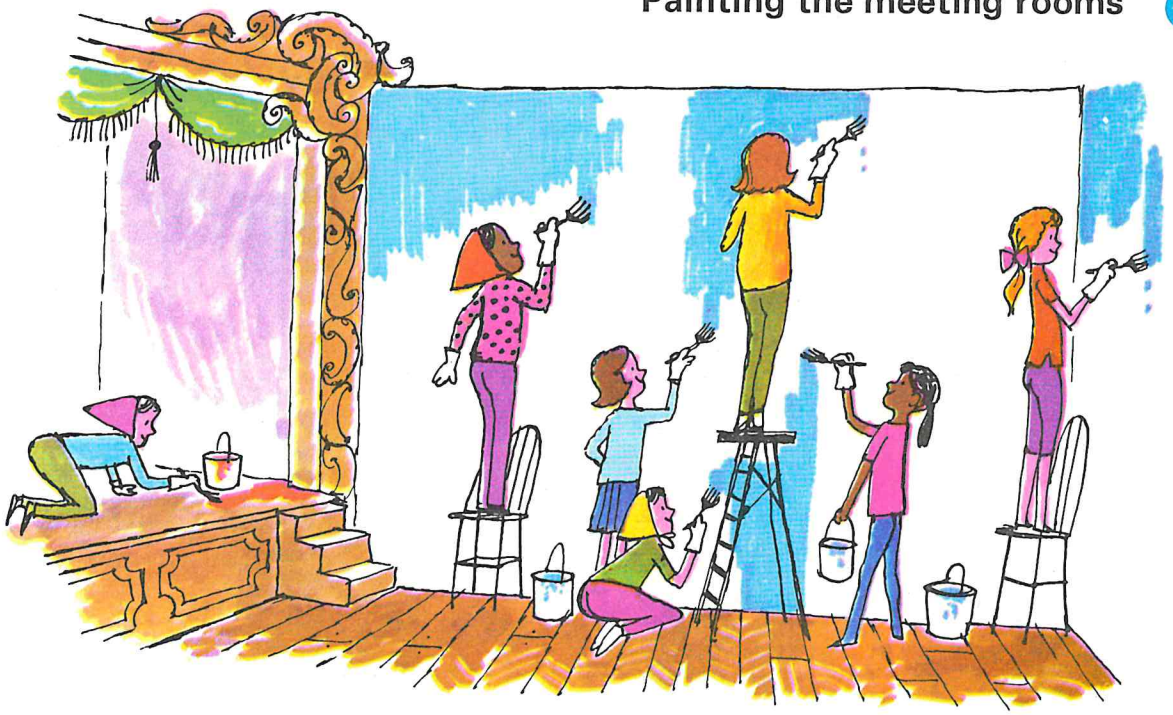
22.

| + | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{3}{5}$ | $\frac{4}{5}$ |
|---|---|---|---|---|
| $\frac{1}{5}$ | *a* | *c* | *e* | *n* |

23.

| + | 0 | *a* | *c* | *e* | *n* |
|---|---|---|---|---|---|
| $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{3}{5}$ | $\frac{4}{5}$ | 1 |

## Painting the meeting rooms



1. Ellen painted $\frac{3}{5}$ of one wall in the morning. She painted $\frac{1}{5}$ more of the same wall in the afternoon. How much of the wall did she paint that day?

2. Laurie and Ruth together painted $\frac{7}{8}$ of one wall in the morning. Laurie painted $\frac{4}{8}$ of the wall. How much of the wall did Ruth paint?

3. Marion painted $\frac{4}{9}$ of the meeting room floor. Sharon painted $\frac{5}{9}$ of the same floor. Did they paint the whole floor?

4. Beverly and Rita painted all of the game room floor. Rita painted $\frac{3}{7}$ of the floor. How much of the floor did Beverly paint?

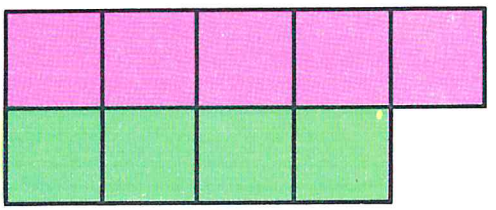
5. Sarah painted $\frac{5}{8}$ of one room. Sandy painted another $\frac{2}{8}$ of the same room. How much of the room did they paint?

6. Rosalind painted $\frac{3}{6}$ of the stage floor before she found out she had used the wrong color. She repainted $\frac{2}{6}$ of the floor. How much of the floor was still the wrong color?

7. Michele painted $\frac{3}{5}$ of one wall. How much of the wall did she have left to paint?

## Fractional numbers greater than 1

The red rod matches 5 of the fourths of the green rod.
The fraction that compares the red rod with the green rod is $\frac{5}{4}$ (five fourths).

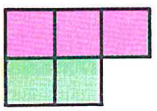


A fraction in the form $1 + \frac{1}{4}$ is an **expanded numeral.**

$$1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$$

### EXERCISES

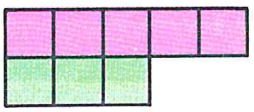
Write fractions to compare the red rods with the green rods.



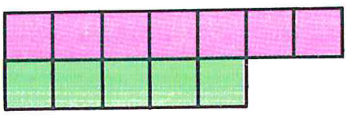
1. $\frac{a}{2}$

2. $\frac{c}{3}$

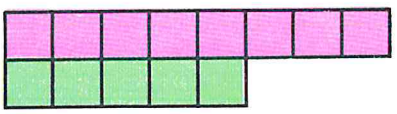
3. $\frac{e}{5}$



4. $\frac{n}{3}$



5. $\frac{r}{5}$



6. $\frac{s}{5}$

Solve the equations.

7. $1 + \frac{1}{3} = \frac{a}{3} + \frac{1}{3} = \frac{c}{3}$

8. $1 + \frac{2}{3} = \frac{a}{3} + \frac{2}{3} = \frac{c}{3}$

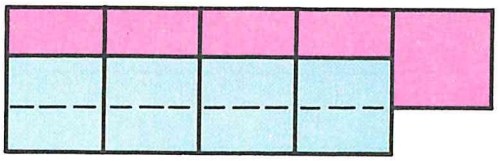
9. $1 + \frac{1}{4} = \frac{a}{4} + \frac{1}{4} = \frac{c}{4}$

10. $1 + \frac{3}{4} = \frac{a}{4} + \frac{3}{4} = \frac{c}{4}$

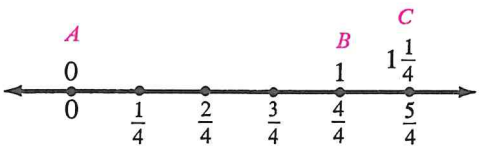
11. $1 + \frac{5}{8} = \frac{a}{8} + \frac{5}{8} = \frac{c}{8}$

12. $1 + \frac{7}{8} = \frac{a}{8} + \frac{7}{8} = \frac{c}{8}$

## Fractional numbers greater than 1



The longer rod is $\frac{5}{4}$ of the shorter rod.




$\overline{AC}$ is $\frac{5}{4}$ of $\overline{AB}$.

$$\frac{5}{4} = \frac{4}{4} + \frac{1}{4} = 1 + \frac{1}{4}$$

## EXERCISES

Name the number of thirds in

**1.** 1 **2.** $1 + \frac{1}{3}$ **3.** $1 + \frac{2}{3}$ **4.** 2 **5.** $2 + \frac{1}{3}$ **6.** $2 + \frac{2}{3}$

Name the number of fourths in

**7.** 1 **8.** $1 + \frac{1}{4}$ **9.** $1 + \frac{2}{4}$ **10.** $1 + \frac{3}{4}$ **11.** 2 **12.** $2 + \frac{1}{4}$

Name the number of fifths in

**13.** 1 **14.** $1 + \frac{1}{5}$ **15.** $1 + \frac{2}{5}$ **16.** $1 + \frac{3}{5}$ **17.** $1 + \frac{4}{5}$ **18.** 2

Solve the equations.

**19.** $1 + \frac{1}{3} = \frac{a}{3}$ **22.** $2 + \frac{2}{3} = \frac{z}{3}$ **25.** $1 + \frac{3}{4} = \frac{r}{4}$ **28.** $\frac{7}{5} = 1 + v$

**20.** $1 + \frac{2}{3} = \frac{c}{3}$ **23.** $1 + \frac{1}{4} = \frac{m}{4}$ **26.** $2 + \frac{1}{4} = \frac{s}{4}$ **29.** $\frac{8}{5} = 1 + x$

**21.** $2 + \frac{1}{3} = \frac{e}{3}$ **24.** $1 + \frac{2}{4} = \frac{n}{4}$ **27.** $\frac{6}{5} = 1 + u$ **30.** $\frac{9}{5} = 1 + z$

Complete the number patterns.

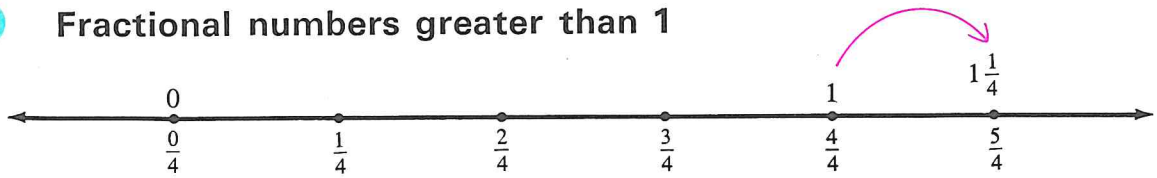
**31.**

| + | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{1}{5}$ |
|---|---|---|---|---|---|---|
| 1 | $\frac{4}{3}$ | $\frac{a}{3}$ | $\frac{c}{4}$ | $\frac{e}{4}$ | $\frac{m}{4}$ | $\frac{n}{5}$ |

**32.**

| + | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{1}{5}$ |
|---|---|---|---|---|---|---|
| 2 | $\frac{7}{3}$ | $\frac{a}{3}$ | $\frac{c}{4}$ | $\frac{e}{4}$ | $\frac{m}{4}$ | $\frac{n}{5}$ |

## Fractional numbers greater than 1



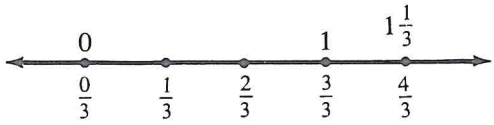

The number line shows that $1 + \frac{1}{4} = \frac{5}{4}$. A short way to write $1 + \frac{1}{4}$ is $1\frac{1}{4}$. Fractions like $1\frac{1}{4}$ are **mixed numerals.**

### EXERCISES

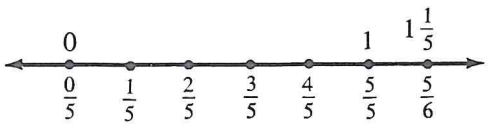
Write the expanded numerals as mixed numerals.

**1.** $1 + \frac{1}{2}$ **2.** $1 + \frac{3}{4}$ **3.** $1 + \frac{1}{3}$ **4.** $1 + \frac{2}{5}$ **5.** $1 + \frac{3}{8}$ **6.** $1 + \frac{2}{5}$ **7.** $1 + \frac{5}{8}$ **8.** $1 + \frac{2}{3}$

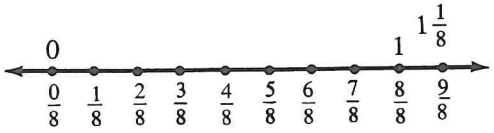
Use the number lines to solve the equations.



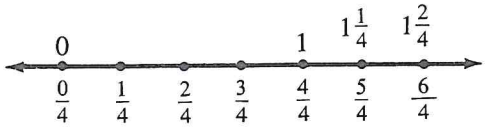

**9.** $1 + \frac{1}{3} = \frac{a}{3}$



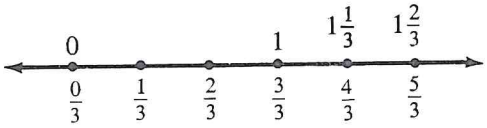

**10.** $1 + \frac{1}{5} = \frac{c}{5}$



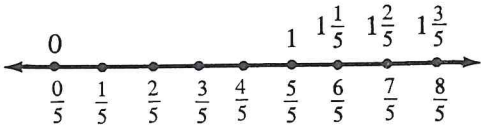

**11.** $1 + \frac{1}{8} = \frac{a}{8}$




**12.** $1 + \frac{2}{4} = \frac{n}{4}$



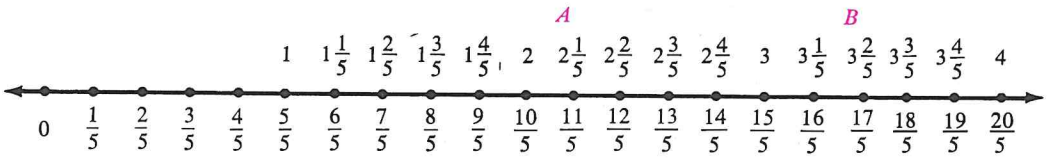

**13.** $1 + \frac{2}{3} = \frac{r}{3}$




**14.** $1 + \frac{3}{5} = \frac{s}{5}$

Solve the equations.

**15.** $1\frac{1}{3} = \frac{a}{3}$ **16.** $1\frac{1}{2} = \frac{c}{2}$ **17.** $1\frac{3}{4} = \frac{e}{4}$ **18.** $1\frac{1}{5} = \frac{m}{5}$ **19.** $1\frac{3}{5} = \frac{n}{5}$ **20.** $1\frac{2}{3} = \frac{r}{3}$ **21.** $1\frac{3}{8} = \frac{s}{8}$ **22.** $1\frac{5}{8} = \frac{u}{8}$




$A$ labels the point $2 + \frac{1}{5} = \frac{(2 \times 5) + 1}{5} = \frac{11}{5}$

$B$ labels the point $3 + \frac{2}{5} = \frac{(3 \times 5) + 2}{5} = \frac{17}{5}$

## EXERCISES

Name the number

| | | | | | | |
|---|---|---|---|---|---|---|
| **1.** | of halves | **a.** in 1 | **b.** in 2 | **c.** in 3 | **d.** in 4 | **e.** in 5 |
| **2.** | of fourths | **a.** in 1 | **b.** in 2 | **c.** in 3 | **d.** in 4 | **e.** in 5 |
| **3.** | of fifths | **a.** in 1 | **b.** in 2 | **c.** in 3 | **d.** in 4 | **e.** in 5 |
| **4.** | of eighths | **a.** in 1 | **b.** in 2 | **c.** in 3 | **d.** in 4 | **e.** in 5 |

Name the number

| | | | | | | |
|---|---|---|---|---|---|---|
| **5.** | of halves | **a.** in $1\frac{1}{2}$ | **b.** in $2\frac{1}{2}$ | **c.** in $3\frac{1}{2}$ | **d.** in $4\frac{1}{2}$ | **e.** in $5\frac{1}{2}$ |
| **6.** | of fourths | **a.** in $1\frac{1}{4}$ | **b.** in $2\frac{1}{4}$ | **c.** in $3\frac{1}{4}$ | **d.** in $4\frac{1}{4}$ | **e.** in $5\frac{1}{4}$ |
| **7.** | of fifths | **a.** in $1\frac{1}{5}$ | **b.** in $2\frac{1}{5}$ | **c.** in $3\frac{1}{5}$ | **d.** in $4\frac{1}{5}$ | **e.** in $5\frac{1}{5}$ |
| **8.** | of eighths | **a.** in $1\frac{1}{8}$ | **b.** in $2\frac{1}{8}$ | **c.** in $3\frac{1}{8}$ | **d.** in $4\frac{1}{8}$ | **e.** in $5\frac{1}{8}$ |

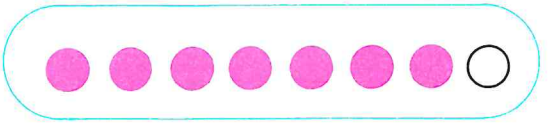
$\frac{23}{4}$ may be renamed as $5+\frac{3}{4}$ or $5\frac{3}{4}$.

Rename each of these in two ways.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **9.** | $\frac{7}{2}$ | **11.** | $\frac{19}{14}$ | **13.** | $\frac{12}{5}$ | **15.** | $\frac{15}{4}$ | **17.** | $\frac{16}{5}$ | **19.** | $\frac{17}{4}$ |
| **10.** | $\frac{8}{3}$ | **12.** | $\frac{11}{4}$ | **14.** | $\frac{14}{5}$ | **16.** | $\frac{17}{3}$ | **18.** | $\frac{18}{5}$ | **20.** | $\frac{19}{4}$ |

## Comparing fractional numbers

$\frac{3}{8}$ of the circles are red.



$\frac{7}{8}$ of the circles are red.

The **numerators** are 3 and 7. The **denominators** are 8.

$$\frac{3}{8} < \frac{7}{8}$$

### EXERCISES

Complete the sentence and the exercise.

1. In the fraction $\frac{3}{4}$, the numerator is ___?___ and the denominator is ___?___.

2. Write the fractions $\frac{6}{7}, \frac{3}{7}, \frac{4}{7}, \frac{2}{7}, \frac{1}{7}, \frac{7}{7}$, and $\frac{5}{7}$ in order from least to greatest.

Complete the number sentences. Write $<$ or $>$ to replace each ◯.

3. $\frac{1}{3} \bigcirc \frac{2}{3}$
4. $\frac{2}{6} \bigcirc \frac{3}{6}$
5. $\frac{4}{9} \bigcirc \frac{5}{9}$
6. $\frac{7}{8} \bigcirc \frac{3}{8}$
7. $\frac{5}{8} \bigcirc \frac{2}{8}$
8. $\frac{3}{8} \bigcirc \frac{5}{8}$
9. $\frac{1}{5} \bigcirc \frac{2}{5}$
10. $\frac{2}{5} \bigcirc \frac{3}{5}$
11. $\frac{4}{5} \bigcirc \frac{1}{5}$
12. $\frac{1}{4} \bigcirc \frac{2}{4}$

$\frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$. Since $\frac{2}{8} > \frac{1}{8}, \frac{1}{4} > \frac{1}{8}$.

Complete the number sentences.

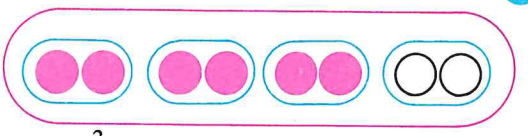
13. $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$, so $\frac{1}{2} \bigcirc \frac{1}{4}$
14. $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$, so $\frac{1}{3} \bigcirc \frac{1}{6}$
15. $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$, so $\frac{2}{4} \bigcirc \frac{3}{4}$
16. $\frac{4}{8} + \frac{1}{8} = \frac{5}{8}$, so $\frac{4}{8} \bigcirc \frac{5}{8}$
17. $\frac{2}{3} + \frac{1}{3} = 1$, so $1 \bigcirc \frac{2}{3}$
18. $\frac{3}{8} + \frac{1}{8} = \frac{1}{2}$, so $\frac{1}{2} \bigcirc \frac{3}{8}$

Complete the sentence.

19. If the denominators remain the same, the greater the numerator, the ___?___ the fractional number.

## Comparing fractional numbers

$\frac{3}{8}$ of the circles are red.



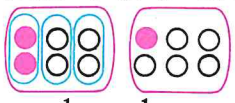
$\frac{3}{4}$ of the circles are red.

The numerators are 3. The denominators are 8 and 4.

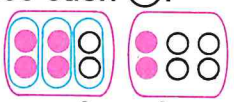
$$\frac{3}{8} < \frac{3}{4}$$

### EXERCISES

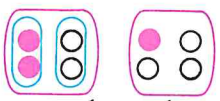
Use the sets to complete the number sentences.
Write $<$ or $>$ to replace each ◯.



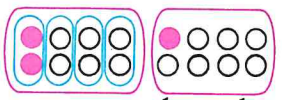
1. $\frac{1}{3}$ ◯ $\frac{1}{6}$



2. $\frac{2}{3}$ ◯ $\frac{2}{6}$

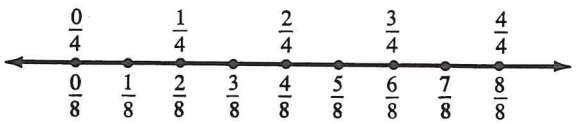


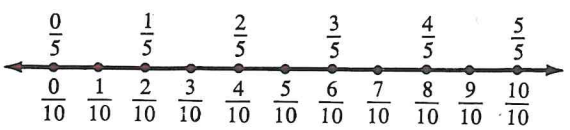
3. $\frac{1}{2}$ ◯ $\frac{1}{4}$



4. $\frac{1}{4}$ ◯ $\frac{1}{8}$

Use the number lines to complete the number sentences.
Write $<$ or $>$ to replace each ◯.



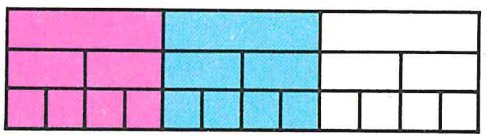

5. $\frac{3}{8}$ ◯ $\frac{3}{4}$

6. $\frac{2}{4}$ ◯ $\frac{2}{8}$

7. $\frac{3}{10}$ ◯ $\frac{3}{5}$

8. $\frac{4}{5}$ ◯ $\frac{4}{10}$

Use the number rods to complete the number sentences.
Write $<$ or $>$ to replace each ◯.



9. $\frac{1}{6}$ ◯ $\frac{1}{3}$

10. $\frac{1}{12}$ ◯ $\frac{1}{6}$

11. $\frac{2}{6}$ ◯ $\frac{2}{3}$

12. $\frac{2}{12}$ ◯ $\frac{2}{6}$

13. $\frac{5}{6}$ ◯ $\frac{5}{12}$

14. $\frac{4}{6}$ ◯ $\frac{4}{12}$

15. $\frac{3}{6}$ ◯ $\frac{3}{12}$

16. $\frac{2}{12}$ ◯ $\frac{2}{3}$

Complete the sentence.

17. If the numerator remains the same, the greater the denominator the ___?___ the fractional number.

## Sharpshooting

Answer set **A** as quickly as you can. Then see if you can answer set **B** more quickly.

### A

1. Name the sum.
   $20{,}376 + 89{,}815$
2. How far can a bird fly in 12 hours, if its speed is 47 miles per hour?
3. Name the greatest number.
   $(31 \times 21)$, 650, $(7 \times 94)$
4. Name the total cost of 8 baseballs, if each ball costs \$1.17.
5. How many quarts are in 7 gallons?
6. Name the perimeter of a square with sides of 3 inches.
7. If thirteen cookies make a baker's dozen, how many cookies are in 7 baker dozens?
8. Two hundred fifty miles in 5 hours is a speed of __?__ miles per hour?
9. Solve the equation.
   $\frac{2}{3} \times 12 = m$
10. Name the sum.
    $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = m$
11. One half of eight is __?__.
12. Name the quotient.
    $9\overline{)112}$

### B

1. Name the difference.
   $98{,}124 - 79{,}367$
2. Name the average of 25, 37, 62, and 48.
3. Name the least number.
   342, $12 \times 29$, $5 \times 70$
4. Name the difference between \$3.97 and \$4.23.
5. How many inches are in 5 yards?
6. What is the area of a rectangle that is 8 feet long and 11 feet wide?
7. If there are five fingers per hand, how many fingers are on 60 hands?
8. Three hundred sixty eggs make how many dozen?
9. Solve the equation.
   $5 \times \frac{1}{7} = n$
10. Name the difference.
    $\frac{7}{8} - \frac{5}{8} = n$
11. Solve the equation.
    $8 \times 5 = n \times 10$
12. Name the remainder.
    $13\overline{)270}$

## Matching sets

John read 6 books in 3 days. He thought of this as matching two sets, a set of 6 books and a set of 3 days. He said, "I was reading *at the rate* of 2 books *per* day." $6 \div 3 = 2$

### EXERCISES

Write the missing numerals to name the rates.

1. 30 cents for 5 candy bars: ___?___ cents per candy bar
2. 12 miles in 3 hours: ___?___ miles per hour
3. 8 apples for 4 children: ___?___ apples per child
4. \$12 for 6 hours: ___?___ per hour
5. 20 passengers in 4 cars: ___?___ passengers per car
6. 120 miles in 3 hours: ___?___ miles per hour
7. 30 children in 3 rooms: ___?___ children per room
8. 18 apples for 6 children: ___?___ apples per child
9. 60 cents for 5 orange drinks: ___?___ cents per orange drink
10. 30 cup cakes for 6 children: ___?___ cup cakes per child

Use the rates to help you name the amounts.

11. 6 cents per candy bar: ___?___ cents for 7 candy bars
12. 20 miles per hour: ___?___ miles in 4 hours
13. 5 passengers per car: ___?___ passengers in 8 cars
14. 30 children per room: ___?___ children in 6 rooms
15. 10 cents per drink: ___?___ cents for 7 drinks
16. \$2 per hour: ___?___ in 8 hours
17. 4 apples per child: ___?___ apples for 100 children
18. 5 fish per fishbowl: ___?___ fish in 4 fishbowls

## Equivalent fractions

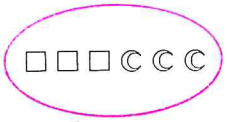
May says, "I have read *half* of the books."
Betty says, "I have read *five tenths* of the books."
Have May and Betty read the same number of books?

### EXERCISES

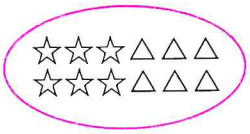
Solve the equations.

**1.**

$\frac{1}{2} = \frac{a}{4}$

**2.** 

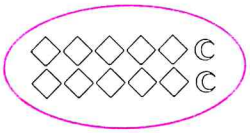
$\frac{1}{2} = \frac{c}{6}$

**3.** 

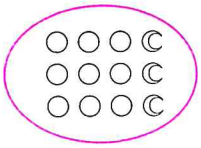
$\frac{1}{2} = \frac{e}{12}$

**4.**

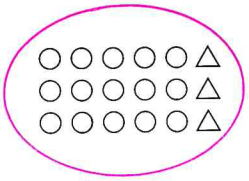
$\frac{4}{5} = \frac{n}{10}$

**5.** 

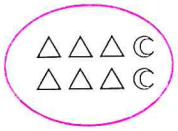
$\frac{5}{6} = \frac{r}{12}$

**6.** 

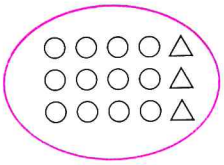
$\frac{3}{4} = \frac{s}{12}$

**7.** 

$\frac{5}{6} = \frac{v}{18}$

**8.** 

$\frac{3}{4} = \frac{m}{8}$

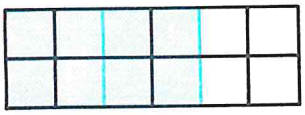
**9.** 

$\frac{4}{5} = \frac{t}{15}$

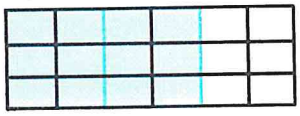
Use the regions to solve the equations.

**10.**

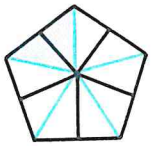
$\frac{2}{3} = \frac{a}{6}$

**11.** 

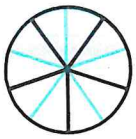
$\frac{2}{3} = \frac{c}{12}$

**12.** 

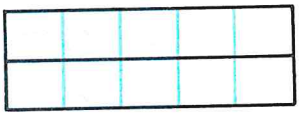
$\frac{2}{3} = \frac{e}{18}$

**13.** 

$\frac{1}{5} = \frac{m}{10}$

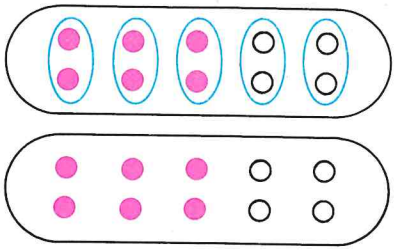
**14.** 

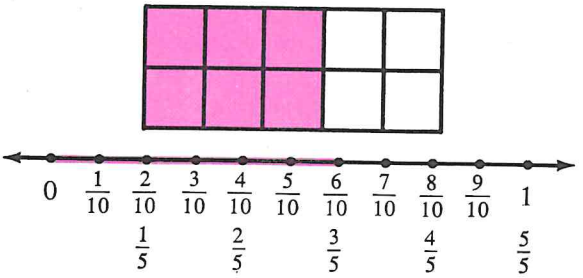
$\frac{2}{5} = \frac{n}{10}$

**15.** 

$\frac{3}{5} = \frac{r}{10}$

Study the sets, the rods, and the number lines.






Here are three ways of showing $\frac{3}{5} = \frac{6}{10}$.

## EXERCISES

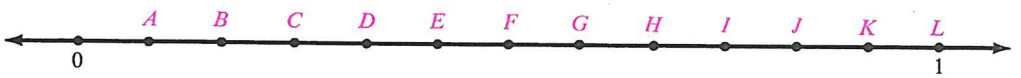
Name the number of red squares in the chart as compared with the total number of squares in the chart.

**1.**

| number of red squares | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| total number of squares | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| fractional number | *a* | *c* | *e* | *i* | *m* | *n* | *r* | *s* | *u* | *v* |

**2.**

| number of red squares | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| total number of squares | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| fractional number | *a* | *c* | *e* | *i* | *m* | *n* | *r* | *s* | *u* | *v* |




Study the number line. Write two fractions for each point.

**3.** *B* **4.** *C* **5.** *D* **6.** *F* **7.** *H* **8.** *I*

Complete the equivalent fractions in each set.

**9.** $\left\{\frac{2}{3}, \frac{4}{6}, \frac{a}{9}, \frac{c}{12}, \frac{e}{15}, \frac{n}{18}\right\}$

**10.** $\left\{\frac{4}{5}, \frac{8}{10}, \frac{a}{15}, \frac{c}{20}, \frac{e}{25}, \frac{n}{30}\right\}$

**11.** $\left\{\frac{3}{4}, \frac{6}{8}, \frac{a}{12}, \frac{c}{16}, \frac{e}{20}, \frac{n}{24}\right\}$

**12.** $\left\{\frac{5}{6}, \frac{10}{12}, \frac{a}{18}, \frac{c}{24}, \frac{e}{30}, \frac{n}{36}\right\}$

## Equivalent fractions

Equivalent fractions name the same fractional number. In adding and subtracting fractional numbers we must work with fractions with the same denominator, so we often have to name equivalent fractions.

Remember how we use the set of names for 1: $\left\{\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \ldots\right\}$.

To name an equivalent fraction we can multiply or divide both numerator and denominator of a fraction by the same number. For example,

$$\frac{3}{4} = \frac{2 \times 3}{2 \times 4} = \frac{6}{8} \qquad \frac{3}{4} = \frac{3 \times 3}{3 \times 4} = \frac{9}{12} \qquad \frac{3}{4} = \frac{6}{8} = \frac{9}{12}$$

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4} \qquad \frac{15}{20} = \frac{15 \div 5}{20 \div 5} = \frac{3}{4} \qquad \frac{12}{16} = \frac{15}{20} = \frac{3}{4}$$

### EXERCISES

Use equations to rename

1. $\frac{2}{3}$ as $\frac{a}{6}$
2. $\frac{4}{5}$ as $\frac{c}{10}$
3. $\frac{5}{9}$ as $\frac{i}{27}$
4. $\frac{6}{7}$ as $\frac{12}{m}$
5. $\frac{6}{9}$ as $\frac{r}{3}$
6. $\frac{5}{10}$ as $\frac{s}{2}$
7. $\frac{15}{20}$ as $\frac{3}{v}$
8. $\frac{30}{50}$ as $\frac{x}{5}$

Complete the equivalent fractions. Name the simplest fraction in each set.

9. $\left\{\frac{1}{4}, \frac{2}{8}, \frac{a}{12}, \frac{c}{16}, \frac{e}{20}\right\}$
10. $\left\{\frac{2}{5}, \frac{d}{10}, \frac{a}{15}, \frac{c}{20}, \frac{e}{25}\right\}$
11. $\left\{\frac{1}{2}, \frac{a}{4}, \frac{3}{c}, \frac{4}{e}, \frac{5}{n}\right\}$
12. $\left\{\frac{3}{8}, \frac{6}{16}, \frac{a}{24}, \frac{c}{32}, \frac{e}{40}\right\}$

Complete the sentences.

13. To rename a fraction with a greater denominator we ___?___ the numerator and denominator by the same factor.
14. To rename a fraction in its simplest form we ___?___ the numerator and denominator by the greatest common factor.

## Renaming one addend

To name the sum $\frac{1}{2} + \frac{1}{8}$ or the difference $\frac{1}{2} - \frac{1}{8}$, we must first rename $\frac{1}{2}$ as $\frac{4}{8}$.

$$\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$$

$$\begin{array}{r} \frac{1}{2} = \frac{4}{8} \\ +\ \frac{1}{8} = +\ \frac{1}{8} \\ \hline \frac{5}{8} \end{array}$$

$$\frac{1}{2} - \frac{1}{8} = \frac{4}{8} - \frac{1}{8} = \frac{3}{8}$$

$$\begin{array}{r} \frac{1}{2} = \frac{4}{8} \\ -\ \frac{1}{8} = -\ \frac{1}{8} \\ \hline \frac{3}{8} \end{array}$$

## EXERCISES

Complete the sentence.

1. When we add or subtract fractional numbers the fractions must have the same ___?___.

Rename the addend. Then name the sum or difference.

2. $\frac{1}{2} + \frac{1}{4} = \frac{a}{4} + \frac{1}{4}$

3. $\frac{1}{4} + \frac{1}{8} = \frac{c}{8} + \frac{1}{8}$

4. $\frac{3}{4} - \frac{1}{8} = \frac{e}{8} - \frac{1}{8}$

5. $\frac{1}{3} + \frac{1}{6} = \frac{m}{6} + \frac{1}{6}$

6. $\frac{2}{3} - \frac{1}{6} = \frac{n}{6} - \frac{1}{6}$

7. $\frac{1}{6} - \frac{1}{12} = \frac{r}{12} - \frac{1}{12}$

8. $\frac{1}{2} + \frac{3}{8} = \frac{s}{8} + \frac{3}{8}$

9. $\frac{1}{4} + \frac{3}{8} = \frac{u}{8} + \frac{3}{8}$

10. $\frac{1}{5} - \frac{1}{10} = \frac{v}{10} - \frac{1}{10}$

Name the sums or differences.

11. $\begin{array}{r} \frac{2}{3} \\ +\ \frac{1}{6} \\ \hline \end{array}$

12. $\begin{array}{r} \frac{1}{3} \\ -\ \frac{1}{6} \\ \hline \end{array}$

13. $\begin{array}{r} \frac{3}{5} \\ +\ \frac{1}{10} \\ \hline \end{array}$

14. $\begin{array}{r} \frac{2}{5} \\ +\ \frac{2}{10} \\ \hline \end{array}$

15. $\begin{array}{r} \frac{3}{4} \\ +\ \frac{1}{8} \\ \hline \end{array}$

16. $\begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{8} \\ \hline \end{array}$

17. $\begin{array}{r} \frac{5}{10} \\ -\ \frac{1}{5} \\ \hline \end{array}$

18. $\begin{array}{r} \frac{3}{4} \\ -\ \frac{5}{8} \\ \hline \end{array}$

19. $\begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{2} \\ \hline \end{array}$

20. $\begin{array}{r} \frac{5}{8} \\ -\ \frac{1}{4} \\ \hline \end{array}$

21. $\begin{array}{r} \frac{5}{8} \\ -\ \frac{1}{2} \\ \hline \end{array}$

22. $\begin{array}{r} \frac{7}{10} \\ -\ \frac{2}{5} \\ \hline \end{array}$

23. $\begin{array}{r} \frac{9}{10} \\ -\ \frac{4}{5} \\ \hline \end{array}$

24. $\begin{array}{r} \frac{7}{8} \\ -\ \frac{3}{4} \\ \hline \end{array}$

## Renaming both addends

To name the sum $\frac{1}{3} + \frac{1}{4}$ or the difference $\frac{1}{3} - \frac{1}{4}$, we must first rename both $\frac{1}{3}$ and $\frac{1}{4}$, to get fractions with the same denominator.

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

$$\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$$

$$\begin{array}{r} \frac{1}{3} = \frac{4}{12} \\ +\ \frac{1}{4} = +\ \frac{3}{12} \\ \hline \frac{7}{12} \end{array} \qquad \begin{array}{r} \frac{1}{3} = \frac{4}{12} \\ -\ \frac{1}{4} = -\ \frac{3}{12} \\ \hline \frac{1}{12} \end{array}$$

### EXERCISES

Name the sums.

1. $\frac{2}{3} + \frac{1}{4} = a$
2. $\frac{1}{3} + \frac{2}{4} = b$
3. $\frac{2}{3} + \frac{1}{8} = c$
4. $\frac{1}{8} + \frac{2}{3} = d$
5. $\frac{1}{4} + \frac{1}{5} = e$
6. $\frac{1}{4} + \frac{3}{5} = f$
7. $\frac{1}{3} + \frac{2}{5} = g$
8. $\frac{2}{3} + \frac{1}{5} = h$

Name the differences.

9. $\frac{2}{3} - \frac{1}{4} = a$
10. $\frac{1}{3} - \frac{1}{8} = b$
11. $\frac{2}{3} - \frac{3}{8} = c$
12. $\frac{2}{3} - \frac{2}{5} = d$
13. $\frac{2}{3} - \frac{1}{8} = e$
14. $\frac{1}{4} - \frac{1}{5} = f$
15. $\frac{3}{5} - \frac{1}{4} = g$
16. $\frac{2}{3} - \frac{1}{5} = h$

Name the sums or differences.

17. $\begin{array}{r} \frac{1}{4} \\ +\ \frac{2}{3} \\ \hline \end{array}$
18. $\begin{array}{r} \frac{1}{2} \\ +\ \frac{1}{3} \\ \hline \end{array}$
19. $\begin{array}{r} \frac{2}{3} \\ -\ \frac{1}{2} \\ \hline \end{array}$
20. $\begin{array}{r} \frac{3}{4} \\ -\ \frac{2}{3} \\ \hline \end{array}$
21. $\begin{array}{r} \frac{2}{3} \\ -\ \frac{3}{5} \\ \hline \end{array}$
22. $\begin{array}{r} \frac{1}{2} \\ -\ \frac{1}{5} \\ \hline \end{array}$
23. $\begin{array}{r} \frac{1}{8} \\ +\ \frac{1}{3} \\ \hline \end{array}$
24. $\begin{array}{r} \frac{3}{8} \\ +\ \frac{1}{3} \\ \hline \end{array}$
25. $\begin{array}{r} \frac{7}{8} \\ -\ \frac{1}{3} \\ \hline \end{array}$
26. $\begin{array}{r} \frac{2}{3} \\ -\ \frac{5}{8} \\ \hline \end{array}$
27. $\begin{array}{r} \frac{3}{5} \\ -\ \frac{1}{2} \\ \hline \end{array}$
28. $\begin{array}{r} \frac{1}{2} \\ -\ \frac{2}{5} \\ \hline \end{array}$

## Using expanded numerals

Remember how we used expanded numerals for naming sums and differences with whole numbers. We can use the same method to name sums and differences with fractional numbers.

$$\begin{array}{rcl} 4\frac{1}{5} & = & 4 + \frac{1}{5} \\ +\ 2\frac{3}{5} & = & +\ 2 + \frac{3}{5} \\ \hline & & 6 + \frac{4}{5} = 6\frac{4}{5} \end{array}$$

$$\begin{array}{rcl} 4\frac{4}{5} & = & 4 + \frac{4}{5} \\ -\ 2\frac{3}{5} & = & -\ 2 - \frac{3}{5} \\ \hline & & 2 + \frac{1}{5} = 2\frac{1}{5} \end{array}$$

## EXERCISES

Copy and complete.

**1.**
$$\begin{array}{rcl} 3\frac{1}{3} & = & 3 + \frac{1}{3} \\ +\ 2\frac{1}{3} & = & +\ 2 + \frac{1}{3} \\ \hline & & a + c = e \end{array}$$

**2.**
$$\begin{array}{rcl} 5\frac{1}{4} & = & 5 + n \\ +\ 3\frac{1}{4} & = & +\ 3 + x \\ \hline & & a + c = e \end{array}$$

**3.**
$$\begin{array}{rcl} 3\frac{2}{3} & = & 3 + \frac{2}{3} \\ -\ 2\frac{1}{3} & = & -\ 2 - \frac{1}{3} \\ \hline & & a + c = e \end{array}$$

**4.**
$$\begin{array}{rcl} 5\frac{5}{8} & = & 5 + n \\ -\ 2\frac{1}{8} & = & -\ 2 - x \\ \hline & & a + c = e \end{array}$$

Name the sums or differences.

**5.** $4\frac{1}{8} + 5\frac{3}{8}$

**6.** $6\frac{1}{4} + 2\frac{1}{4}$

**7.** $7\frac{1}{8} + 5\frac{5}{8}$

**8.** $2\frac{1}{5} + 3\frac{1}{5}$

**9.** $2\frac{2}{6} + 9\frac{3}{6}$

**10.** $4\frac{1}{9} + 3\frac{7}{9}$

**11.** $6\frac{3}{5} - 2\frac{1}{5}$

**12.** $5\frac{7}{8} - 2\frac{3}{8}$

**13.** $5\frac{5}{6} - 2\frac{1}{6}$

**14.** $4\frac{5}{8} - 3\frac{3}{8}$

**15.** $4\frac{7}{9} - 2\frac{2}{9}$

**16.** $13\frac{5}{6} - 4\frac{2}{6}$

## Addition and subtraction of fractional numbers

Remember how we used the three step method for adding and subtracting whole numbers. We can use this method to add and subtract fractional numbers. Here are two examples.

$$\begin{array}{r} 4\frac{1}{5} \\ +\ 2\frac{3}{5} \\ \hline \frac{4}{5} \\ 6 \\ \hline 6\frac{4}{5} \end{array} \qquad \begin{array}{r} 4\frac{4}{5} \\ -\ 1\frac{3}{5} \\ \hline \frac{1}{5} \\ 3 \\ \hline 3\frac{1}{5} \end{array}$$

### EXERCISES

Name the sums or differences.

| | | | | | |
|---|---|---|---|---|---|
| **1.** $4\frac{1}{8} + 5\frac{3}{8}$ | **6.** $3\frac{1}{4} + 7\frac{1}{4}$ | **11.** $2\frac{2}{6} + 3\frac{3}{6}$ | **16.** $3\frac{2}{8} + 6\frac{5}{8}$ | **21.** $6\frac{3}{9} + 5\frac{4}{9}$ | **26.** $2\frac{1}{8} + 3\frac{3}{8}$ |
| **2.** $6\frac{1}{8} + 5\frac{5}{8}$ | **7.** $3\frac{4}{8} + 4\frac{3}{8}$ | **12.** $4\frac{1}{5} + 3\frac{3}{5}$ | **17.** $4\frac{1}{6} + 3\frac{4}{6}$ | **22.** $7\frac{4}{7} + 3\frac{2}{7}$ | **27.** $5\frac{1}{3} + 4\frac{1}{3}$ |
| **3.** $6\frac{6}{7} - 3\frac{2}{7}$ | **8.** $5\frac{7}{8} - 4\frac{3}{8}$ | **13.** $3\frac{5}{8} - 2\frac{3}{8}$ | **18.** $8\frac{7}{9} - 4\frac{2}{9}$ | **23.** $3\frac{4}{5} - 1\frac{3}{5}$ | **28.** $2\frac{3}{4} - 1\frac{1}{4}$ |
| **4.** $3\frac{3}{5} + 5\frac{1}{5}$ | **9.** $6\frac{3}{8} + 2\frac{3}{8}$ | **14.** $3\frac{1}{4} - 1\frac{1}{4}$ | **19.** $3\frac{1}{11} + 4\frac{4}{11}$ | **24.** $4\frac{5}{9} - 3\frac{2}{9}$ | **29.** $4\frac{4}{8} + 3\frac{3}{8}$ |
| **5.** $4\frac{1}{7} + 2\frac{2}{7}$ | **10.** $7\frac{8}{13} - 2\frac{5}{13}$ | **15.** $8\frac{5}{8} + 4\frac{2}{8}$ | **20.** $4\frac{3}{4} - 2\frac{1}{4}$ | **25.** $6\frac{2}{8} + 2\frac{3}{8}$ | **30.** $7\frac{3}{10} + 2\frac{5}{10}$ |

## Fractional numbers greater than 1

In these examples an extra step is needed. Can you see why?

$$3\tfrac{1}{2} = 3\tfrac{2}{4} = 3 + \tfrac{2}{4}$$
$$+\ 1\tfrac{1}{4} = \quad +\ 1 + \tfrac{1}{4}$$
$$4 + \tfrac{3}{4} = 4\tfrac{3}{4}$$

$$3\tfrac{1}{2} = 3\tfrac{2}{4} = 3 + \tfrac{2}{4}$$
$$-\ 1\tfrac{1}{4} = \quad -\ 1 - \tfrac{1}{4}$$
$$2 + \tfrac{1}{4} = 2\tfrac{1}{4}$$

### EXERCISES

Use this method to name the sums or differences.

**1.** $3\frac{1}{2} + 1\frac{1}{4}$ **2.** $4\frac{1}{4} + 2\frac{2}{4}$ **3.** $3\frac{1}{8} + 2\frac{3}{4}$

**4.** $7\frac{3}{8} + 2\frac{1}{4}$ **5.** $12\frac{1}{2} - 3\frac{1}{4}$ **6.** $6\frac{1}{4} + 2\frac{3}{8}$

**7.** $4\frac{3}{4} - 2\frac{5}{8}$ **8.** $8\frac{7}{8} - 2\frac{1}{2}$ **9.** $4\frac{5}{8} + 2\frac{1}{8}$

**10.** $4\frac{3}{7} + 2\frac{2}{7}$ **11.** $4\frac{1}{5} + 1\frac{3}{5}$ **12.** $2\frac{5}{8} - 1\frac{1}{4}$

**13.** $4\frac{3}{4} + 2\frac{1}{8}$ **14.** $6\frac{1}{3} + 2\frac{3}{6}$ **15.** $6\frac{2}{3} + 2\frac{1}{6}$

**16.** $3\frac{7}{8} - 2\frac{1}{2}$ **17.** $6\frac{1}{2} + 2\frac{3}{8}$ **18.** $6\frac{1}{2} - 2\frac{3}{8}$

After some practice you may be able to do the extra step in your head. Your examples then look like this.

$$3\tfrac{1}{2} = 3 + \tfrac{2}{4}$$
$$+\ 1\tfrac{1}{4} = +\ 1 + \tfrac{1}{4}$$
$$4 + \tfrac{3}{4} = 4\tfrac{3}{4}$$

$$3\tfrac{1}{2} = 3 + \tfrac{2}{4}$$
$$-\ 1\tfrac{1}{4} = -\ 1 - \tfrac{1}{4}$$
$$2 + \tfrac{1}{4} = 2\tfrac{1}{4}$$

**19–36.** Use this method to name the sums and differences in exercises 1 to 18.

## Trading marbles

1. Vernon had 12 marbles. He told Glen he would trade $\frac{3}{4}$ of them for some of Glen's marbles. How many marbles would Vernon trade?

2. Stephen had 10 red and 9 yellow marbles. He traded $\frac{1}{5}$ of his red marbles and $\frac{2}{3}$ of his yellow marbles.
   a. How many red marbles did he trade?
   b. How many yellow marbles did he trade?

3. Pete had 10 marbles. John had 12 marbles. John traded $\frac{1}{2}$ of his marbles for $\frac{4}{5}$ of Pete's.
   a. Who got more marbles in the trade?
   b. How many more?

4. Donald had 8 orange and 15 green marbles. He traded $\frac{3}{4}$ of his orange marbles and $\frac{1}{3}$ of his green marbles.
   a. Did he trade more orange marbles or more green marbles?
   b. How many marbles did he trade in all?

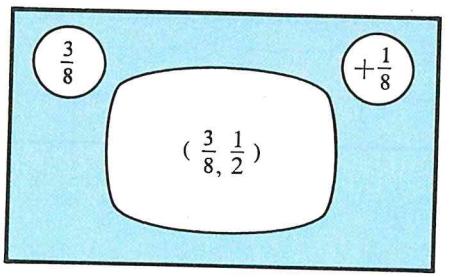
*5. George had 18 marbles. He traded $\frac{1}{3}$ of them for 7 of Vernon's. How many marbles does George have now?

*6. James was willing to trade $\frac{3}{5}$ of his 10 marbles for 2 of Sam's large yellow marbles.
   a. How many marbles was James willing to trade?
   b. How many marbles would James have after the trade?

## Practice with fractional numbers

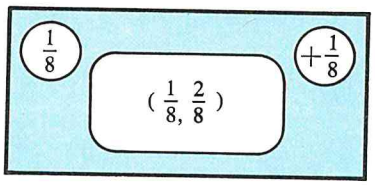
Our TV can show functions with fractional numbers. This TV shows the rule *add* $\frac{1}{8}$.

If the first number is $\frac{3}{8}$, the number pair is $\left(\frac{3}{8}, \frac{4}{8}\right)$ or $\left(\frac{3}{8}, \frac{1}{2}\right)$.

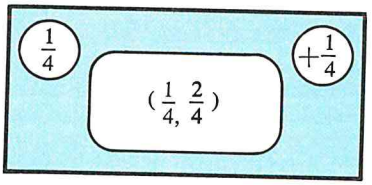



### EXERCISES

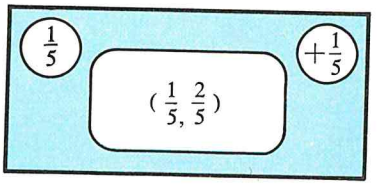
Use the TV to find the rule. Then complete the number pairs in each set.



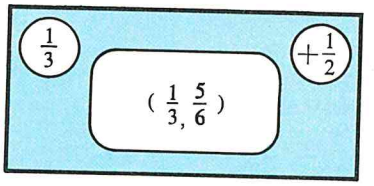

1. $\left\{\left(\frac{1}{8}, \frac{2}{8}\right), \left(\frac{1}{4}, \frac{3}{8}\right), \left(\frac{3}{8}, a\right), \left(\frac{1}{2}, c\right), \left(\frac{5}{8}, e\right), \left(\frac{7}{8}, r\right)\right\}$



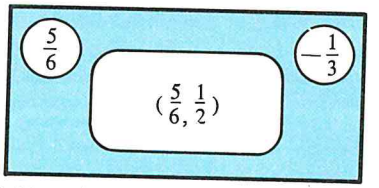

2. $\left\{\left(\frac{1}{4}, \frac{2}{4}\right), \left(\frac{2}{4}, \frac{3}{4}\right), \left(\frac{3}{4}, a\right), \left(1, c\right), \left(1\frac{1}{4}, e\right), \left(1\frac{3}{4}, r\right)\right\}$



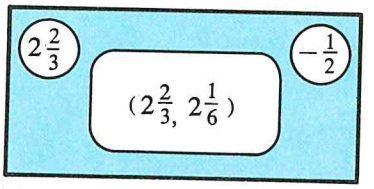

3. $\left\{\left(\frac{1}{5}, \frac{2}{5}\right), \left(\frac{2}{5}, \frac{3}{5}\right), \left(\frac{3}{5}, a\right), \left(\frac{4}{5}, c\right), \left(1, e\right), \left(1\frac{1}{5}, n\right)\right\}$




4. $\left\{\left(\frac{1}{3}, \frac{5}{6}\right), \left(\frac{1}{4}, a\right), \left(\frac{1}{5}, b\right), \left(\frac{2}{3}, c\right), \left(\frac{3}{4}, d\right), \left(\frac{5}{6}, f\right)\right\}$



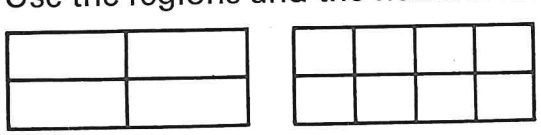

5. $\left\{\left(\frac{5}{6}, \frac{1}{2}\right), \left(\frac{2}{3}, a\right), \left(\frac{5}{9}, b\right), \left(\frac{7}{12}, c\right), \left(\frac{7}{15}, d\right), \left(\frac{17}{21}, f\right)\right\}$




6. $\left\{\left(2\frac{2}{3}, 2\frac{1}{6}\right), \left(1\frac{1}{2}, a\right), \left(1\frac{1}{3}, b\right), \left(2\frac{2}{5}, c\right), \left(5\frac{5}{6}, e\right)\right\}$

## CHECKING UP

Use the regions and the number lines to solve the equations.

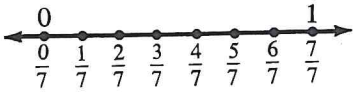


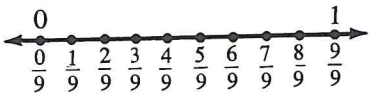
1. $\frac{1}{4} + \frac{1}{4} = a$

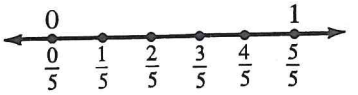
2. $\frac{7}{8} - \frac{3}{8} = b$

3. $\frac{5}{10} - \frac{4}{10} = c$

4. $\frac{3}{6} + \frac{2}{6} = d$




5. $\frac{4}{7} + \frac{2}{7} = e$

6. $\frac{8}{9} - \frac{5}{9} = f$

7. $1 - \frac{3}{5} = g$

Name the sums or differences.

8. $\frac{3}{7} + \frac{2}{7}$

9. $\frac{7}{8} - \frac{5}{8}$

10. $\frac{5}{9} + \frac{2}{9}$

11. $\frac{7}{11} - \frac{6}{11}$

12. $\frac{9}{10} - \frac{1}{10}$

13. $\frac{3}{4} + \frac{1}{4}$

14. $\frac{4}{5} - \frac{2}{5}$

15. $\frac{8}{12} + \frac{3}{12}$

Name the sums or differences.

16. $\begin{array}{r} \frac{5}{7} \\ +\ \frac{1}{7} \\ \hline \end{array}$

17. $\begin{array}{r} \frac{5}{9} \\ -\ \frac{1}{9} \\ \hline \end{array}$

18. $\begin{array}{r} \frac{2}{4} \\ +\ \frac{2}{4} \\ \hline \end{array}$

19. $\begin{array}{r} \frac{8}{10} \\ -\ \frac{6}{10} \\ \hline \end{array}$

20. $\begin{array}{r} \frac{5}{9} \\ -\ \frac{4}{9} \\ \hline \end{array}$

21. $\begin{array}{r} \frac{6}{7} \\ -\ \frac{2}{7} \\ \hline \end{array}$

Rename

22. $\frac{4}{5}$ as $\frac{a}{10}$

23. $\frac{3}{4}$ as $\frac{c}{12}$

24. $\frac{6}{9}$ as $\frac{e}{3}$

25. $\frac{5}{10}$ as $\frac{n}{2}$

Name the sums or differences.

26. $\frac{1}{5} + \frac{3}{5} = \frac{a}{5}$

27. $\frac{7}{8} - \frac{1}{8} = \frac{c}{8}$

28. $\frac{3}{8} + \frac{5}{8} = e$

29. $1 - \frac{3}{7} = m$

30. $\frac{1}{4} + \frac{1}{2} = n$

31. $\frac{1}{4} + \frac{5}{8} = r$

32. $\frac{3}{8} + \frac{1}{4} = s$

33. $\frac{5}{8} - \frac{1}{4} = x$

34. $\begin{array}{r} 4\frac{1}{4} \\ +\ 2\frac{1}{2} \\ \hline \end{array}$

35. $\begin{array}{r} 3\frac{1}{2} \\ +\ 2\frac{3}{8} \\ \hline \end{array}$

36. $\begin{array}{r} 5\frac{1}{6} \\ +\ 3\frac{1}{3} \\ \hline \end{array}$

37. $\begin{array}{r} 4\frac{1}{8} \\ +\ 2\frac{1}{4} \\ \hline \end{array}$

38. $\begin{array}{r} 7\frac{1}{2} \\ -\ 5\frac{1}{4} \\ \hline \end{array}$

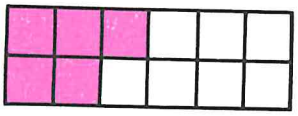
39. $\begin{array}{r} 8\frac{3}{4} \\ -\ 4\frac{1}{2} \\ \hline \end{array}$

A fraction is a numeral for a fractional number. A fractional number compares a subset with a set. There are many fractions for each fractional number.

Name the fractional numbers shown by the red subsets and region.

1.

2.

3. 

Complete the equivalent fractions in each set.

4. $\left\{\frac{2}{3}, \frac{a}{6}, \frac{c}{9}, \frac{8}{e}, \frac{10}{n}\right\}$

5. $\left\{\frac{1}{4}, \frac{2}{a}, \frac{3}{c}, \frac{e}{16}, \frac{n}{20}\right\}$

When we join subsets we add the fractional numbers. When we separate subsets we subtract the fractional numbers.

Solve the equations.

6. $\frac{1}{8} + \frac{4}{8} = \frac{a}{8}$

7. $\frac{1}{5} + \frac{3}{5} = \frac{c}{5}$

8. $\frac{1}{12} + \frac{10}{12} = \frac{e}{12}$

9. $\frac{5}{7} - \frac{2}{7} = \frac{n}{7}$

10. $1 - \frac{2}{5} = \frac{r}{5}$

11. $\frac{9}{10} - \frac{2}{10} = \frac{s}{10}$

The distributive property may be used to name the sum or difference of fractional numbers. For example:

$$\frac{2}{7} + \frac{4}{7} = \left(2 \times \frac{1}{7}\right) + \left(4 \times \frac{1}{7}\right)$$
$$= (2 + 4) \times \frac{1}{7} = \frac{6}{7}$$

Solve the equations.

12. $\frac{2}{8} + \frac{3}{8} = \left(a \times \frac{1}{8}\right) + \left(3 \times \frac{1}{8}\right)$
$= (b + 3) \times \frac{1}{8} = \frac{c}{8}$

13. $\frac{2}{10} + \frac{5}{10} = \left(2 \times \frac{1}{10}\right) + \left(a \times \frac{1}{10}\right)$
$= (2 + b) \times \frac{1}{10} = \frac{c}{10}$

## LOOKING WAY BACK

Name the sums and differences.

**1.** $14 + 28 = a$ **3.** $48 + 29 = c$ **5.** $41 - 29 = a$ **7.** $84 - 29 = c$

**2.** $39 + 57 = b$ **4.** $63 + 87 = d$ **6.** $37 - 18 = b$ **8.** $91 - 57 = d$

Name the sums and differences.

**9.**
$$\begin{array}{r} 46 \\ 39 \\ 29 \\ 56 \\ \underline{41} \end{array}$$

**11.**
$$\begin{array}{r} 927 \\ 386 \\ 497 \\ 285 \\ \underline{461} \end{array}$$

**13.**
$$\begin{array}{r} 570 \\ 234 \\ 506 \\ 483 \\ \underline{925} \end{array}$$

**15.**
$$\begin{array}{r} 286 \\ -\ 179 \end{array}$$

**19.**
$$\begin{array}{r} 6924 \\ -\ 4739 \end{array}$$

**16.**
$$\begin{array}{r} 407 \\ -\ 296 \end{array}$$

**20.**
$$\begin{array}{r} 5804 \\ -\ 4730 \end{array}$$

**10.**
$$\begin{array}{r} 578 \\ 263 \\ 485 \\ 507 \\ \underline{482} \end{array}$$

**12.**
$$\begin{array}{r} 385 \\ 279 \\ 436 \\ 384 \\ \underline{527} \end{array}$$

**14.**
$$\begin{array}{r} 7231 \\ 4654 \\ 3825 \\ 4629 \\ \underline{3704} \end{array}$$

**17.**
$$\begin{array}{r} 407 \\ -\ 297 \end{array}$$

**21.**
$$\begin{array}{r} 7631 \\ -\ 4198 \end{array}$$

**18.**
$$\begin{array}{r} 3896 \\ -\ 1289 \end{array}$$

**22.**
$$\begin{array}{r} 8390 \\ -\ 5486 \end{array}$$

Name the products.

**23.** $3 \times 46 = a$

**24.** $4 \times 27 = b$

**25.** $3 \times 89 = c$

**26.** $4 \times 87 = d$

**27.**
$$\begin{array}{r} 384 \\ \times\ 7 \end{array}$$

**28.**
$$\begin{array}{r} 289 \\ \times\ 9 \end{array}$$

**29.**
$$\begin{array}{r} 407 \\ \times\ 8 \end{array}$$

**30.**
$$\begin{array}{r} 473 \\ \times\ 6 \end{array}$$

**31.**
$$\begin{array}{r} 279 \\ \times\ 7 \end{array}$$

**32.**
$$\begin{array}{r} 973 \\ \times\ 5 \end{array}$$

**33.**
$$\begin{array}{r} 863 \\ \times\ 8 \end{array}$$

**34.**
$$\begin{array}{r} 754 \\ \times\ 6 \end{array}$$

**35.**
$$\begin{array}{r} 916 \\ \times\ 9 \end{array}$$

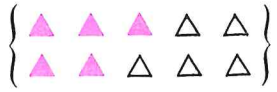
Write a compact numeral for

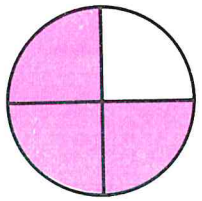
**36.** 4 million + 436 thousand + 745

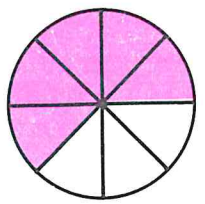
**37.** 748 thousand, 2 hundred, 50

**38.** nineteen million, nine hundred forty three thousand, six hundred eighty two

Write fractions for the red regions.

**39.** 

**40.** 

**41.** 

## EXTRA FOR EXPERTS

**1.** Use the digits 0 to 9 to name 100. Each digit may be used only once. For example,

$$81 + 9 + (2 \times 5) = 100$$

Here is a new method for multiplying. Multiply from the left and cross out any digit as another number is added to it. Here is how we name the product $8326 \times 7$.

| | | | |
|---|---|---|---|
| 8 3 2 6 | 8 3 2 6 | 8 3 2 6 | 8 3 2 6 |
| × 7 | × 7 | × 7 | × 7 |
| 5 6 | 5 6 1 | 5 6 1 4 | 5 6 1 4 2 |
| | 2 | 2 1 | 2 1 4 |
| | 8 | 8 2 | 8 2 8 |

$$8326 \times 7 = 58{,}282$$

Use this method to name the products.

**2.** $6192 \times 4$ **4.** $3874 \times 9$ **6.** $6924 \times 6$ **8.** $9675 \times 7$ **10.** $8421 \times 8$ **12.** $9610 \times 4$

**3.** $6235 \times 7$ **5.** $4873 \times 5$ **7.** $7386 \times 3$ **9.** $7452 \times 9$ **11.** $3976 \times 6$ **13.** $8514 \times 3$

Copy and continue the patterns until there are 9 rows.

**14.**
$$37 \times (3 \times 1) = 111$$
$$37 \times (3 \times 2) = 222$$
$$37 \times (3 \times 3) = 333$$
$$37 \times (3 \times 4) = 444$$

**15.**
$$(1 \times 9) - 1 = 8$$
$$(21 \times 9) - 1 = 188$$
$$(321 \times 9) - 1 = 2888$$
$$(4321 \times 9) - 1 = 38888$$

*Just For Fun*

**16.** "*Quotient* means *How many*," Liza said, "To divide 48 by 6, we have to find how many sixes there are in 48." Lorraine said, "There are no sixes in 48." What did Lorraine mean?

**17.** Here's a problem: If a man can run 100 yards in 10 seconds, how long will it take him to run 100 miles (a mile is 1760 yards)?
Does multiplication give a sensible answer?

## Different kinds of numbers

We have studied different kinds of numbers such as whole numbers, fractional numbers, prime numbers, even numbers, and odd numbers. We may list some of these as sets.

$W$ = {whole numbers} = {0, 1, 2, 3, 4, 5, . . .}
$P$ = {prime numbers} = {2, 3, 5, 7, 11, 13, . . .}
$O$ = {odd numbers} = {1, 3, 5, 7, 9, 11, . . .}

### EXERCISES

For $W$, $P$, and $O$ above

1. List the next six members of each set.
2. List the first five members of $P \cap O$.

For $F$ = {fractional numbers}, $E$ = {even numbers}, and $T$ = {multiples of 3}

3. List any six members of each set.
4. List the first five members of $E \cap T$.

Write *T* or *F* to show whether each statement is *true* or *false*.

5. All prime numbers are whole numbers.
6. All fractional numbers are whole numbers.
7. All multiples of 3 are odd numbers.
8. No even number is an odd number.
9. There is only one even prime number.
10. Some whole numbers are multiples of 3.

Write *F* or *I* to show whether each set is *finite* or *infinite*.

11. $W$ = {whole numbers}
12. $P$ = {prime numbers}
13. $O$ = {odd numbers}
14. $F$ = {fractional numbers}
15. $E$ = {even numbers}
16. $T$ = {multiples of 3}
17. $V$ = {a, e, i, o, u}
18. $C$ = {letters of the alphabet}

## Different uses for numbers

We use numbers in many different ways. For example,

in $6 + 4 = 10$ 4 is an addend,

in $3 + 1 = 4$ 4 is the sum,

in $4 \times 6 = 24$ 4 is a factor.

### EXERCISES

Match each equation or numeral on the left with its description on the right.

1. $18 \div 3 = 6$
2. $\frac{3}{4}$
3. $14 = 2 \times 7$
4. $3 \times 11 = 33$
5. $105 \div 15 = 7$
6. $12 \times 7 = 84$
7. $\frac{5}{12}$
8. $6 + 9 = 15$

a. 3 as a divisor

b. 12 as a factor

c. 5 as a numerator

d. 15 as a sum

e. 7 as a quotient

f. 33 as a product

g. 4 as a denominator

h. 14 as a multiple

Complete these number charts.

9.

| | | | | |
|---|---|---|---|---|
| **addend** | 7 | 4 | *c* | 23 |
| **addend** | 9 | *r* | 7 | 15 |
| **sum** | *a* | 13 | 15 | *m* |

10.

| | | | | |
|---|---|---|---|---|
| **factor** | 3 | 9 | *c* | 18 |
| **factor** | 7 | *s* | 8 | 5 |
| **product** | *a* | 54 | 64 | *v* |

11.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **numerator** | 1 | 2 | 3 | *a* | 5 | *r* | 6 | 7 | 8 | *x* | 10 | 3 |
| **denominator** | 7 | 5 | *c* | 9 | 3 | 8 | *m* | 9 | 10 | 10 | 12 | 14 |
| **fractional number** | *n* | *u* | $\frac{3}{4}$ | $\frac{7}{9}$ | *e* | $\frac{5}{8}$ | $\frac{6}{7}$ | *s* | *w* | $\frac{9}{10}$ | *z* | *t* |

12.

| **number** | **multiples of $n$** | | | | | |
|---|---|---|---|---|---|---|
| $n$ | $n \times 0$ | $n \times 1$ | $n \times 2$ | $n \times 3$ | $n \times 4$ | $n \times 5$ |
| 2 | 0 | 2 | *a* | *r* | *c* | *w* |
| 5 | *n* | *e* | 10 | *v* | *u* | *s* |

## Properties of the number operations

**Closure**

| $3 \times 4 = n$ | $3 \div 4 = n$ |
| --- | --- |
| The product of two whole numbers is always a whole number. | The quotient when one whole number is divided by another is not always a whole number. |
| We say that {whole numbers} is **closed** for multiplication. | We say that {whole numbers} is **not closed** for division. |

### EXERCISES

Match each statement with an equation.

1. The sum of two even numbers is always an even number.
2. The sum of two primes greater than 2 is always an even number.
3. The sum of two multiples of 5 is always a multiple of 5.
4. The product of two even numbers is an even number.
5. The difference between two multiples of 3 is a multiple of 3.
6. Some numbers are multiples of both 3 and 5.

a. $11 + 3 = 14$

b. $5 \times 3 = 15$

c. $15 - 9 = 6$

d. $10 + 15 = 25$

e. $24 + 8 = 32$

f. $2 \times 8 = 16$

Write *T* or *F* to show whether each statement is *true* or *false*.

7. The sum of two whole numbers is always a whole number.
8. The sum of two odd numbers is always an odd number.
9. The sum of two fractional numbers is always a whole number.
10. The difference between two whole numbers is always a whole number.
11. The product of two whole numbers is always a whole number.
12. If we divide a number by a greater number the quotient is a fractional number less than one.

## Properties of the number operations

| commutative property | associative property |
|---|---|
| $6 + 9 = 9 + 6 = 15$<br>$5 \times 8 = 8 \times 5 = 40$<br>The order of the numbers *does not* change the sum or product. | $(3 + 4) + 5 = 3 + (4 + 5) = 12$<br>$(3 \times 4) \times 5 = 3 \times (4 \times 5) = 60$<br>The way the numbers are grouped *does not* change the sum or product. |
| $9 - 6 = 3$ because $3 + 6 = 9$<br>$6 - 9 \neq 3$ because $3 + 9 \neq 6$<br>$9 - 6 \neq 6 - 9$<br><br>$4 \div 8 = \frac{1}{2}$ and $8 \div 4 = 2$<br>$4 \div 8 \neq 8 \div 4$<br>The order of the numbers *does* change the difference or quotient. | $8 - (4 - 3) = 8 - 1 = 7$<br>$(8 - 4) - 3 = 4 - 3 = 1$<br>$8 - (4 - 3) \neq (8 - 4) - 3$<br><br>$16 \div (4 \div 2) = 16 \div 2 = 8$<br>$(16 \div 4) \div 2 = 4 \div 2 = 2$<br>$16 \div (4 \div 2) \neq (16 \div 4) \div 2$<br>The way the numbers are grouped *does* change the difference or quotient. |

### EXERCISES

Write *T* or *F* to show whether each statement is true or false.

**1.** $18 + 7 = 7 + 18$

**2.** $18 - 7 = 7 - 18$

**3.** $5 \times 20 = 20 \times 5$

**4.** $20 \div 5 = 5 \div 20$

**5.** $3 \times 18 = 18 \times 3$

**6.** $18 \div 3 = 3 \div 18$

**7.** $\frac{3}{8} + \frac{5}{8} = \frac{5}{8} + \frac{3}{8}$

**8.** $\frac{5}{8} - \frac{3}{8} = \frac{3}{8} - \frac{5}{8}$

**9.** $386 + 14 = 14 + 386$

**10.** $386 - 14 = 14 - 386$

**11.** $(156 + 287) + 13 = 156 + (287 + 13)$

**12.** $(23 \times 13) \times 6 = 23 \times (13 \times 6)$

**13.** $(13 - 9) - 1 = 13 - (9 - 1)$

**14.** $(4 \times 3) \times 6 = 4 \times (3 \times 6)$

**15.** $(12 - 4) + 3 = 12 - (4 + 3)$

**16.** $(16 \div 4) \div 2 = 16 \div (4 \div 2)$

**17.** $(42 \times 6) \times 7 = 42 \times (6 \times 7)$

**18.** $18 - (6 - 2) = (18 - 6) - 2$

Do both ways give the same answer? Write *Yes* or *No*.

**19.**
$$\begin{array}{r} 13 \\ +\ 47 \\ \hline \end{array} \qquad \begin{array}{r} 47 \\ +\ 13 \\ \hline \end{array}$$

**20.**
$$\begin{array}{r} 26 \\ -\ 14 \\ \hline \end{array} \qquad \begin{array}{r} 14 \\ -\ 26 \\ \hline \end{array}$$

**21.** $3\overline{)15}$ $\quad$ $15\overline{)3}$

## Properties of the number operations

| Zero as an addend | One as a factor |
|---|---|
| $4 + 0 = 4$<br>The sum of zero and any addend is that addend. | $4 \times 1 = 4$<br>The product of one and any factor is that factor. |
| **Addition and subtraction are opposites**<br>$(4 + 3) - 3 = 4 + (3 - 3)$<br>$= 4 + 0 = 4$<br>Adding and then subtracting the same number is the same as adding zero.<br>$2 + 5 = 7$<br>$7 - 5 = 2$<br>Subtraction *undoes* addition. | **Multiplication and division are opposites**<br>$(4 \times 3) \div 3 = 4 \times (3 \div 3)$<br>$= 4 \times 1 = 4$<br>Multiplying and then dividing by the same number is the same as multiplying by one.<br>$5 \times 2 = 10$<br>$10 \div 2 = 5$<br>Division *undoes* multiplication. |

### EXERCISES

Solve the equations.

1. $3743 + 0 = a$
2. $2946 - 0 = b$
3. $7879 - 7879 = c$
4. $6896 \times 1 = d$
5. $7398 \div 1 = e$
6. $5564 \div 5564 = f$
7. $38 + 7 - 7 = g$
8. $46 - 8 + 8 = h$
9. $2 + 393 - 393 = i$
10. $(9 \times 6) \div 6 = j$
11. $(54 \div 6) \times 6 = k$
12. $(4 \times 874) \div 874 = m$
13. $7 + 8 = n$
14. $15 - 7 = p$
15. $15 - 8 = q$
16. $42 + 15 = r$
17. $57 - 15 = s$
18. $57 - 42 = t$
19. $6 \times 9 = u$
20. $54 \div 9 = v$
21. $54 \div 6 = w$
22. $60 \times 7 = x$
23. $420 \div 7 = y$
24. $420 \div 60 = z$
25. $0 + 0 = a$
26. $2 \times 0 = b$
27. $2 + 0 = c$

Study the examples. Then complete the sentence.

28. **a.** $21 - 7 = 14$, $14 - 7 = 7$, $7 - 7 = 0$, so $21 \div 7 = 3$
    **b.** $21 - 0 = 21$, $21 - 0 = 21$, $21 - 0 = 21$, . . .

    We cannot find a quotient when ___?___ is the divisor.

### distributive property

To name a sum where the addends have a *common factor*, we may first multiply by the *common factor* and then add

$$(3 \times 4) + (7 \times 4) = 12 + 28 = 40$$

*or* we may add and then multiply by the *common factor*

$$(3 \times 4) + (7 \times 4) = (3 + 7) \times 4 = 10 \times 4 = 40$$

## EXERCISES

Name the sums and products.

1. $(3 \times 6) + (7 \times 6) = a$
2. $(3 \times 10) + (7 \times 10) = c$
3. $\left(3 \times \frac{1}{4}\right) + \left(7 \times \frac{1}{4}\right) = e$
4. $(3 + 7) \times 6 = b$
5. $(3 + 7) \times 10 = d$
6. $(3 + 7) \times \frac{1}{4} = f$

Solve the equations.

7. $3 \times 20 = a$
   $3 \times 4 = r$
   $3 \times 24 = c$

8. $20 \times 7 = m$
   $5 \times 7 = n$
   $25 \times 7 = z$

9. $16 \times 20 = u$
   $16 \times 4 = v$
   $16 \times 24 = i$

10. $8 \times 13 = s$
    $9 \times 13 = n$
    $17 \times 13 = u$

11. $3 \times 400 = e$
    $3 \times 8 = a$
    $3 \times 408 = r$

12. $12 \times 10 = v$
    $13 \times 10 = w$
    $25 \times 10 = x$

13. $\frac{1}{2} \times 20 = x$
    $\frac{1}{2} \times 4 = e$
    $\frac{1}{2} \times 24 = n$

14. $8 \times \frac{1}{2} = s$
    $16 \times \frac{1}{2} = u$
    $24 \times \frac{1}{2} = r$

The distributive property is used in this example.

$$\begin{array}{r} 37 \\ \times\ 8 \\ \hline \end{array} = \begin{array}{r} 30 + 7 \\ \times\ 8 \\ \hline 240 + 56 = 296 \end{array}$$

Use the distributive property to name the products.

15. $\begin{array}{r} 48 \\ \times\ 6 \\ \hline \end{array}$
16. $\begin{array}{r} 59 \\ \times\ 4 \\ \hline \end{array}$
17. $\begin{array}{r} 105 \\ \times\ 3 \\ \hline \end{array}$
18. $\begin{array}{r} 131 \\ \times\ 5 \\ \hline \end{array}$
19. $\begin{array}{r} 283 \\ \times\ 2 \\ \hline \end{array}$

## The distributive property

Study these examples of the distributive property.

| Multiplying whole numbers | Dividing whole numbers | Adding fractional numbers |
|---|---|---|
| $\begin{array}{r} 43 \\ \times\ 7 \\ \hline 21 \\ 280 \\ \hline 301 \end{array}$ | $4\overline{)68} = 4\overline{)40 + 28}$ | $\frac{3}{4} + \frac{5}{4} = (3 + 5) \times \frac{1}{4}$ $= 8 \times \frac{1}{4}$ $= 2$ |

### EXERCISES

Use the distributive property to name the products.

1. $8 \times 46 = a$
2. $7 \times 496 = b$
3. $3 \times 8457 = c$

4. $\begin{array}{r} 486 \\ \times\ 9 \\ \hline \end{array}$
5. $\begin{array}{r} 7504 \\ \times\ 8 \\ \hline \end{array}$
6. $\begin{array}{r} 4517 \\ \times\ 6 \\ \hline \end{array}$
7. $\begin{array}{r} 8716 \\ \times\ 15 \\ \hline \end{array}$
8. $\begin{array}{r} 3486 \\ \times\ 37 \\ \hline \end{array}$

**9–16.** Name the common factors in exercises 1 to 8.

Copy and complete the divisions.

17. $\dfrac{c}{4\overline{)84}} = \dfrac{a + e}{4\overline{)80 + 4}}$
18. $\dfrac{a}{5\overline{)75}} = \dfrac{c + e}{5\overline{)50 + 25}}$
19. $\dfrac{r}{4\overline{)108}} = \dfrac{s + e}{4\overline{)100 + 8}}$
20. $\dfrac{n}{3\overline{)78}} = \dfrac{m + w}{3\overline{)60 + 18}}$
21. $\dfrac{u}{6\overline{)90}} = \dfrac{m + n}{6\overline{)60 + 30}}$
22. $\dfrac{u}{12\overline{)156}} = \dfrac{v + w}{12\overline{)120 + 36}}$
23. $\dfrac{i}{4\overline{)92}} = \dfrac{s + r}{4\overline{)80 + 12}}$
24. $\dfrac{v}{7\overline{)91}} = \dfrac{x + z}{7\overline{)70 + 21}}$

**25–32.** Name the common factors in exercises 17 to 24.

Name the sums or differences.

33. $\frac{3}{4} - \frac{1}{4} = \frac{a}{4} = b$
34. $\frac{1}{2} + \frac{1}{4} = \frac{a}{4}$
35. $\frac{2}{3} + \frac{1}{3} = i$
36. $\frac{7}{8} - \frac{2}{8} = \frac{c}{8}$
37. $\frac{1}{3} + \frac{1}{6} = \frac{c}{6}$
38. $\frac{3}{8} + \frac{1}{4} = \frac{n}{8}$
39. $\frac{3}{4} + \frac{1}{4} = \frac{m}{4} = e$
40. $\frac{5}{6} + \frac{1}{12} = \frac{m}{12}$
41. $\frac{1}{3} + \frac{7}{9} = \frac{x}{9}$

## Practice with addition, subtraction, and multiplication

Study the examples.

$$\begin{array}{r} {\scriptstyle 111111} \\ 428673925 \\ +\ 437869284 \\ \hline 866543209 \end{array} \qquad \begin{array}{r} {\scriptstyle 852\ 3} \\ 29631487 \\ -\ 7892394 \\ \hline 21739093 \end{array} \qquad \begin{array}{r} {\scriptstyle 3527243} \\ 24629354 \\ \times\ 8 \\ \hline 197034832 \end{array}$$

EXERCISES

Name the sums.

**1.** $\begin{array}{r} 328756439 \\ +\ 276583016 \\ \hline \end{array}$ **2.** $\begin{array}{r} 298765438 \\ +\ 412854691 \\ \hline \end{array}$ **3.** $\begin{array}{r} 208571498 \\ +\ 373468291 \\ \hline \end{array}$

**4.** $\begin{array}{r} 3843 \\ 2965 \\ 4763 \\ 8296 \\ +\ 5741 \\ \hline \end{array}$ **5.** $\begin{array}{r} 2965 \\ 3821 \\ 8765 \\ 3307 \\ +\ 2541 \\ \hline \end{array}$ **6.** $\begin{array}{r} 3607 \\ 2953 \\ 5784 \\ 2689 \\ +\ 3147 \\ \hline \end{array}$ **7.** $\begin{array}{r} 2303 \\ 4719 \\ 3864 \\ 7540 \\ +\ 2707 \\ \hline \end{array}$ **8.** $\begin{array}{r} 4869 \\ 3179 \\ 4672 \\ 4063 \\ +\ 8174 \\ \hline \end{array}$

Name the differences.

**9.** $\begin{array}{r} 286834796 \\ -\ 172946318 \\ \hline \end{array}$ **10.** $\begin{array}{r} 478196073 \\ -\ 421817526 \\ \hline \end{array}$ **11.** $\begin{array}{r} 483129584 \\ -\ 27463171 \\ \hline \end{array}$

Name the products.

**12.** $\begin{array}{r} 7256483 \\ \times\ 7 \\ \hline \end{array}$ **14.** $\begin{array}{r} 68934168 \\ \times\ 8 \\ \hline \end{array}$ **16.** $\begin{array}{r} 394281754 \\ \times\ 6 \\ \hline \end{array}$

**13.** $\begin{array}{r} 3917274 \\ \times\ 6 \\ \hline \end{array}$ **15.** $\begin{array}{r} 23928614 \\ \times\ 5 \\ \hline \end{array}$ **17.** $\begin{array}{r} 735286409 \\ \times\ 9 \\ \hline \end{array}$

Solve the problems.

**18.** If there are 604,800 seconds in 1 week, how many seconds are in 4 weeks?

**19.** If your heart beats 36,792,000 times in one year, how many times does it beat in 9 years?

**20.** If a large city uses 19,550,000 gallons of water in a week, how many gallons does it use in 8 weeks?

## Division

Study this method for the division $5882 \div 52$.

$$\begin{array}{r|l} 52\overline{)5882} & \\ 5200 & 100 \\ \hline 682 & \\ 520 & 10 \\ \hline 162 & \\ 156 & 3 \\ \hline 6 & 113 \end{array}$$

$$\begin{aligned} 100 \times 52 &= 5200 \\ 5882 - 5200 &= 682 \\ 10 \times 52 &= 520 \\ 682 - 520 &= 162 \\ 3 \times 52 &= 156 \\ 162 - 156 &= 6 \\ 100 + 10 + 3 &= 113 \end{aligned}$$

The quotient is 113, and the remainder is 6.

### EXERCISES

Name the quotients and remainders.

**1.** $47\overline{)5326}$

| | |
|---|---|
| 4700 | 100 |
| 626 | |
| 470 | $a$ |
| 156 | |
| 141 | $b$ |
| $c$ | $e$ |

**3.** $35\overline{)3957}$

| | |
|---|---|
| 3500 | $e$ |
| 457 | |
| 350 | 10 |
| 107 | |
| 105 | $x$ |
| $g$ | $z$ |

**5.** $56\overline{)6197}$

| | |
|---|---|
| 5600 | $i$ |
| 597 | |
| 560 | $n$ |
| $a$ | $r$ |

**2.** $27\overline{)3098}$

| | |
|---|---|
| 2700 | $m$ |
| 398 | |
| 270 | $n$ |
| 128 | |
| 108 | 4 |
| $z$ | $r$ |

**4.** $67\overline{)7542}$

| | |
|---|---|
| 6700 | $u$ |
| 842 | |
| 670 | 10 |
| 172 | |
| 134 | $r$ |
| $s$ | $a$ |

**6.** $43\overline{)4927}$

| | |
|---|---|
| 4300 | $u$ |
| 627 | |
| 430 | $v$ |
| 197 | |
| 172 | $w$ |
| $x$ | $z$ |

Name the quotients and remainders.

**7.** $24\overline{)2793}$

**8.** $61\overline{)6842}$

**9.** $38\overline{)4267}$

**10.** $73\overline{)8158}$

**11.** $46\overline{)5152}$

**12.** $27\overline{)3102}$

**13.** $56\overline{)6367}$

**14.** $31\overline{)3948}$

## Estimating the quotient

To **estimate** the quotient $74\overline{)3821}$, we may use these steps:

1. $10 \times 74 = 740$ and $100 \times 74 = 7400$. Since $740 < 3821 < 7400$, the quotient is between 10 and 100.
2. 3821 is about 38 hundreds. 74 is about 7 tens. $38 \div 7$ is about 5. Our **estimate** is 50.

We complete the division like this:

$$\begin{array}{r|l} 74\overline{)3821} & \\ 3700 & 50 \\ \hline 121 & \\ 74 & 1 \\ \hline 47 & 51 \end{array}$$

Check

$$\begin{aligned} 50 \times 74 &= 3700 \\ 1 \times 74 &= \quad 74 \\ \hline 51 \times 74 &= 3774 \\ 3821 - 3774 &= 47 \end{aligned}$$

The quotient is 51 and the remainder is 47.

### EXERCISES

Use estimates to name the quotients and remainders.

**1.** $43\overline{)1479}$
**2.** $32\overline{)2278}$
**3.** $52\overline{)4396}$
**4.** $27\overline{)2451}$
**5.** $48\overline{)4662}$
**6.** $72\overline{)1944}$
**7.** $58\overline{)2088}$
**8.** $46\overline{)3785}$
**9.** $53\overline{)3762}$
**10.** $31\overline{)2654}$
**11.** $81\overline{)5947}$
**12.** $73\overline{)6875}$
**13.** $69\overline{)2967}$
**14.** $46\overline{)2438}$
**15.** $3378 \div 62$
**16.** $3064 \div 41$
**17.** $93\overline{)5862}$
**18.** $42\overline{)3521}$
**19.** $2637 \div 38$
**20.** $1152 \div 64$
**21.** $4161 \div 57$
**22.** $2965 \div 28$
**23.** $4598 \div 71$
**24.** $5162 \div 57$
**25.** $7541 \div 88$
**26.** $6752 \div 79$
**27.** $1092 \div 39$
**28.** $4048 \div 88$

**29.** Each week Joe's class saved the same amount for the class library. At the end of 23 weeks they had saved \$86.25. How much did they save each week?

## Division

Study the methods for completing the division $67\overline{)4992}$, and the method for checking the work.

| A | Check | B |
|---|---|---|
| $67\overline{)4992}$<br>$4690 \mid 70$<br>$302$<br>$268 \mid 4$<br>$34 \mid 74$ | $70 \times 67 = 4690$<br>$4 \times 67 = 268$<br>$74 \times 67 = 4958$<br>$4992 - (74 \times 67) = 34$ | $74$<br>$67\overline{)4992}$<br>$4690$<br>$302$<br>$268$<br>$34$ |

### EXERCISES

Complete and check the divisions.

**1.** $54\overline{)4494}$

$4320 \mid x$

$174$

$162 \mid y$

$12 \mid z$

$80 \times 54 = d$

$3 \times 54 = r$

$83 \times 54 = a$

$4494 - (83 \times 54) = g$

**2.** $42\overline{)3973}$

$3780 \mid x$

$193$

$168 \mid y$

$25 \mid z$

$90 \times 42 = k$

$4 \times 42 = e$

$94 \times 42 = m$

$3973 - (94 \times 42) = n$

Use method A to complete the divisions. Check your work.

**3.** $63\overline{)4126}$

$3780$

$346$

$315$

$31$

**4.** $23\overline{)1993}$

$1840$

$153$

$138$

$15$

**5.** $37\overline{)2552}$

$2220$

$332$

$296$

$36$

**6.** $81\overline{)4673}$

$4050$

$623$

$567$

$56$

Use method B to name the quotients and remainders.

**7.** $46\overline{)2663}$

$2300$

$363$

$322$

**8.** $35\overline{)3076}$

**9.** $92\overline{)4837}$

**10.** $83\overline{)3775}$

$3320$

$455$

$415$

**11.** $28\overline{)2768}$

**12.** $61\overline{)4938}$

**13.** $68\overline{)2591}$

$2040$

$551$

$544$

**14.** $3592 \div 53$

**15.** $2981 \div 78$

**16.** $54\overline{)4153}$

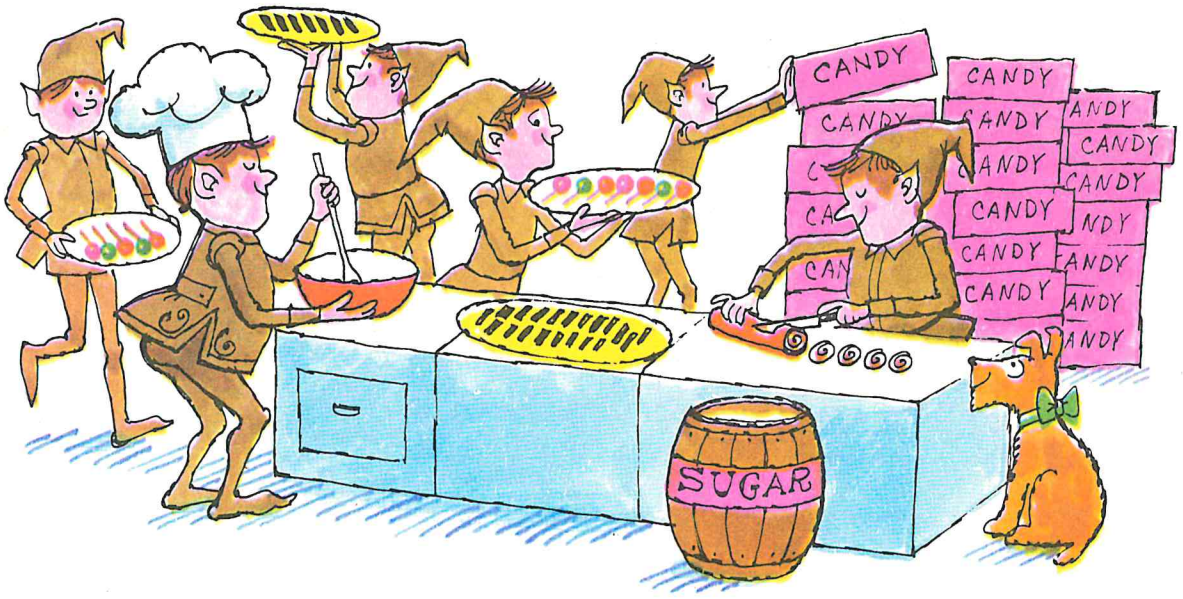
$3780$

$373$

$324$

**17.** $4109 \div 89$

**18.** $2467 \div 42$




1. The candy factory makes 6794 pounds of licorice a day. If 32 pounds of licorice are packed in each box, how many full boxes of licorice do they make each day?

2. If 8 licorice sticks can be made from each pound of licorice, how many sticks can be packed into each box?

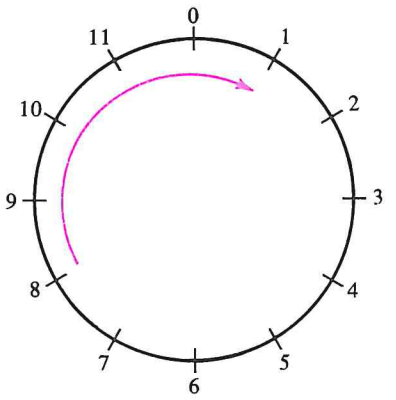
3. The candy factory makes 5493 lollipops each day. If 24 lollipops are packed in each box, how many full boxes of lollipops do they make each day?

4. The candy factory makes 9925 pieces of bubble gum each day. If 48 pieces of bubble gum are packed in each box, how many full boxes of bubble gum do they make each day?

5. At the end of one day, the 41 elves that work at the candy factory made 4997 boxes of candy. If each elf helped make the same number of boxes, how many full boxes did each elf make?

6. If the elves wanted to put the same number of boxes of candy on 17 trucks and they had 3264 boxes, how many boxes would they put on each truck?

## Clock equations



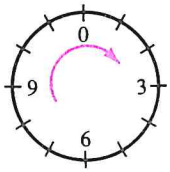

The arrow shows that 1 o'clock is 5 hours after 8 o'clock. We can write a **clock equation** for this fact.

$$8 + 5 \overset{12}{=} 1$$

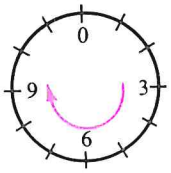
The $\overset{12}{=}$ means that we are working on a clock with only 12 numbers.

### EXERCISES

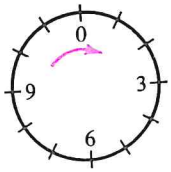
Write the clock equations for the moves shown on these clocks. The first one has been done for you.



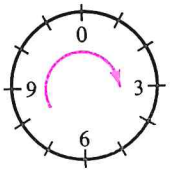

**1.** $8 + 6 \overset{12}{=} 2$




**2.** $3 + a \overset{12}{=} 9$




**3.** $10 + n \overset{12}{=} 1$




**4.** $8 + 7 \overset{12}{=} z$

Complete the sentences and solve the clock equations.

**5.** 6 hours after 8 o'clock is 2 o'clock. $8 + 6 \overset{12}{=} 2$

**6.** 9 hours after 4 o'clock is $a$ o'clock. $4 + 9 \overset{12}{=} x$

**7.** 8 hours after 8 o'clock is $c$ o'clock. $8 + 8 \overset{12}{=} d$

**8.** 4 hours after 10 o'clock is $e$ o'clock. $10 + 4 \overset{12}{=} r$

**9.** 3 hours after 11 o'clock is $n$ o'clock. $11 + 3 \overset{12}{=} h$

**10.** 11 hours after 8 o'clock is $i$ o'clock. $8 + 11 \overset{12}{=} j$

**11.** 3 hours before 2 o'clock is $k$ o'clock. $2 - 3 \overset{12}{=} w$

**12.** 6 hours before 1 o'clock is $m$ o'clock. $1 - 6 \overset{12}{=} n$

**13.** 10 hours before 5 o'clock is $s$ o'clock. $5 - 10 \overset{12}{=} p$

**14.** 9 hours before 3 o'clock is $u$ o'clock. $3 - 9 \overset{12}{=} r$

Solve the clock equations.

**15.** $4 + 7 \overset{12}{=} a$

**16.** $7 + 7 \overset{12}{=} e$

**17.** $8 + i \overset{12}{=} 10$

**18.** $3 - 5 \overset{12}{=} m$

**19.** $6 + 7 \overset{12}{=} b$

**20.** $9 + 9 \overset{12}{=} f$

**21.** $7 + j \overset{12}{=} 1$

**22.** $6 - 9 \overset{12}{=} n$

**23.** $9 + 3 \overset{12}{=} c$

**24.** $10 + 10 \overset{12}{=} g$

**25.** $9 + k \overset{12}{=} 7$

**26.** $10 - 11 \overset{12}{=} p$

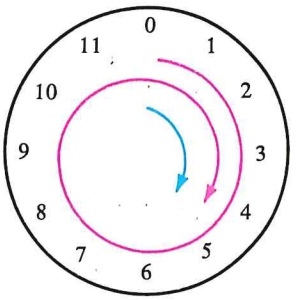
**27.** $6 + 10 \overset{12}{=} d$

**28.** $11 + 11 \overset{12}{=} h$

**29.** $l + 11 \overset{12}{=} 6$

**30.** $2 - 11 \overset{12}{=} q$

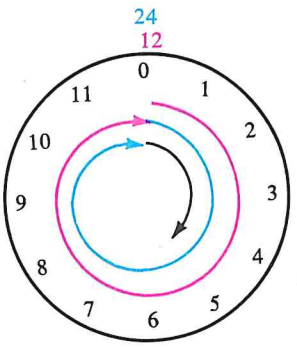
The red arrow shows 16 moves around the clock starting from 0. The blue arrow shows 4 moves around the clock starting from 0. 16 moves and 4 moves on the clock both stop at 4.




We say 16 is **equivalent** to 4. We write the clock equation $16 \overset{12}{\equiv} 4$.

In this arithmetic we use only the numbers 0 through 11.

We can name numbers greater than 11 by subtracting 12 until the remainder is less than 12. The remainder names the number.




| | | |
|---|---|---|
| $16$ | $12$ | $29$ |
| $-\ 12$ | $-\ 12$ | $-\ 12$ |
| $4$ | $0$ | $17$ |
| | | $-\ 12$ |
| | | $5$ |
| $16 \overset{12}{\equiv} 4$ | $12 \overset{12}{\equiv} 0$ | $29 \overset{12}{\equiv} 5$ |

## EXERCISES

Name the numbers on the clock equivalent to

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | 15 | **3.** | 20 | **5.** | 24 | **7.** | 30 | **9.** | 36 | **11.** | 40 |
| **2.** | 47 | **4.** | 64 | **6.** | 75 | **8.** | 100 | **10.** | 144 | **12.** | 200 |

Use the equations to name the equivalent numbers.

For example, $53 - (4 \times 12) = 5$, so $53 \overset{12}{\equiv} 5$.

| | | | | | |
|---|---|---|---|---|---|
| **13.** | $22 - (1 \times 12) = 10$ | **15.** | $62 - (5 \times 12) = 2$ | **17.** | $79 - (6 \times 12) = 7$ |
| **14.** | $90 - (7 \times 12) = 6$ | **16.** | $111 - (9 \times 12) = 3$ | **18.** | $120 - (10 \times 12) = 0$ |

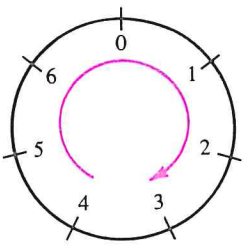
Solve the clock equations.

For example, $7 + 7 = 14$ and $14 \overset{12}{\equiv} 2$, so $2 \times 7 \overset{12}{\equiv} 2$.

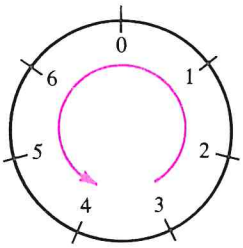
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **19.** | $2 \times 5 \overset{12}{\equiv} a$ | **22.** | $3 \times 5 \overset{12}{\equiv} m$ | **25.** | $2 \times 8 \overset{12}{\equiv} v$ | **28.** | $2 \times 6 \overset{12}{\equiv} z$ |
| **20.** | $4 \times 5 \overset{12}{\equiv} r$ | **23.** | $7 \times 3 \overset{12}{\equiv} u$ | **26.** | $9 \times 5 \overset{12}{\equiv} x$ | **29.** | $11 \times 8 \overset{12}{\equiv} e$ |
| **21.** | $5 \times 10 \overset{12}{\equiv} c$ | **24.** | $9 \times 9 \overset{12}{\equiv} s$ | **27.** | $10 \times 10 \overset{12}{\equiv} w$ | **30.** | $11 \times 11 \overset{12}{\equiv} n$ |

## More about clock arithmetic

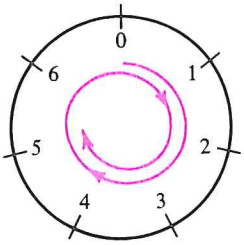
Study the equations shown on these 0 to 6 clocks.




$4 + 6 \overset{7}{=} 3$



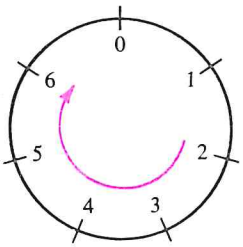

$3 - 6 \overset{7}{=} 4$



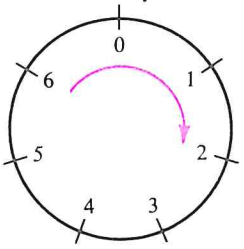

$3 \times 4 \overset{7}{=} 5$

### EXERCISES

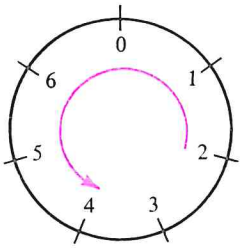
Use the clocks to solve the equations.



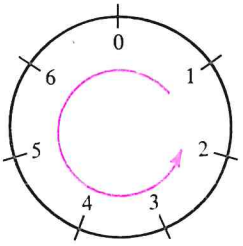

**1.** $2 + 4 \overset{7}{=} a$



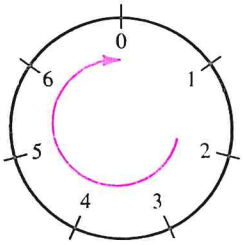

**3.** $6 + 3 \overset{7}{=} b$



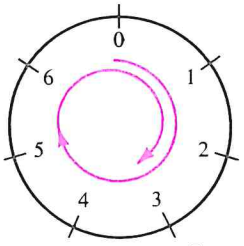

**5.** $2 - 5 \overset{7}{=} c$



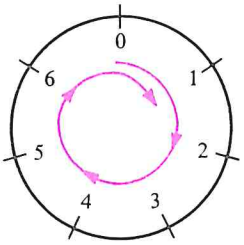

**7.** $1 - 6 \overset{7}{=} d$



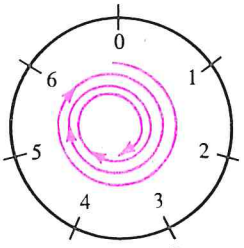

**2.** $2 + 5 \overset{7}{=} e$




**4.** $2 \times 5 \overset{7}{=} f$




**6.** $4 \times 2 \overset{7}{=} g$




**8.** $4 \times 6 \overset{7}{=} h$

Solve the 7-clock equations.

**9.** $3 + 4 \overset{7}{=} a$

**10.** $2 + 5 \overset{7}{=} e$

**11.** $6 + 2 \overset{7}{=} i$

**12.** $3 + 5 \overset{7}{=} u$

**13.** $6 + 3 \overset{7}{=} n$

**14.** $6 + 6 \overset{7}{=} m$

**15.** $5 + 4 \overset{7}{=} r$

**16.** $3 \times 4 \overset{7}{=} p$

**17.** $5 \times 4 \overset{7}{=} r$

**18.** $5 \times 2 \overset{7}{=} a$

**19.** $2 \times 6 \overset{7}{=} i$

**20.** $4 \times 4 \overset{7}{=} s$

**21.** $3 \times 5 \overset{7}{=} e$

**22.** $3 \times 6 \overset{7}{=} d$

**23.** $2 - 5 \overset{7}{=} q$

**24.** $6 - 3 \overset{7}{=} u$

**25.** $1 - 3 \overset{7}{=} i$

**26.** $5 - 6 \overset{7}{=} e$

**27.** $4 - 2 \overset{7}{=} t$

**28.** $3 - 1 \overset{7}{=} l$

**29.** $2 - 4 \overset{7}{=} y$

## Basic facts in clock arithmetic

Copy and complete the tables to list the basic addition and multiplication facts for 7-clock arithmetic.

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | | | | | | | |
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | | | | | | | |
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |

Use the tables to solve the 7-clock equations.

**1.** $5 + 3 \overset{7}{=} a$ **3.** $4 + 5 \overset{7}{=} i$ **5.** $6 + 1 \overset{7}{=} v$ **7.** $6 + 6 \overset{7}{=} w$

**2.** $3 \times 4 \overset{7}{=} e$ **4.** $5 \times 5 \overset{7}{=} u$ **6.** $6 \times 2 \overset{7}{=} x$ **8.** $5 \times 6 \overset{7}{=} z$

Complete the sentences for arithmetic on the 7-clock.

**9.** If 0 is an addend, then the sum is the same as the other ___?___.

**10.** If 0 is a factor, then the product is ___?___.

**11.** If 1 is a factor, then the product is the same as the other ___?___.

Write *T* or *F* to show whether each 7-clock equation is *true* or *false*.

**12.** $3 + 4 \overset{7}{=} 4 + 3$

**13.** $6 + 5 \overset{7}{=} 5 + 6$

**14.** $6 \times 3 \overset{7}{=} 3 \times 6$

**15.** $3 + (4 + 5) \overset{7}{=} (3 + 4) + 5$

**16.** $3 \times (4 \times 5) \overset{7}{=} (3 \times 4) \times 5$

**17.** $3 \times (4 + 5) \overset{7}{=} (3 \times 4) + (3 \times 5)$

Answer the questions.

**18.** Are the commutative properties of addition and of multiplication true for 7-clock arithmetic?

**19.** Are the associative properties of addition and of multiplication true for 7-clock arithmetic?

**20.** Can we use the distributive property in 7-clock arithmetic?

## Another number system

Here is another system we have made up just for fun. The numerals for the numbers in the system are $\triangle$, $\bigcirc$, $\otimes$, $\varnothing$, $\triangledown$. The two number operations are $+$ and $\times$. The basic addition and multiplication facts are shown on these tables.

| $+$ | $\triangle$ | $\bigcirc$ | $\otimes$ | $\varnothing$ | $\triangledown$ |
|---|---|---|---|---|---|
| $\triangle$ | $\triangle$ | $\bigcirc$ | $\otimes$ | $\varnothing$ | $\triangledown$ |
| $\bigcirc$ | $\bigcirc$ | $\otimes$ | $\varnothing$ | $\triangledown$ | $\triangle$ |
| $\otimes$ | $\otimes$ | $\varnothing$ | $\triangledown$ | $\triangle$ | $\bigcirc$ |
| $\varnothing$ | $\varnothing$ | $\triangledown$ | $\triangle$ | $\bigcirc$ | $\otimes$ |
| $\triangledown$ | $\triangledown$ | $\triangle$ | $\bigcirc$ | $\otimes$ | $\varnothing$ |

| $\times$ | $\triangle$ | $\bigcirc$ | $\otimes$ | $\varnothing$ | $\triangledown$ |
|---|---|---|---|---|---|
| $\triangle$ | $\triangle$ | $\triangle$ | $\triangle$ | $\triangle$ | $\triangle$ |
| $\bigcirc$ | $\triangle$ | $\bigcirc$ | $\otimes$ | $\varnothing$ | $\triangledown$ |
| $\otimes$ | $\triangle$ | $\otimes$ | $\triangledown$ | $\bigcirc$ | $\varnothing$ |
| $\varnothing$ | $\triangle$ | $\varnothing$ | $\bigcirc$ | $\triangledown$ | $\otimes$ |
| $\triangledown$ | $\triangle$ | $\triangledown$ | $\varnothing$ | $\otimes$ | $\bigcirc$ |

### EXERCISES

Use the tables to solve these equations.

**1.** $\bigcirc + \otimes = a$ **4.** $\varnothing + \varnothing = d$ **7.** $\triangledown + \varnothing = g$ **10.** $\bigcirc + \triangle = i$

**2.** $\bigcirc \times \otimes = b$ **5.** $\varnothing \times \varnothing = e$ **8.** $\triangledown \times \varnothing = f$ **11.** $\bigcirc \times \varnothing = j$

**3.** $\triangledown + \triangledown = c$ **6.** $\triangle \times \varnothing = f$ **9.** $\triangledown \times \triangledown = h$ **12.** $\otimes + \otimes = k$

The addition table shows that $\otimes + \triangledown = \bigcirc$ so $(\otimes + \triangledown) + \varnothing = \bigcirc + \varnothing = \triangledown$. Use this method to solve these equations.

**13.** $(\bigcirc + \triangle) + \otimes = a$ **15.** $(\otimes + \varnothing) + \triangle = c$ **17.** $(\varnothing + \varnothing) + \varnothing = e$ **19.** $(\bigcirc + \triangledown) + \otimes = g$

**14.** $\triangledown + (\otimes + \triangle) = b$ **16.** $\varnothing + (\triangledown + \otimes) = d$ **18.** $(\otimes + \varnothing) + \triangledown = f$ **20.** $\otimes + (\varnothing + \triangledown) = h$

The multiplication table shows that $\otimes \times \varnothing = \bigcirc$ so $(\otimes \times \varnothing) \times \varnothing = \bigcirc \times \varnothing = \varnothing$. Use this method to solve these equations.

**21.** $(\otimes \times \bigcirc) \times \varnothing = a$ **23.** $(\triangledown \times \varnothing) \times \varnothing = c$ **25.** $(\triangle \times \varnothing) \times \triangledown = e$ **27.** $(\triangledown \times \triangledown) \times \triangledown = g$

**22.** $\varnothing \times (\otimes \times \otimes) = b$ **24.** $(\otimes \times \triangledown) \times \triangle = d$ **26.** $\triangledown \times (\otimes \times \varnothing) = f$ **28.** $(\triangledown \times \otimes) \times \varnothing = h$

## The metric system of measures

In our system of measures we have many different units. For example, we have yards, feet, inches, pounds, ounces, cups, pints and quarts.

Our system of measures uses many different bases. For example, there are 12 inches in 1 foot, 3 feet in 1 yard, and 16 ounces in 1 pound.

Another system of measures, invented by French scientists more than 150 years ago, uses only base ten. Each unit is one tenth of a larger unit. This system is called the **metric system.**

This chart compares the metric system with ours.

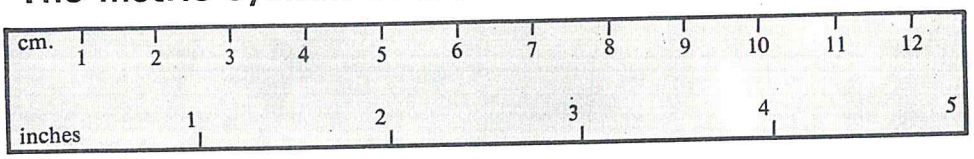
| |
|---|
| A **meter** is a little more than a yard.<br>1 meter is about 39 inches. |
| A **centimeter** $\left(\frac{1}{100} \text{ meter}\right)$ is about $\frac{2}{5}$ inch. |
| A **kilometer** (1000 meters) is about 1100 yards or $\frac{5}{8}$ mile. |
| A **kilogram** is about $2\frac{2}{10}$ pounds. |

### EXERCISES

Complete the sentences.

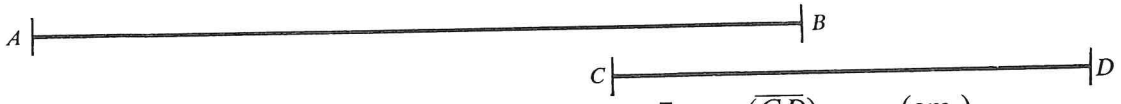
1. 1 meter is about ___?___ inches, so 3 meters are about ___?___ inches.
2. 1 centimeter is about ___?___ inch, so 5 centimeters are about ___?___ inches.
3. 1 kilometer is about ___?___ miles, so 8 kilometers are about ___?___ miles.
4. 1 kilogram is about ___?___ pounds, so 4 kilograms are about ___?___ pounds.
5. Ten meters is about 11 yards. Eighty meters is about ___?___ yards.

## The metric system of measures

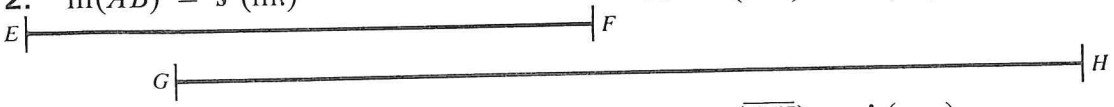



On this ruler the units on one side are centimeters. The other side is marked in inch units.

On one side of a strip of cardboard mark inch units; on the other side mark centimeter units. Use the strip to measure the line segments, first in centimeters and then in inches.




1. $m(\overline{AB}) = a$ (cm.)
2. $m(\overline{AB}) = s$ (in.)

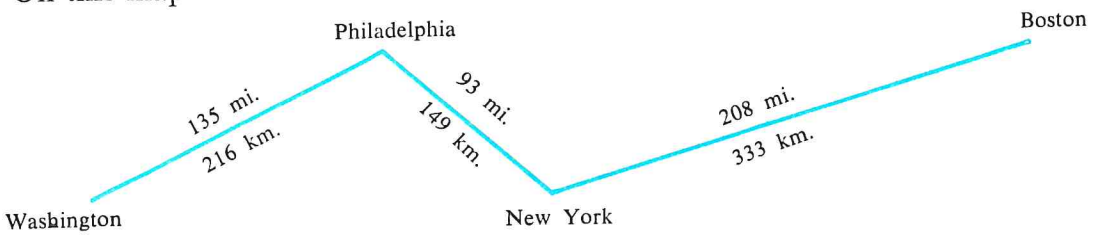



3. $m(\overline{EF}) = n$ (cm.)
4. $m(\overline{EF}) = u$ (in.)

5. $m(\overline{CD}) = c$ (cm.)
6. $m(\overline{CD}) = r$ (in.)

7. $m(\overline{GH}) = i$ (cm.)
8. $m(\overline{GH}) = b$ (in.)

On this map the distances are shown in kilometers and in miles.




Name the distances, in kilometers and in miles between

9. Washington and Philadelphia.
10. Washington and Boston.
11. Washington and New York.
12. Philadelphia and Boston.

In the metric system the unit for measuring liquids is a **liter.** It is about the same as an American quart.

Name these quantities:

13. in the American system **a.** 5 liters **b.** 3 liters **c.** 10 liters
14. in the metric system **a.** 3 quarts **b.** 1 gallon **c.** 3 gallons

| country | area (in square miles) |
|---|---|
| Australia | 2,971,081 |
| Chile | 286,396 |
| France | 212,659 |
| Japan | 142,688 |
| New Zealand | 103,736 |
| Nigeria | 356,869 |
| U.S.S.R. | 8,655,890 |
| U.S.A. | 3,628,150 |

## EXERCISES

Use the chart to answer the questions.

1. What is the difference in area between the largest and the smallest of these countries?
2. What is the sum of the areas of the largest and the smallest of these countries?
3. What is the difference in area between
   a. U.S.A. and U.S.S.R.?
   b. Chile and France?
   c. Nigeria and Chile?
   d. Japan and New Zealand?
4. What is the difference between the area of U.S.S.R. and
   a. 8 times the area of New Zealand?
   b. 6 times the area of Japan?
5. Is the combined area of France, Chile, Nigeria, Japan and New Zealand more or less than a million square miles?
6. Is the area of U.S.S.R. more or less than the combined areas of the other countries on the chart? How much more or less?
7. What is the total area of the countries on the chart?

Solve the problem.

8. If a city is 10 miles long and 4 miles wide, how many square miles does it cover?

## CHECKING UP

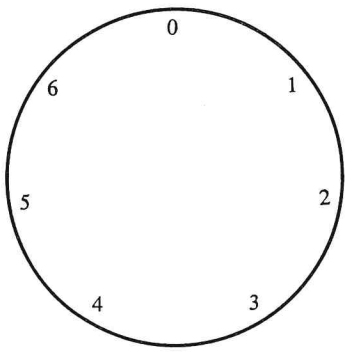
Write *T* or *F* to show whether each statement is *true* or *false*.

1. A whole number divided by a whole number always gives a whole number as the quotient.
2. In the numeral $\frac{5}{6}$, 6 is a denominator.
3. All prime numbers are odd numbers.
4. The associative property says that $6 + 3 = 3 + 6$.
5. All multiples of 4 are even numbers.
6. All multiples of 3 are odd numbers.
7. If 3 is a factor of a number, then 6 is a factor of the number.
8. Some multiples of 11 are even numbers.
9. If $A$ = {the letters of the alphabet}, then $A$ is an infinite set.
10. If $M$ = {all the men in the world}, then $M$ is an infinite set.
11. If $T$ = {the hairs on your head}, then $T$ is a finite set.
12. If $S$ = {the desks in your room}, then $S$ is an infinite set.

Name the quotients and remainders.

13. $25\overline{)1600}$
14. $65\overline{)5432}$
15. $56\overline{)2128}$
16. $76\overline{)3291}$
17. $47\overline{)1081}$
18. $57\overline{)5284}$
19. $36\overline{)2478}$
20. $93\overline{)8729}$

Use the circular number line on the right to help you solve the 7-clock equations.




21. $2 + 6 \overset{7}{=} a$
22. $b + 4 \overset{7}{=} 2$
23. $3 - 6 \overset{7}{=} c$
24. $1 - 5 \overset{7}{=} d$
25. $2 \times 4 \overset{7}{=} e$
26. $3 \times 5 \overset{7}{=} f$
27. $6 \times 6 \overset{7}{=} g$
28. $4 \times h \overset{7}{=} 3$

Name the sums or differences.

29. $\frac{3}{4} + \frac{1}{4} = a$
30. $\frac{1}{8} + \frac{5}{8} = n$
31. $\frac{4}{8} + \frac{1}{4} = r$
32. $\frac{1}{3} + \frac{4}{6} = i$
33. $\frac{5}{6} - \frac{1}{3} = m$
34. $\frac{7}{8} - \frac{1}{4} = c$
35. $\frac{5}{6} - \frac{5}{6} = s$
36. $\frac{5}{9} - \frac{1}{3} = e$

Some of the different kinds of numbers we have studied are {fractional numbers}, {odd numbers}, {even numbers}, and {prime numbers}.

Describe each set of numbers.

1. $\{0, 4, 8, 12, 16, 20, \ldots\}$
2. $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
3. $\{0, 2, 4, 6, 8, 10, 12, \ldots\}$
4. $\{2, 3, 5, 7, 11, 13, \ldots\}$

We have studied properties of the number operations.

Name the property stated in each sentence.

5. The order of the factors does not affect the product.
6. The product of 1 and any number is equal to the number.
7. The way the addends are grouped does not affect the sum.

**Clock arithmetic** is a different kind of number system. On a 0 to 11 clock (12-clock arithmetic), there are only 12 numerals: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.

Solve the 12-clock equations.

8. $7 + 8 \overset{12}{=} a$
9. $5 + 9 \overset{12}{=} b$
10. $8 + 9 \overset{12}{=} c$
11. $3 - 7 \overset{12}{=} d$
12. $4 \times 5 \overset{12}{=} e$
13. $7 \times 7 \overset{12}{=} r$
14. $10 + 8 \overset{12}{=} s$
15. $2 - 6 \overset{12}{=} m$

The **metric system** is a system of measurement used in many parts of the world. All units are based on powers of 10.

On a strip of cardboard, mark off the centimeters as shown on the ruler on page 336.

Use the cardboard to draw the line segments.

16. $m(\overline{AB}) = 5$ (cm.)
17. $m(\overline{RS}) = 9\frac{1}{2}$ (cm.)
18. $m(\overline{GH}) = 6$ (cm.)
19. $m(\overline{MN}) = 14$ (cm.)
20. Draw rectangle $RSTU$ with sides 3 cm. and 12 cm. in length.

## LOOKING WAY BACK

Name the sums.

**1.** $\begin{array}{r} 37 \\ 19 \\ 46 \\ 25 \\ 89 \\ +\ 75 \\ \hline \end{array}$ **2.** $\begin{array}{r} 42 \\ 30 \\ 52 \\ 68 \\ 60 \\ +\ 98 \\ \hline \end{array}$ **3.** $\begin{array}{r} 327 \\ 192 \\ 344 \\ 734 \\ 893 \\ +\ 302 \\ \hline \end{array}$ **4.** $\begin{array}{r} 8517 \\ 6492 \\ 7088 \\ 9256 \\ 9765 \\ +\ 5004 \\ \hline \end{array}$ **5.** $\begin{array}{r} 93764 \\ 28376 \\ 18549 \\ 90327 \\ 26494 \\ +\ 61721 \\ \hline \end{array}$ **6.** $\begin{array}{r} 110276 \\ 349270 \\ 276993 \\ 178954 \\ 975321 \\ +\ 618903 \\ \hline \end{array}$

Name the differences.

**7.** $\begin{array}{r} 49 \\ -\ 34 \\ \hline \end{array}$ **8.** $\begin{array}{r} 83 \\ -\ 65 \\ \hline \end{array}$ **9.** $\begin{array}{r} 803 \\ -\ 413 \\ \hline \end{array}$ **10.** $\begin{array}{r} 9261 \\ -\ 5208 \\ \hline \end{array}$ **11.** $\begin{array}{r} 11230 \\ -\ 9547 \\ \hline \end{array}$ **12.** $\begin{array}{r} 56801 \\ -\ 32678 \\ \hline \end{array}$

Name the sums or differences.

**13.** $\begin{array}{r} 394716 \\ -\ 285986 \\ \hline \end{array}$ **14.** $\begin{array}{r} 810274 \\ +\ 734573 \\ \hline \end{array}$ **15.** $\begin{array}{r} 9076457 \\ -\ 6990765 \\ \hline \end{array}$ **16.** $\begin{array}{r} 7392815 \\ +\ 3729763 \\ \hline \end{array}$

Name the products.

**17.** $\begin{array}{r} 37 \\ \times\ 9 \\ \hline \end{array}$ **19.** $\begin{array}{r} 672 \\ \times\ 18 \\ \hline \end{array}$ **21.** $\begin{array}{r} 7231 \\ \times\ 26 \\ \hline \end{array}$ **23.** $\begin{array}{r} 3650 \\ \times\ 29 \\ \hline \end{array}$ **25.** $\begin{array}{r} 9658 \\ \times\ 376 \\ \hline \end{array}$

**18.** $\begin{array}{r} 46 \\ \times\ 23 \\ \hline \end{array}$ **20.** $\begin{array}{r} 397 \\ \times\ 23 \\ \hline \end{array}$ **22.** $\begin{array}{r} 1357 \\ \times\ 35 \\ \hline \end{array}$ **24.** $\begin{array}{r} 8653 \\ \times\ 132 \\ \hline \end{array}$ **26.** $\begin{array}{r} 9764 \\ \times\ 457 \\ \hline \end{array}$

Name the quotients and remainders.

**27.** $9\overline{)307}$ **28.** $4\overline{)2356}$ **29.** $35\overline{)2726}$ **30.** $29\overline{)19178}$

Name the sums or differences.

**31.** $\begin{array}{r} \frac{1}{2} \\ +\ \frac{1}{2} \\ \hline \end{array}$ **33.** $\begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{4} \\ \hline \end{array}$ **35.** $\begin{array}{r} \frac{7}{8} \\ -\ \frac{5}{8} \\ \hline \end{array}$ **37.** $\begin{array}{r} \frac{2}{3} \\ +\ \frac{1}{6} \\ \hline \end{array}$ **39.** $\begin{array}{r} \frac{3}{4} \\ +\ \frac{5}{12} \\ \hline \end{array}$ **41.** $\begin{array}{r} \frac{7}{9} \\ +\ \frac{2}{3} \\ \hline \end{array}$

**32.** $\begin{array}{r} \frac{2}{3} \\ -\ \frac{1}{3} \\ \hline \end{array}$ **34.** $\begin{array}{r} \frac{1}{4} \\ +\ \frac{1}{8} \\ \hline \end{array}$ **36.** $\begin{array}{r} \frac{7}{9} \\ -\ \frac{6}{9} \\ \hline \end{array}$ **38.** $\begin{array}{r} \frac{2}{3} \\ -\ \frac{1}{6} \\ \hline \end{array}$ **40.** $\begin{array}{r} \frac{5}{9} \\ -\ \frac{1}{3} \\ \hline \end{array}$ **42.** $\begin{array}{r} \frac{1}{2} \\ -\ \frac{2}{10} \\ \hline \end{array}$

Use base eight numerals to complete the number patterns.

**1.** Addition

| + | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | *a* | *b* | 4 | *c* | *d* | 7 | 10 |
| 2 | 3 | 4 | *e* | 6 | *f* | 10 | 11 |
| 3 | 4 | *g* | 6 | *h* | 10 | *i* | *j* |
| 4 | 5 | 6 | *k* | *l* | 11 | 12 | *m* |
| 5 | 6 | *n* | *p* | 11 | *r* | 13 | *s* |
| 6 | 7 | 10 | *t* | *u* | *v* | 14 | *w* |
| 7 | *x* | 11 | 12 | *y* | 14 | 15 | *z* |

**2.** Multiplication

| × | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | *a* | *b* | 3 | *c* | 5 | *d* | 7 |
| 2 | 2 | 4 | *e* | 10 | *f* | 14 | 16 |
| 3 | 3 | *g* | 11 | *h* | 17 | *i* | 25 |
| 4 | 4 | 10 | *j* | 20 | *k* | *l* | *m* |
| 5 | *n* | 12 | *p* | 24 | *q* | 36 | *r* |
| 6 | 6 | *s* | 22 | *t* | *u* | *v* | 52 |
| 7 | *w* | 16 | *x* | 34 | *y* | *z* | 61 |

Use base six numerals to complete the number patterns.

**3.** Addition

| + | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 10 |
| 2 | 3 | *a* | *b* | 10 | 11 |
| 3 | *c* | *d* | *e* | 11 | 12 |
| 4 | *f* | *g* | *h* | 12 | *i* |
| 5 | 10 | *j* | *k* | *l* | 14 |

**4.** Multiplication

| × | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 | *a* | 3 | *b* | 5 |
| 2 | *c* | 4 | *d* | 12 | 14 |
| 3 | 3 | *e* | *f* | 20 | 23 |
| 4 | 4 | *g* | *h* | 24 | *i* |
| 5 | *j* | *k* | 23 | *l* | 41 |

Use base seven numerals to complete the number patterns.

**5.** Addition

| + | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 10 |
| 2 | *a* | 4 | *b* | *c* | *d* | 11 |
| 3 | 4 | 5 | *e* | *f* | 11 | 12 |
| 4 | *g* | *h* | *i* | *j* | 12 | 13 |
| 5 | *k* | *l* | *m* | *n* | 13 | *p* |
| 6 | *q* | *r* | 12 | *s* | *t* | 15 |

**6.** Multiplication

| × | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | *a* | 2 | *b* | 4 | 5 | 6 |
| 2 | 2 | *c* | 6 | 11 | *d* | 15 |
| 3 | *e* | *f* | *g* | *h* | 21 | *i* |
| 4 | 4 | *j* | 15 | *k* | *l* | *m* |
| 5 | *n* | 13 | *z* | 26 | *p* | 42 |
| 6 | 6 | *q* | 24 | *r* | *s* | 51 |

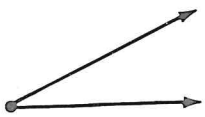
# GLOSSARY

**Abacus** (plural: abaci) (p. 14)
Rods and beads used to show place value.

**Addends** (p. 28)
In addition, the numbers used to name the sum.

**Addition** (p. 28)
The renaming of 2 addends as a sum.

**Angle** (p. 71)
The union of two rays with a common end point.



**Area** (p. 88)
The measure of a region together with the unit used for measurement.

**Associative Property of Addition** (p. 38)
The way addends are grouped does not affect the sum, as

$$7 + (3 + 5) = (7 + 3) + 5$$

**Associative Property of Multiplication** (p. 115)
The way the factors are grouped does not affect the product, as

$$2 \times (3 \times 5) = (2 \times 3) \times 5$$

**Average** (p. 280)
The average of a set of numbers is the number all the numbers would be if they were all the same and their sum did not change.

**Bar graph** (p. 278)
A graph using columns to picture data.

**Base** (p. 216)
The number on which a place-value system is built. The base of our system is ten.

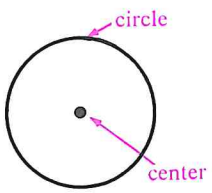
**Basic facts** (p. 30)
Additions and subtractions with each addend less than 10.

**Braces { }** (p. 2)
Symbols used to enclose the members of a set.

**Cardinal number** (p. 4)
The number that tells how many objects are in a set.

**Center of a circle** (p. 78)
A point in the circular region which is the same distance from all points on the circle.

**Circle** (p. 78)
A simple closed curve with all its points a fixed distance from the center.




**Clock arithmetic** (p. 330)
A set of finite numbers shown on a clock.

**Closed curve** (p. 75)
A curve that begins and ends at the same point.

**Closed number sentence** (p. 33)
A number sentence that is true or false.

# GLOSSARY

**Closed surface** (p. 260)
A surface which separates space into three sets of points, those inside, on, and outside the surface.

**Closure Property of {Whole Numbers} for Addition** (p. 320)
The sum of two whole numbers is always a whole number.

**Closure Property of {Whole Numbers} for Multiplication** (p. 320)
The product of two whole numbers is always a whole number.

**Common factor** (p. 99)
A number which is a factor of 2 or more other numbers. 6 is a common factor of 12 and 18.

**Common Factor Property** (p. 99)
To name the sum of two addends with a common factor, such as $(4 \times 7) + (4 \times 3)$, we may multiply, then add $(28 + 12 = 40)$ *or* add then multiply

$$(7 + 3 = 10, 4 \times 10 = 40).$$

**Common multiple** (p. 196)
A number which is a multiple of two or more numbers.

**Commutative Property of Addition** (p. 36)
The order of the addends does not affect the sum, as $4 + 5 = 5 + 4$.

**Commutative Property of Multiplication** (p. 97)
The order of the factors does not affect the product, as $4 \times 8 = 8 \times 4$.

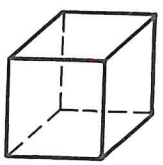
**Compact numeral** (p. 13)
The usual, short form of a numeral, using place values, as 395.

**Composite number** (p. 223)
A whole number greater than 1, which has more than two factors.

**Cone** (p. 263)
A closed surface formed by the union of a circular region and a curved surface and coming to a point at the top.

**Counting numbers** (p. 5)
The numbers in $\{1, 2, 3, 4, \ldots\}$.

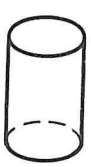
**Cube** (p. 261)
A closed surface formed by the union of 6 congruent square regions with common sides.



**Cubic unit** (p. 268)
A cube used as a unit of measure to name the volume inside a space figure.

**Curve** (p. 66)
A set of connected points that form a path.

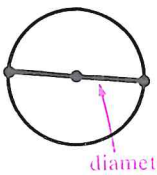
**Cylinder** (p. 262)
A space figure formed by the union of 2 circular regions and a curved rectangular region, as a can.



**Data** (p. 274)
A set of facts or numbers about one thing.

**Denominator** (p. 132)
4 is the denominator of the fraction $\frac{3}{4}$. It names the number of objects in the total set.

**Diameter** (p. 78)
*A* diameter of a circle is any line segment that passes through the center and has both end points on the circle. *The* diameter is the length of such a line segment.




**Difference** (p. 29)
The missing addend in a subtraction.

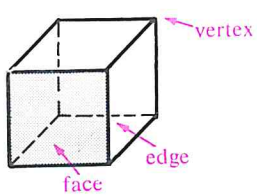
**Digit** (p. 13)
A symbol used to write numerals. In our system of writing numerals {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} is the set of digits.

**Distributive Property of Multiplication** (p. 99)
(See the Common Factor Property.)

**Division** (p. 94)
The renaming of a product and one factor as the other factor, as

$$28 \div 4 = 7$$

**Edge** (p. 260)
In certain space figures, the intersection of the plane regions (faces) of simple closed curves.




**Ellipse** (p. 93)
An ellipse is an oval closed curve.

**Empty set** (p. 3)
The set that has no elements. Its cardinal number is zero.

**Equal sets** (p. 3)
Sets that have exactly the same members.

**Equation** (p. 10)
A mathematical sentence stating that two numerals name the same number.

**Equivalent fractions** (p. 142)
Fractions that name the same fractional number, as $\frac{1}{2}$ and $\frac{3}{6}$.

**Equivalent sets** (p. 4)
Sets with the same cardinal number.

**Estimate** (p. 327)
A guess about what the answer to a problem will be.

**Expanded numeral** (p. 13)
A numeral written out as the sum of the total value of its digits, as $300 + 90 + 5$.

**Exponent** (p. 210)
A numeral written above and to the right of another numeral to show how many times the base number is to be used as a factor, as

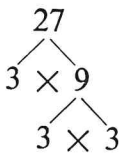
$$3^4 = 3 \times 3 \times 3 \times 3 = 81.$$

**Faces** (p. 260)
(See "Edge.") The plane regions which form the sides of a simple closed surface.

## GLOSSARY

**Factor** (p. 198)
A whole number that is multiplied by another whole number and gives a product.

**Factor tree** (p. 205)
A tree — like arrangement of factors to show prime factorization, as




**Finite set** (p. 5)
A set whose members can be listed and counted.

**Fraction** (p. 128)
A numeral for a fractional number. $\frac{3}{6}$, $\frac{1}{2}$, $\frac{2}{4}$ are all fractions for the fractional number one-half.

**Fractional number** (p. 128)
Numbers used to compare the cardinal number of a subset with the cardinal number of the set, as $\frac{1}{2}$, $\frac{7}{8}$, etc.

**Function** (p. 177)
A set of ordered pairs like $\{(0, 2), (1, 3), (2, 4), (3, 5), \cdots\}$. In this function we have the rule "add 2."

**Graph** (p. 275)
A picture used to show data. Different types of graphs are bar, line, and circle.

**Inequality** (p. 10)
A numerical statement that one number is greater (or less) than another.

**Infinite set** (p. 5)
A non-empty set which is not finite. All of its members cannot be listed and counted.

**Intersection** (p. 67)
The point or points common to two or more lines or planes.

**Intersection** (of sets) (p. 193)
The intersection of $A$ and $B$, written $A \cap B$, is the set of all elements that are common to both $A$ and $B$.

**Kilogram** (p. 335)
A unit of weight used in the Metric System. One kilogram is about $2\frac{2}{10}$ pounds.

**Least Common Multiple** (p. 197)
The least nonzero member of the set of common multiples of two numbers.

**Line** (p. 66)
A straight curve which has no end points and goes on in either direction without end.

**Line segment** (p. 66)
A part of a line with two end points. $\overline{MN}$ names line segment $MN$.

**Liter** (p. 336)
A unit of liquid measure in the Metric System. A liter is about a quart.

**Measure** (p. 68)
A number that tells how many units match the object for the property being measured: length, area or volume.

**Members of a set** (p. 2)
The objects in a set.

**Meter** (p. 335)
A unit of length used in the Metric System. One meter is about one yard.

**Metric system** (p. 335)
A system of units used in many countries throughout the world. It uses 10 as its base.

**Mixed numeral** (p. 298)
A numeral with part naming a whole number and part naming a fractional number less than 1. $2\frac{3}{7}$ is a mixed numeral.

**Multiple** (p. 195)
The multiple of a number is a product of that number and a whole number. The multiples of 3 may be found by multiplying 3 by the members of the set $\{0, 1, 2, 3, \ldots\}$.

**Multiplication** (p. 94)
Renaming two or more factors as a product, as $7 \times 5 = 35$.

**Number** (p. 5)
An idea of "how many".

**Number line** (p. 11)
A line with points labeled by numbers.

**Number plane** (p. 85)
A plane with points labeled by number pairs.

**Numeral** (p. 9)
A name for a number as $6 + 1$, $5 + 2$, seven, $9 - 2$, are other numbers for 7.

**Numerator** (p. 132)
The numerator of the fraction $\frac{3}{4}$ is 3. It names the cardinal number of a subset.

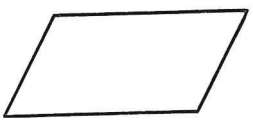
**One Property** (p. 322)
The product of one and any factor is that factor.

**Open number sentence** (p. 33)
A number sentence that cannot be said to be true or false.

**Ordered Pair** (p. 238)
A pair of numbers in which the order is important. For example, (3, 6) is not the same as (6, 3).

**Parallel lines** (p. 73)
Lines in the same plane that never meet. The distance between parallel lines is always the same.

**Parallelogram** (p. 77)
A quadrilateral with opposite sides parallel and congruent.



**Perimeter** (p. 77)
The distance around a polygon.

**Perpendicular lines** (p. 72)
Lines that intersect to form square (right) angles.

**Place value** (p. 55)
The value of each place in a compact numeral. In 532, 5 is in the hundred's place, 3 is in the ten's place, and 2 is in the one's place.

**Plane** (p. 70)
A flat surface that goes on in all directions without end.

**Point** (p. 256)
A point may be thought of as the intersection of two lines and may be represented by a dot.

**Polygon** (p. 76)
A simple closed curve formed by the union of 3 or more line segments.

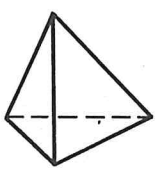
**Power** (p. 212)
Any product shown by a base and an exponent. $5 \times 5 = 5^2$ is the second power of 5.

**Prime factorization** (p. 204)
A numeral which names a number as a product of its prime factors.

**Prime number** (p. 202)
A number with only 2 factors, 1 and the number itself.

**Product** (p. 95)
When two numbers are multiplied together, the result is a product, as 81 is the product in $9 \times 9 = 81$.

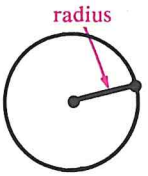
**Pyramid** (p. 260)
A simple closed surface made up of 3 or more triangular regions and a base.



**Quadrilateral** (p. 77)
A polygon with 4 sides.

**Quotient** (p. 179)
The number resulting from the division of one number by another. In $12 \div 4 = 3$, 3 is the quotient.

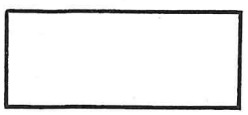
**Radius** (p. 79)
A radius of a circle is any line segment with one end point on the circle and the other at the center of the circle.




**Range** (p. 282)
The difference between the greatest and least members of a set of data.

**Ray** (p. 66)
A part of a line that has one end point and goes on without end in the other direction. $\overrightarrow{RS}$ names ray $RS$.

**Rectangle** (p. 77)
A quadrilateral with four right angles.

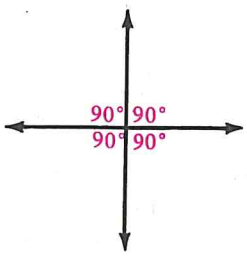


**Region** (p. 78)
The plane area inside a simple closed curve.

**Remainder** (p. 181)
The number remaining in a division when the greatest multiple of the divisor has been subtracted from the dividend. 5 is the remainder in

$$\begin{array}{r} 4 \\ 6\overline{)29} \\ \underline{24} \\ 5 \end{array}.$$

**Right angle** (p. 72)
The angle formed by the intersection of two perpendicular rays. A right angle has a measure of 90°.



**Set** (p. 1)
A collection of objects or ideas.

**Simple closed curve** (p. 75)
A closed curve that does not intersect itself.

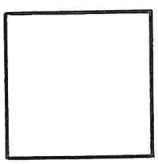
**Simple closed surface** (p. 260)
A closed surface that does not intersect itself.

**Solution** (p. 33)
The number that makes an open number sentence true.

**Space figure** (p. 260)
A figure formed by the union of plane regions.

**Sphere** (p. 264)
A round closed surface. Every point on the surface is an equal distance from a point called the center.

**Square** (p. 77)
A rectangle with all 4 sides congruent.



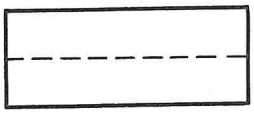
**Square unit** (p. 88)
A unit of measure used for naming the area of a plane region. A square unit (such as a square inch) is a square with each side equal to one unit (inch) in length.

**Subset** (p. 6)
$A$ is a subset of $B$ if the members of $A$ are all members of $B$.

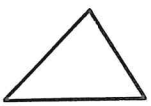
**Subtraction** (p. 28)
Renaming a sum and one addend as the missing addend, or difference.

**Sum** (p. 28)
The number that is the result of renaming addends.

**Symmetrical curve** (p. 80)
A curve that can be separated into two matching parts.



**Triangle** (p. 76)
A simple closed curve formed as the union of three line segments. (A polygon with 3 sides.)



**Union of sets** (p. 8)
The union of set $A$ and set $B$, written $A \cup B$, is the set of all the elements that are in $A$ or $B$.

**Universal set** (p. 7)
One set is a universal set of another, or others, if all the members of the others can be found in the universal set.

**Vertex (plural: vertices)** (p. 71)
(See "Edge.") The common end point of two rays or the point in which 2 sides of a polygon intersect.

**Volume** (p. 268)
The measure of a closed surface together with the units used for measurement.

**Whole numbers** (p. 11)
The numbers in $\{0, 1, 2, 3, \ldots\}$.

**Zero property** (p. 322)
If zero is one addend, the sum and the other addend are the same.

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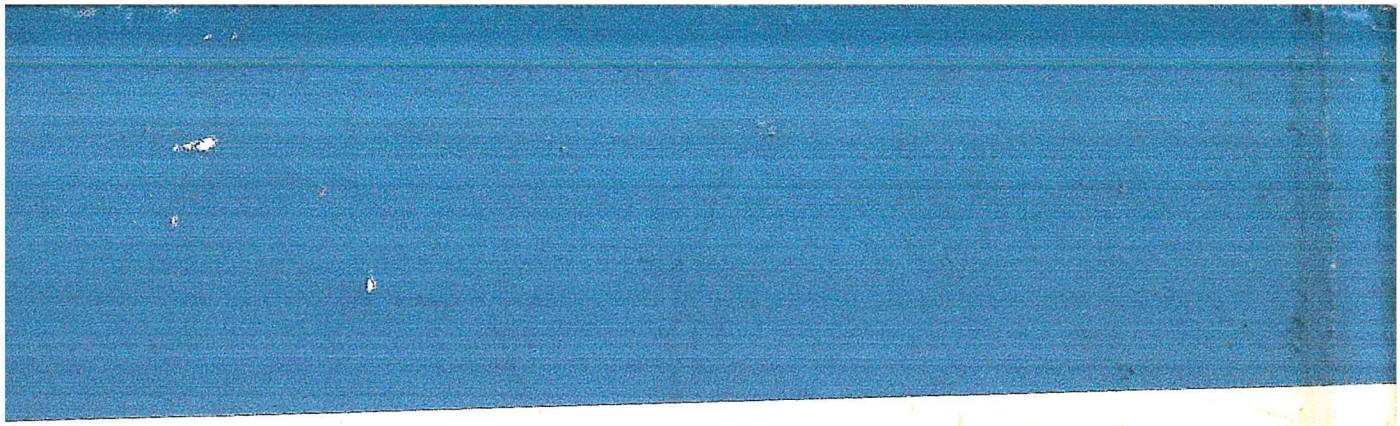
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